OLNEY'S ELEMENTS OF ARITHMETIC

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Stillman F. Allen.

THE ELEMENTS OF ARITHMETIC,

FOR

INTERMEDIATE, GRAMMAR, AND COMMON SCHOOLS;

IN WHICH

THE ANALYTICAL PROCESSES KNOWN AS MENTAL ARITHMETIC ARE ASSIMILATED AND INCORPORATED WITH THE MORE MECHANICAL AND FORMAL PROCESSES CALLED WRITTEN ARITHMETIC, THUS AFFORDING IN ONE BOOK A SUFFICIENT AMOUNT OF THEORETICAL AND PRACTICAL ARITHMETIC FOR A GOOD ENGLISH EDUCATION.

BY EDWARD OLNEY,

PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF MICHIGAN, AND AUTHOR OF A SERIES OF MATHEMATICAL TEXT-BOOKS.

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PREFACE.

FELLOW TEACHERS, can we not contrive to teach Arithmetic so as to leave some time in our Public School course for something else? Do we need *Two Species* of Arithmetic? Is it absolutely necessary that pupils go through from four to six different volumes to obtain a respectable knowledge of this subject? If we must continue to take one-third of the time spent in school, for eight or ten years, to teach the Arithmetic which is necessary to a good English Education, my attempt in preparing this volume is futile, and you need read no further. But, if you agree with me, that after a child has learned to count, and knows the elementary combinations of the digits, *i. e.*, the Addition, Subtraction, Multiplication, and Division Tables, he can learn all the Arithmetic, of all kinds, mental, written, theoretical, practical, philosophical, commercial, etc., etc., that is necessary to a good common school education, or that is consistent with a well-balanced course, by pursuing Arithmetic as one of his studies for from *Three* to *Six Years*, according to his maturity, then please examine the following pages.

In Graded Schools this book will follow the Author's Primary; but it is so constructed that it may be used alone with entire satisfaction in our rural or district schools, where but a single book is practicable. It contains all the Arithmetical topics which can properly or profitably be included in a common school course. (See "Contents.")

There is no important process of analysis usually given in our *Mental Arithmetics*. which is not to be found in this treatise, but not in detached places. The analytical methods of our old mental arithmetics have been assimilated and made the foundation of the more mechanical and formal methods which have been called *Written*, or *Practical Arithmetic*.

The plan of the book recognizes the growth in mental power which ought to be presumable in the three years during which this subject will constitute one of the pupil's studies in school. There

are *Three Stages* of mental development which, in our elementary teaching, we do well to mark :

1. ***The earliest stage,***—in which the faculties chiefly exercised are observation (perception) and memory. At this stage the pupil is neither competent to formulate thought, nor to derive benefit from abstract, formal statements of principles, definitions, or processes.

2. ***An intermediate stage,***—in which the reasoning faculties (abstraction, judgment) are coming into prominence. At this stage the pupil needs to be shown the truth, so that he has a clear perception of it, *before* a formal, abstract statement is presented to him. But the work is not concluded until he can state the truth (definition, principle, proposition, rule) intelligently, in good language, and in general (abstract) terms.

3. ***The ultimate stage,***—or that in which the mental powers are so matured and trained that one is competent to receive truth from the general, abstract, or formal statement of it. At this stage, definitions, principles, propositions, and statements of processes may be given *first,* and illustrated, demonstrated, or applied afterward.

The period covered by the study of this book is presumed to be the second, and an entrance upon the third. Hence, in the first part, all definitions, principles, and rules *follow* an inductive process of which they are a condensed statement. At first, no more formal demonstrations are given than are included in these inductions (see Fundamental Rules). But, as we proceed, the inductions are less extended, and the formal demonstrations are more relied upon (see Common Fractions); until, finally, the inductions are dropped entirely (see Denominate Numbers, *et sq.*), and the formal statement is made at the outset and followed by the appropriate illustration, application, or demonstration.

Again, at first, the pupil is not presumed to be capable of *making definitions, or rules ;* but, as he proceeds, he is occasionally required to write the rule for himself (in subordinate cases first), and, finally, to write both rule and demonstration in due form, after the subject has been fully developed (see pp. 139, 143, 152, 221, 294). And still again, some subjects are treated exclusively upon the analytical method used in our mental arithmetics, without any allusions whatever to formal rules and demonstrations (see pp. 240–244, 271–275, 275–278). Thus that variety of discipline is secured which is indispensable to good training.

Few rules, and such as are comprehensive and practical, is a

motto which has been closely adhered to. (See Common Fractions, Denominate Numbers, Business Rules, etc.)

While the book furnishes an unusually large number of exercises, for both oral and written work, a *Teacher's Hand-Book of Arithmetical Exercises* will accompany it, which will be found to furnish abundant exercises for mental (oral) and written drill in the class-room, as well as afford means from which the teacher can draw in extending the several topics when he finds it expedient.

As this book does not assume to be a scientific treatise, no attempt is made at *Synoptical Analyses.* These must be based upon a philosophical treatment, or be caricatures. Yet, notwithstanding this *practical* character of the work, it will be found that the several subjects presented are thoroughly analyzed, and that no principle or process is given that is not accompanied with a thoroughly logical exposition. The same principle which has given form to other members of this series will be found to have moulded this; viz., that *it is of far more importance that a pupil be trained to think, than that he become simply expert in solving problems.*

A specimen *Teacher's Analysis* will be found on the next page. It is given as an illustration of what the Teacher needs to make and to fix in his mind, as a basis for questioning the class, or for teaching each subject.

Thus have I attempted to put in compact form a thoroughly practical and at the same time a logical exposition of the elements of Arithmetic, in the full belief that by such a treatment a better knowledge of Arithmetic can be attained, in much less time, and at less cost for books, than has been done by the methods which have prevailed for years past. Whether I have been successful or not, the judgment of my fellow-teachers, and the test of the class-room must determine: to these I hopefully commend the book.

EDWARD OLNEY.

ANN ARBOR, *June* 1, 1875.

This ENLARGED EDITION has been greatly enriched in the Business Rules (see pp. 245–315, 351–358), Denom. Nos. and problems in Mensuration (see pp. 193–213, 358–361), and by a very copious list of drill and test exercises in Fractions (see pp. 348–351). More practical cases are provided for in Discount than in any other treatise known to the author. *Arts.* 195, 233, 271, 307, as well as problems 6–9, pp. 284, 285, and problems 8–15, pp. 292–294, present exceedingly practical matters not heretofore found in our common arithmetics.

TEACHER'S ANALYSIS.

SUBJECT OF CHAPTER.—Fundamental Principles and Rules.

SECTION I.—Reading numbers is taught in *Four Steps.*

FIRST STEP. What numbers and figures are.
SECOND STEP. How numbers are grouped.
THIRD STEP. To read numbers represented by two or three figures each.
FOURTH STEP. To read numbers represented by more than three figures each.

SECTION II.—Writing numbers is taught in *Three Steps.*

FIRST STEP. To write any number less than 10.
SECOND STEP. To write any number less than 1000.
THIRD STEP. To write any number whatever.

SECTION III.—Addition is taught in *Six Steps.*

FIRST STEP. To add a number not exceeding 9 to a greater or equal number not exceeding 9.
SECOND STEP. To add a number not exceeding 9 to another less than itself.
THIRD STEP. To memorize the Addition Table.
FOURTH STEP. To add a number represented by one figure to one represented by two figures.
FIFTH STEP. To add a column of figures.
SIXTH STEP. To add numbers represented by several figures each.

SECTION IV.—Subtraction is taught in *Three Steps.*

FIRST STEP. To ascertain the remainder when any number represented by one figure is taken from one not less than itself, but less than itself + 10.
SECOND STEP. When minuend and subtrahend are each represented by several figures, and no figure in the subtrahend exceeds the figure in the same order in the minuend.
THIRD STEP. When there are figures in the subtrahend which exceed those of the like orders in the minuend.

SECTION V.—Multiplication is taught in *Six Steps.*

FIRST STEP. To ascertain the product when any number not greater than 12 is multiplied by any number not greater than itself.
SECOND STEP. To ascertain the product when any number not greater than 12 is multiplied by any number greater than itself, but less than 12.
THIRD STEP. Three Principles on which the general problem is based.
FOURTH STEP. When multiplicand has several figures, and multiplier but one.
FIFTH STEP. When the factors are represented by several figures each.
SIXTH STEP. When there are 0's at the right of both factors.

SECTION VI.—Division is taught in *Seven Steps.*

FIRST STEP. Definitions and how to make the Division Table.
SECOND STEP. Two fundamental principles.
THIRD STEP. Short Division.
FOURTH STEP. To find how many times a large divisor is contained in a number less than 10 times itself.
FIFTH STEP. Long Division.
SIXTH STEP. To divide by 10, 100, 1000, etc.
SEVENTH STEP. To divide by a composite number.

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CHAPTER I.

FUNDAMENTAL PRINCIPLES AND RULES.

SECTION I.

READING NUMBERS.

FIRST STEP.*

What Numbers and Figures are.

1. A Unit is one.

How many *units* are there in three? How many in eight? In one? In six?

2. Number is an answer to the question, How many? A unit, or any collection of units, is a number.

---

* These *steps* are not things to be learned by the pupil, but are the divisions of the subjects to be marked by the teacher. Thus the pupil is to know *first*, What numbers and figures are; *second*, How they are grouped, etc.

1. Is six a number? Why?

Answer. Six is a number because it is an answer to the question, How many?

2. Is fifteen a number? Why? Is one a number? Why?

3. **Arithmetic** is the elementary branch of the science of number.

---

4. **Figures** are characters used to represent numbers. There are ten figures, viz.,

0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

5. The last nine of these figures are called **Digits**, and the first, 0, is called **Zero**, or *Cipher.*

6. The name of any digit, and the number of units which it represents are the same. Hence to read a number represented by a single figure, simply pronounce the name of the figure.

What is the *name* of this digit, 6? How many units does it represent? What is the name of this, 7? How many units does it represent? Is 0 a digit?

7. **The Value** of a figure is the number which it represents.

What is the value of 8? Of 6? Of 1? Of 7?

8. **Zero**, 0, is an auxiliary* character, and in itself has no value. When it stands alone it signifies *naught*, or *nothing;* that is, no number.

The digits are often called *Significant Figures,* because they signify something, that is, some number, when standing alone.

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* Auxiliary means helping. The character 0 is called an auxiliary character because it *helps* the digits in representing certain other numbers, as 10, 100, 30, 205, etc.

SECOND STEP.

How Numbers are Grouped.

1. Here are three children putting up counters in packages. The girl at the right is Mary. She is putting them up in packages, as the doctor puts up powders. She puts up ten counters in each package.

The boy, George, puts up larger packages. He puts ten of Mary's packages in one of his. See what a pile he has at his right.

The other girl is Jane. She is putting up George's packages into bundles, with ten of George's packages in each of hers. What large packages Jane makes!

2. How many counters are there in one of Mary's packages? What, then, are Mary's packages? *Answer,* Tens.

3. How many *tens* are there in one of George's packages?

How many single counters are there in one of George's packages?

What, then, are George's packages? *Answer,* Hundreds.

4. How many packages of hundreds are there in one of Jane's packages? We call ten hundreds a *thousand.*

What, then, are Jane's packages?

5. How many units * make *one ten?*

6. How many tens make *one hundred?*

7. How many hundreds make *one thousand?*

8. How many hundreds are there in a thousand?

9. How many tens are there in a hundred?

10. How many units are there in a ten?

9. Numbers are grouped into tens, and this way of grouping them is called the *Decimal System.*

Decimal means by tens; so that *Decimal System* means a system by tens.

Principle.

10. *When figures are written side by side, like the letters of a word, and without any other marks, the right-hand figure represents* UNITS, *the next one* TENS, *and the next* HUNDREDS.

1. In 485 which figure represents units?

Which figure represents tens?

Which figure represents hundreds?

2. In 485 which figure represents the packages of counters which George puts up? What are these?

Which figure represents the packages which Mary puts up? What are these?

Which figure represents single counters?

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* It will be best to confine the use of the word units to its primary signification. To introduce the idea of units of different orders will only confuse.

3. In 728 which figure represents hundreds? How many hundreds does it represent?

Which figure represents tens? How many tens?

Which figure represents units? How many units?

4. Name the hundreds, tens, and units represented in 467. This is done thus: 4 hundreds, 6 tens, and 7 units.

5. Name the hundreds, tens, and units in 582. In 741. In 777. In 981. In 222. In 111. In 342.

THIRD STEP.

Reading Numbers represented by Two or Three Figures.

To Read a Number represented by two Figures when the left hand figure is 1.

11. Rule.—*If represented thus,* 10, *read* TEN, *since one in the second place means one ten* (10).

If represented thus, 11, *read* ELEVEN.*

If represented thus, 12, *read* TWELVE.*

In all other cases pronounce the number in units place first and then say TEEN—*signifying "and ten"; observing to contract three to* THIR, *five to* FIF, *and eight to* EIGH.

1. How is 13 read? What does the *thir* mean? What the *teen?* How many tens in 13? How many units?

---

* The words *eleven* and *twelve* are of Saxon origin, and their etymological significance is not recognizable by a mere English student. *Eleven* is from an old Saxon word *ein-lif*, which some say signified *one-left*, *i. e.*, ten and one more; and others make *lif* equivalent to *teen*, so that eleven would mean *one-teen*, as thirteen means three-teen. The etymological significance of *twelve* is disposed of in like manner.

2. Read the following, telling what the word for each number means:

13, 17, 16, 11, 15, 14, 12, 10, 18, 19.

3. In reading the numbers from 10 to 19, which do you name first, the tens or the units?

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1. How many tens, and how many units are represented by 20? *Answer,* Two tens and *no* units. So 20 means 2 tens, or *twenty.*

Observe that the *twen* means *two,* and the *ty* tens. In our reading of numbers we have, therefore, three expressions for *ten,* viz., ten, teen, and ty.

2. Read as above and explain the meaning of the words:

50, 40, 80, 70, 60, 90, 30.

---

1. How many tens and how many units are represented by 23? What are two tens called?

Then 23 is twenty (or two tens) and three: leaving out the *and,* we read it twenty-three.

To Read a Number represented by two Figures when the left hand figure is any other than 1.

12. **Rule.**—*Pronounce the number of tens first, contracting the word* TENS *to* TY, TWO *to* TWEN, THREE *to* THIR, FOUR *to* FOR, FIVE *to* FIF, *and* EIGHT *to* EIGH; *then pronounce the number of units.*

4. Read the following, giving the meaning of each expression: 54, 73, 82, 48, 71, 32, 23, 56, 65, 33, 55, 91, 98, 36, 43, 34, 41, 21, 26, 62, 78, 64, 31, 71, 17, 50, 13, 80, 61, 16, 81, 18, 88, 99, 47, 74, 96, 69, 79, 89, 77, 51, 15, 28, 68, 86.

Since *eigh* is the contraction for *eight,* and *ty* for *tens,* 86 is read *eighty-six.*

1. In 576 what does the 5 represent?

What do the other two figures taken together represent? Then how do we read 576?

Answer, Five hundred seventy-six.

2. In 700 how many hundreds are represented? How many tens? How many units?

If there are no tens or units represented in 700, and there are 7 hundreds, how will you read it?

3. In 503 how many hundreds are represented? How many tens? How many units? Then how is 503 read?

Answer, Five hundred three.

To Read a Number represented by Three Figures.

13. Rule.—*Pronounce the number of hundreds first, and then read the other two figures as directed in the preceding rules.*

4. Read the following: 834, 235, 482, 576, 821, 763, 222, 343, 988, 889, 898, 888, 111, 644, 446, 279, 927, 972.

5. Read 708, 604, 303, 101, 105, 107, 109, 408, 404, 802, 801, 703, 602, 808, 609, 708, 906, 507, 801, 108, 406.

6. Read 530, 640, 760, 990, 810, 960, 870, 440, 110, 250, 340, 820, 910, 750, 450, 340, 880, 730, 690.

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14. The groups of ten each into which numbers are collected are called ORDERS.

We have now become acquainted with *Units Order, Tens Order,* and *Hundreds Order,* and have seen that ten hundreds make a thousand, which is the next higher order.

Ex. In 6827 what order is 7 in? What 2? 8? 6?

[Illustrate by the packages of counters as in the picture, page 3.]

FOURTH STEP.

Reading Numbers represented by more than Three Figures.

15. When we wish to read a number represented by a large number of figures, as 764312895, we first point the figures off into sets by putting a comma between the third and fourth, sixth and seventh, etc., that is, after every third figure from the right.

16. Each set of figures thus pointed off is called a PERIOD. Each period, except the one at the left, must have three figures in it. The left-hand period may have one, two, or three figures in it.

17. The second period from the right is called *Thousands Period;* the next, or third from the right, is called *Millions Period;* the next, or the fourth from the right, is called *Billions Period.**

1. What is the thousands period in 467312895?

Answer, 312.

What is the millions period? *Answer,* 467.

2. What is the billions period in 24876403287?

Answer, 24.

3. What is the thousands period in the last number? What the millions?

4. Point off 53212671. What is the number which stands in millions period? What in thousands period?

5. Point off 3412756, and then name in order the number standing in each period, beginning at the left.

When pointed off, this is 3,412,756.

---

* The successive periods beyond billions are trillions, quadrillions, quintillions, sextillions, septillions, and octillions, etc.

Hence we read 3 million, 412 thousand, 756.

6. Point off and read 245642.

When pointed off, this is 245,642.

Hence it is read 245 thousand, 642.

7. Point off and read 50300806.

This is read 50 million, 300 thousand, 806.

8. Point off and read the following numbers:

348256	12350246710	452506481835
2957643	881100604	2648300500
54203	18203700	30825605100

9. Read 2070582. What are the figures in thousands period? *Answer,* 070. How much does 070 represent? How many units? How many tens? How many hundreds? When these three figures, 070, stand alone and in this order, what more do they represent than 70?

If 070 is only 70, might we not omit the 0 before the 7 in the number 2070582 without altering the value? Why not? Read 270582. Read 2070582. Are they the same?

To Read a Number represented by more than Three Figures.

18. **Rule.**—*First separate the number into periods, by placing a comma before every third figure from the right. Then, beginning at the left, read the number in each period in succession according to the rule for reading a number represented by three figures, and after the number represented in any period, pronounce the name of that period. Pass over periods filled by zeros in silence.*

Examples for Practice.

Point off and read the following:

21050707	505050	700000	3003000001
283004005	5050505	1500000	5000011006

2000200	5010210	2000004	6000000112
50034	53457	11111111	50000000
4785	8888888	223344	4260000
124080	30608	4375682	4008000
340077	400000	370000	70060080
3000010	7200000	20009	600000008

The last is pointed off thus: 600,000,008; and read six hundred million, eight.

---

19. Numeration is naming the orders of the figures in a number for the purpose of reading it.*

The following table will show the names of the orders and of the periods at a glance:

NUMERATION TABLE.

		Billions.			Millions.			Thousands.			Units.	Names of the PERIODS.
8	5	3,	4	2	6,	5	7	4,	3	1	9	NUMBER.
Hundreds of Billions	Tens of Billions	Billions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units	Names of the ORDERS.

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* It will be observed that we have taught how to read numbers without the use of the old process of "numerating." Nevertheless, it is important for the pupil to get the view of the relations of the periods and orders which this process gives.

1. Numerate, that is, name the orders in 72156437851, beginning at the right.

2. Numerate the following:

421561304	142345685087	426005832
31020567	32564875190	31110050

Principle.

20. *In the Decimal System* 10 *of any order always makes one of the next higher order, and a thousand of any period always makes one of the next higher period.*

1. How many tens make a hundred?
2. How many hundreds make a thousand?
3. How many thousands make a tens-of-thousands?
4. How many tens-of-thousands make a hundreds-of-thousands?
5. How many thousands make a million?
6. How many millions make a billion?
7. How many billions make a tens-of-billions?
8. How many tens-of-billions make a hundreds-of-billions?

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21. Two very important practical observations are to be made in closing this section.

1. *Any number of figures at the right may be read as so many units.*

2. *Any number of figures at the left may be read as so many of the lowest order of those figures.*

Thus in 2536: 1. We may consider the 3 and 6 as representing 36 units; for 3 tens and 6 units are 36 units. In like manner, the 5, 3, and 6 may be considered as 536 units, since 5 hundreds, 3 tens, and 6 units make 536 units. 2. The 25 may be considered as 25 hundreds, for

2 thousands and 5 hundreds make 25 hundreds. So also the 253 may be regarded as so many *tens*, since 2 thousands, 5 hundreds, and 3 tens make 253 tens.

Ex. Explain in this manner the various ways in which the following may be read:

348, 4285, 785401, 600820.

[The teacher cannot be too careful to see that these ideas are clearly perceived and firmly fixed. They are at the foundation of most of the operations in the fundamental rules.]

SECTION II.

WRITING NUMBERS.

FIRST STEP.

To Write in Figures any Number less than 10.

22. *Simply write the figure of the same name as the number.*

1. Write seven, nine, six, four, nine, and two in figures

SECOND STEP.

To Write any Number less than 1000.

1. Write in figures seven hundred forty-six. What is the highest order mentioned? How many are there of this order? What is the next lower order? How many of this order are there in the number to be written? How many of the next lower order are there?

To Write in Figures any Number less than 1000.

23. Rule.—*First consider what is the highest order to be represented, and write the figure representing the required number of this order. Then consider how many are to be represented of each of the lower orders, and write the figures representing each of these at the right, tens occupying the first place at the right of hundreds, and units the next, or right-hand place. When the number lacks any order lower than the highest, the corresponding place is to be filled with a* 0.

2. Write in figures the following numbers:

forty-eight,	eighty-two,	seventy-four,
sixty-three,	thirty-one,	seventy-seven,
ninety-seven,	fifty-five,	forty-four.

3. Write in figures the following numbers:

four hundred thirty-five,
seven hundred twenty-eight,
three hundred sixty-two,
one hundred thirty-four,
two hundred twenty-six,
nine hundred forty-one,
seven hundred seventy-seven,
one hundred eleven,
five hundred fourteen,
eight hundred thirteen,
two hundred twelve,
three hundred thirty,
seven hundred seventeen.

4. Write six hundred; five hundred; eight hundred; three hundred. Which orders will be filled with zeros in writing these numbers? Why?

5. Write four hundred twenty; seven hundred fifty; six hundred seventy; two hundred eighty. Which order will be filled with a zero in writing these numbers? Why?

6. Write four hundred seven,
six hundred twenty,
eight hundred one,
three hundred thirty,
one hundred seven,
one hundred ten,
five hundred fifty-five.

In writing these numbers, what order will be filled by a zero? Why?

24. This method of writing numbers in figures is often called the ARABIC NOTATION, and the ten figures used are called *Arabic Characters.*

[This is because these figures were introduced into Europe by the Moors, or Arabs, and were then thought to have been invented by the Arabs. It is now known that they came from farther East, perhaps from Thibet.]

Examples for Practice.

1. Write all the numbers from one to two hundred.

2. Write the following:

three hundred fifty-six,
two hundred twenty-two,
seven hundred ninety,
seven hundred nine,
seven hundred ninety-nine,
six hundred,
six hundred five,

six hundred fifty,
six hundred fifty-four,
eight hundred eighteen,
nine hundred sixteen,
seven hundred,
seven hundred five,
seven hundred sixty-five,
three hundred eighty,
three hundred eighty-eight,
six hundred sixty-six.

3. Write all the numbers from four hundred to six hundred.

4. Write all the numbers from three hundred twenty-six to five hundred forty-two.

5. Write as many different numbers as you can with the three digits 5, 8, and 3, and read them. With 6, 4, and 2. 1, 3, and 9. 7, 0, and 2. 6, 0, and 0.

[The teacher can readily multiply exercises of this kind at pleasure.]

THIRD STEP.

To Write any Number whatever.

1. In seven hundred thirty-five million, four hundred ninety-two thousand, two hundred thirty-six, how many periods are there? What are their names?

How many millions are there? Write in figures the number of millions.

How many thousands are there? Write in figures the number of thousands, placing it at the right of millions.

How many are now left of the number to be represented? Write this at the right of thousands.

Thus we have 735,492,236. Read this number.

2. In seventy-two thousand five hundred twenty-eight, how many periods are there? What are their names?

How many are there in thousands period?

How many are there in units period?

Write the number of thousands.

At the right of the thousands write the number required in units period.

To Write in the Arabic Notation any Number whatever.*

25. Rule.—*Note the highest period mentioned in the number, and write the number required in this period according to* **(23)**. *Then note the next lower period mentioned, and write in like manner the number specified in it. Thus continue the operation till the entire number is written, observing to fill with zeros any periods or orders which would otherwise be vacant.*

Examples for Practice.

1. Write twenty-three million, four hundred fifty-six thousand, five hundred thirty-nine.

2. Write forty-five thousand, seven hundred sixty.

3. Write seventeen billion, one hundred eighty-one million, five hundred sixty-two thousand, two hundred seventy-eight.

4. Write four thousand, one hundred four.

5. Write seventy-five thousand, seventy-five.

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* When the word number is thus used, a whole number is usually implied. The pupil will get no other idea from it at this stage of his progress.

6. Write six hundred five thousand, one hundred twenty-three.

7. Write eight hundred seventy-two thousand, five hundred twelve.

8. Write nine million, seven hundred sixty-five thousand, four hundred thirty-two.

9. Write three hundred forty million, forty-three thousand, five hundred sixty-seven.

10. Write three hundred seventy-four billion, four hundred thirty-eight million, eight hundred sixty-two thousand, eight hundred forty-seven.

11. Write seventy-two million, eighty-three thousand, twenty-seven.

12. Write seventeen thousand, seventeen hundred seventeen.

13. Write 2 hundred 89 million, 1 thousand.

14. Write one million, one thousand, one.

15. Write ten million, ten thousand, ten.

16. Write one hundred sixty-seven thousand, nine hundred thirty-eight.

17. Write ten billion.

18. Write 4 hundred 69 billion, 9 hundred 31 million, 7 hundred seventy-seven.

19. Write nine hundred ninety-nine million, nine hundred ninety-nine thousand, nine hundred ninety-nine.

20. Write one hundred two billion, two hundred thousand, seven.

21. Write 3 hundred forty-seven million, 5 hundred twenty-one thousand, 8 hundred ninety-six.

22. Write 10 billion, 2 hundred 47 million, 327.

23. Write 3 hundred 4 thousand, 26.

24. Write 504 billion, 627 million, 17 thousand, 2.

25. Write 12 million, 8 thousand, six.

26. Write two hundred sixty-one billion, five hundred seventy-eight million, nine hundred thirteen thousand, one hundred twelve.

27. Write ten million, ten.

28. Write 1 million, 1 thousand, 1 hundred 1.

29. Write twenty-two million, two hundred twenty-two thousand, two hundred twenty-two.

30. Write 1 hundred 1 thousand, 2 hundred 2.

SECTION III.

*THE ROMAN NOTATION.**

26. Dates, numbers of sections and chapters, and of the pages of an introduction to a book, are often represented by the seven letters

I, V, X, L, C, D, M.

27. When used to represent numbers, the values of these letters are as follows:

I, *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* and M, *one thousand.*

28. This method of representing numbers is called the ROMAN NOTATION.

To Read a Number represented by the Roman Notation.

29. Rule.—*Add the values of the letters, observing that when a letter is followed by one of greater value*

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* This section is placed here in deference to custom. It should be omitted by pupils unacquainted with addition and subtraction until they have studied those subjects.

than itself, the difference between these two is to be taken in making up the same.

Examples for Practice.

1. Read XXVIII.

Suggestion. X is 10; V, 5; and I, 1. Then adding the values of *all* the letters, we have 28. Hence XXVIII is 28.

2. Read XIX.

Suggestion.—Here the I before X diminishes the value of the latter making IX *nine*. Hence XIX is 10 and 9, or 19. X, 10.

3. Read MDCCXLVIII.

Suggestion.—Here we have 1000, 500, 100, 100, 40 (the XL is 40), 5, 1, 1, and 1. Adding these, there results 1748.

30. By observing what *orders* are represented, we can read such expressions at sight.

4. Read MMDCCCLXXVII.

Suggestion.—Here the two M's represent *thousands;* D and the three C's, *hundreds;* L and the two X's, *tens;* V and the two I's, *units.* Hence we read 2 thousand, 8 hundred seventy-seven.

5. Read the following:

I.	MDCCCLXXV.	XIV.
IV.	MMCCXXII.	XXIX.
LX.	CXL.	XXXIV.
VIII.	MCCVIII.	XLVIII.
XX.	XLV.	CCLXXI.
XL.	MDCXXVII	MMCLVIII.
XXX.	MCDXCII.	MDCCCXL.
LXXX.	IV.	MDCXX.
CII.	V.	III.
VII.	XIX.	L.

VIII.	XXXVIII.	LII.
IX.	XI.	CCCXLIX.
DVI.	MMMDCCLXXXI.	XXXIX.

To Write any Number less than Four Thousand, in the Roman Notation.

31. Rule.—I. *Write the letter of highest value which does not exceed the number to be written. Repeat this letter as many times as possible without exceeding the number.*

II. *Observe how much remains to be represented, and treat it in the same manner, annexing these letters to the former. Continue this process till the entire number is represented.*

Observing that

IV	is written	instead of	IIII for four,
IX	"	"	VIIII for nine,
XL	"	"	XXXX for forty,
XC	"	"	LXXXX for ninety,
CD	"	"	CCCC for four hundred,
CM	"	"	DCCCC for nine hundred;

that is, no letter is written four times in succession.

Examples for Practice.

1. Write 327 in the Roman Notation.

Suggestion. C represents the highest value lower than 327, and can be written three times, thus CCC, which is 300. There now remains 27 to be represented. X is the letter next lower in value, and can be written twice, thus XX, which is 20. 7 now remains, and V is the letter next lower in value, but cannot be repeated, hence we write V. There now remains 2, which is written II. Collecting the letters, we have CCCXXVII for 327.

2. Write 2738 in the Roman Notation.
3. Write 1875 in the Roman Notation.
4. Write 17, 28, 51, 123, 571, 120, 115, 731.
5. Write 949, 494, 3489, 2974, 1740, 1620, 1492, 1776.

[**Note.**—Observe that 4 and 9 are the only digits whose values are represented by differences.]

6. Write all the numbers from 1 to 200.

SECTION IV.

ADDITION.

FIRST STEP.

1. How many oranges are 5 oranges and 4 oranges? 5 and 4 make what number? What is the sum of 5 and 4?

32. The Sum of two or more numbers is the number they make when united.

33. The First Step in Addition is to find the Sum of two numbers, each not exceeding nine, when the one to be added does not exceed the one to which it is to be added.

[This very elementary step is mentioned here simply for completeness. The method of effecting it is fully presented in the PRIMARY ARITHMETIC.]

34. The sign $+$ is called *plus,* and when placed between two numbers indicates that they are to be added.* Hence it is called the sign of addition.*

35. The sign $=$ is called the sign of equality, and signifies that what is written before it is equal to that which is written after it.

Thus $5+4=9$, is read "5 plus 4 equals 9." This means the same as "5 and 4 are nine."

6. Read the following: $6 + 3 = 9$; $5 + 2 = 7$; $8 + 4 = 12$; $3 + 1 = 4$; $7 + 3 = 10$; $1 + 1 = 2$; $9 + 8 = 17$; $6 + 6 = 12$.

If the pupil does not know the Addition Table, he should fill out the following and *commit it to memory.* If he does, he should be allowed to pass to the 4th step.

ADDITION TABLE.

$1 + 1 =$ ——		
$2 + 1 =$ ——	$2 + 2 =$ ——	
$3 + 1 =$ ——	$3 + 2 =$ ——	$3 + 3 =$ ——
$4 + 1 =$ ——	$4 + 2 =$ ——	$4 + 3 =$ ——
$5 + 1 =$ ——	$5 + 2 =$ ——	$5 + 3 =$ ——
$6 + 1 =$ ——	$6 + 2 =$ ——	$6 + 3 =$ ——
$7 + 1 =$ ——	$7 + 2 =$ ——	$7 + 3 =$ ——
$8 + 1 =$ ——	$8 + 2 =$ ——	$8 + 3 =$ ——
$9 + 1 =$ ——	$9 + 2 =$ ——	$9 + 3 =$ ——

* The pupil is supposed to know the meaning of these words from his study of Primary Arithmetic, or because they are words in common use; we are not yet prepared to give the formal definition of *Addition* and to distinguish it from *Counting*. (See **40**.)

$4 + 4 =$ —— $5 + 4 =$ —— $6 + 4 =$ —— $7 + 4 =$ —— $8 + 4 =$ —— $9 + 4 =$ ——	$5 + 5 =$ —— $6 + 5 =$ —— $7 + 5 =$ —— $8 + 5 =$ —— $9 + 5 =$ ——	$6 + 6 =$ —— $7 + 6 =$ —— $8 + 6 =$ —— $9 + 6 =$ ——
$7 + 7 =$ —— $8 + 7 =$ —— $9 + 7 =$ ——	$8 + 8 =$ —— $9 + 8 =$ ——	$9 + 9 =$ ——

SECOND STEP.

To Add a Number not exceeding 9 to another less than itself.

1. If the girl in the picture, page 21, should give the boy her 5 oranges, how many would the boy have? 4 and 5 more are how many?

If the boy should give the girl his oranges, how many would she have? 5 and 4 more are how many?

Which is the more, 4 and 5, or 5 and 4?

2. How many are $4 + 3$? Then how many are $3 + 4$?

3. How many are $6 + 4$? Then how many are $4 + 6$?

[A practical knowledge of the fact that the sum of two numbers is the same in whatever order they are taken, diminishes the work of learning the Addition Table nearly one-half.]

4. In this way fill the blanks in the following:

$3 + 5 =$ —; $4 + 5 =$ —; $1 + 6 =$ —; $2 + 6 =$ —;
$3 + 6 =$ —; $4 + 6 =$ —; $5 + 6 =$ —; $5 + 6 =$ —;
$1 + 7 =$ —; $2 + 7 =$ —; $3 + 7 =$ —; $4 + 7 =$ —;

$5 + 7 = -$; $6 + 7 = -$; $1 + 8 = -$; $2 + 8 = -$;
$3 + 8 = -$; $4 + 8 = -$; $5 + 8 = -$; $6 + 8 = -$;
$7 + 8 = -$; $1 + 9 = -$; $2 + 9 = -$; $3 + 9 = -$;
$4 + 9 = -$; $5 + 9 = -$; $6 + 9 = -$; $7 + 9 = -$;
$8 + 9 = -$; $6 + 9 = -$; $3 + 5 = -$; $1 + 4 = -$.

THIRD STEP.

To Commit the Addition Table to Memory.

[Persistent, direct, and determined effort is necessary to this task. As long as there is any temptation to *count* instead of *add*, this work has not been well done. Drill, *drill*, habitual, long-continued DRILL, in adding *at sight*, is necessary. The devices suggested in the following exercises are very helpful.]

Fill the blanks, and then commit to memory each of the following 5 exercises:*

1. $1 + 1 = -$; $2 + 2 = -$; $3 + 3 = -$; $4+4 = -$; $5 + 5 = -$; $6 + 6 = -$; $7 + 7 = -$; $8 + 8 = -$; $9 + 9 = -$.

2. $2 + 1 = -$; $3 + 2 = -$; $3 + 1 = -$: $4 + 2 = -$; $4 + 1 = -$; $4 + 3 = -$; $5 + 4 = -$; $5 + 2 = -$; $5 + 1 = -$; $5 + 3 = -$.

3. $6 + 4 = -$; $6 + 2 = -$; $6 + 5 = -$; $6 + 1 = -$; $6 + 3 = -$; $7 + 2 = -$; $7 + 5 = -$; $7 + 1 = -$; $7 + 4 = -$; $7 + 6 = -$; $7 + 3 = -$.

4. $8 + 5 = -$; $8 + 2 = -$; $8 + 7 = -$; $8 + 4 = -$; $8 + 6 = -$; $8 + 3 = -$; $8 + 1 = -$.

---

* These 5 exercises thoroughly mastered give a knowledge of the entire Addition Table. There is little or no use in learning this table *in order*, or in repeating it so.

5. $9+3=$ —; $9+7=$ —; $9+1=$ —; $9+6=$ —; $9+4=$ —; $9+2=$ —; $9+8=$ —; $9+5=$ —.

---

1. Commit to memory, so as to recognize *instantly*, the pairs of digits which give 0 as the units of their sum, as $1+9$, $2+8$, $3+7$, etc. Those which give 1 as the units, as $2+9$, $3+8$, $4+7$, etc. Those which give 2. Those which give 3, 4, 5, 6, 7, 8, 9.

This should be persisted in until the pupil can recognize the units figure of any combination, at sight*—instantly. Cards filled with pairs of digits arranged promiscuously, or similar arrangements on the blackboard, afford excellent drill lessons for this purpose, the pupils being required to speak the units of each sum as fast as the teacher can point out the pairs.

2. What figure with 4 gives 2 as units? What 3? What 7? etc.

3. What figure with 7 gives 2 as units? What 5? What 9? What 6? etc.

4. What figure with 2 gives 8 as units? What 1? What 7? What 5? etc.

5. 3 and what make 7? 3 and what make 9? 5? 8? 10? 12? 4? 11?

6. 7 and what make 12? 7 and what make 15? 13? 11? 8? 16? 14? 9?

Such drill should be continued until these combinations are perfectly familiar. They will be seen to be the basis of subtraction. No pupil should be allowed to pass to the *Fourth Step* till he can recognize *instantly* the sum of any two digits.

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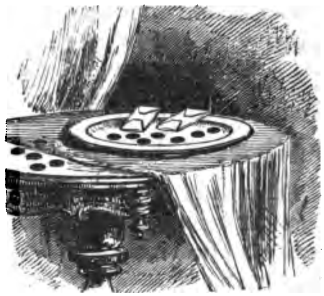
* The teacher should bear in mind that recognition by sight and by ear are two distinct things, and in this case each is essential.

FOURTH STEP.

To Add any Number represented by One Figure to any Number represented by Two Figures.



1. How many packages of 10 counters each are there on the plate? How many single counters on the plate? How many single counters on the table? If you put all the counters together (add them), how many single counters will there be? Can you make another package of tens then? Then how many packages of tens, and how many single counters will there be?

2. Add 27 and 5.

Suggestion.—How many units* in 27? 7 units and 5 units make how many? 12 units make how many tens, and how many units? Then 27 and 5 are how many tens, and how many units?

3. If you have 4 tens packages of counters and 8 single counters, how many counters have you? If you now add 7 single counters to your stock, how many packages of tens can you make, and how many counters will you have in all?

4. Add 48 and 7.

To Add a Number represented by One Figure to one represented by Two Figures.

36. Rule.—*First add the units, and see how many tens and units they make. Then add this number of tens to*

*** The only idea of units which should have been given to the pupil up to this point is that a unit is *one*; so that he will not think of tens as units in any sense.**

the given number of tens, and observe how many tens and units there are in all.

The following will suggest a class of very useful exercises. See also Author's *Primary Arithmetic*, pp. 53, 54. It will be seen that it is about as easy to recognize the sum of two numbers when one is represented by two digits and the other by one, as to recognize the sum of two digits. If this point is well made, it will greatly facilitate subsequent work.

2	2	2	2 etc.		4	4	4 etc.
3	13	23	33		5	15	25
8	8	8	8 etc.		7	7	7 etc.
7	17	27	37		6	16	26
5	5	5	5	5	5	5	5 etc.
8	18	28	38	48	58	68	78

Examples for Practice.

1. How many are 37 and 6? 35 and 4? 39 and 7?
2. How many are 45 and 8? 46 and 9? 48 and 3?
3. How many are 64 and 7? 61 and 8? 67 and 6?
4. How many are 89 and 6? 85 and 7? 84 and 9?
5. How many are 57 and 8? 53 and 9? 56 and 8?
6. How many are 17 and 6? 18 and 9? 19 and 7?
7. How many are 78 and 5? 79 and 4? 74 and 7?
8. How many are 16 and 8? 13 and 8? 13 and 6?
9. How many are 13 and 9? 85 and 7? 76 and 8?
10. How many are 23 and 7? 43 and 6? 55 and 5?
11. How many are 42 and 8? 54 and 6? 13 and 7?
12. How many are 13 and 2? 12 and 7? 15 and 4?
13. Copy the following on your slates and fill the blanks without counting:

23 + 6 = ——	11 + 2 = ——	20 + 2 = ——
28 + 5 = ——	17 + 8 = ——	20 + 4 = ——
45 + 5 = ——	10 + 2 = ——	28 + 6 = ——
49 + 7 = ——	18 + 5 = ——	29 + 6 = ——
81 + 9 = ——	19 + 7 = ——	38 + 7 = ——
87 + 8 = ——	11 + 9 = ——	59 + 8 = ——
87 + 9 = ——	12 + 9 = ——	62 + 9 = ——

FIFTH STEP.

To Add a Column of Figures.

37. A line of figures running up and down the page is called a COLUMN. Here is a column of six figures, which we will proceed to add.

$$\begin{array}{r} 3 \\ 4 \\ 2 \\ 5 \\ 6 \\ \underline{7} \end{array}$$

1. 7 and 6 are how many? 13 and 5 are how many? 18 and 2 are how many? 20 and 4 are how many? 24 and 3 are how many?

Then how many are 7 and 6 and 5 and 2 and 4 and 3?

To Add a Column of Figures.

38. Rule.—*Begin at the bottom, and add the lowest figure to the next above it, to this sum add the third figure, to this sum add the next figure, and so continue until all are added.*

We begin at the bottom because it is customary. It really makes no difference where we begin if only we add all the numbers.

2. Add the column in the margin.

$$\begin{array}{r} 4 \\ 5 \\ 6 \\ 8 \\ 5 \\ \underline{2} \\ 30 \end{array}$$

Suggestions. 2 and 5 are how many? 7 and 8 are how many? 15 and 6 are how many? 21 and 5 are how many? 26 and 4 are how many?

3. Copy the following columns of large figures on your slates, and add them, writing the sum of each column right under it:

4 33	8 33	6	7	2	6	4
2 29	3 25	4	3	4	5	7
5 27	2 22	8	8	6	4	8
6 22	4 20	3	2	5	7	5
3 16	7 16	7	6	3	3	7
7 13	3 9	5	4	7	2	6
2 6	5 6	4	5	6	1	9
4	1	2	8	4	5	3
		39	3	1	33	4

4. Add the following columns of large figures twice each; first add from bottom to top, and then from top to bottom, and see if the sum is the same when you add *up* as when you add *down* the column:

8 53	7	9	8	7	3	6
15 7 45	9	9	8	6	4	6
20 5 38	8	8	8	4	2	8
24 4 33	6	7	7	5	6	8
27 3 29	4	5	6	8	7	9
29 2 26	5	8	6	9	5	9
37 8 24	8	4	7	7	8	7
44 7 16	6	6	4	8	9	7
53 9	7	9	8	9	9	7
	60					67

1. Count 100 by twos; thus, 2, 4, 6, 8, 10, etc.*

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* Let the teacher illustrate this process, and show that it is the same as adding a column of 2's.

2. Count 101 by twos, beginning with 3; thus, 3, 5, 7, 9, 11, etc.

3. Count 102 by 3's; thus, 3, 6, 9, 12, etc.

4. Count 100 by 3's, beginning with 1; thus, 1, 4, 7, 10, 13, etc.

5. Count 101 by 3's, beginning with 2; thus, 2, 5, 8, 11, 14, etc.

6. Count 100 by 4's. Count by 4's from 1 to 101, from 2 to 102, from 3 to 103.

7. Count by 5's from 1 to 101, from 2 to 102, from 3 to 103, from 4 to 104, from 5 to 100.

8. Count by 6's from 1 to 103, from 2 to 104, from 3 to 105, from 4 to 100, from 5 to 101, from 6 to 102.

9. Count by 7's from 1 to 106, from 2 to 100, from 3 to 101, from 4 to 102, from 5 to 103, from 6 to 104, from 7 to 105.

10. Count by 8's from 1 to 105, from 2 to 106, from 3 to 107, from 4 to 100, from 5 to 101, from 6 to 102, from 7 to 103, from 8 to 104.

[It is essential that pupils be trained to add rapidly *at sight*. Any figures which may chance to stand on the blackboard will afford drill exercises, the pupils naming the sums as the teacher points to the digits.]

SIXTH STEP.

To Add several Numbers represented by several Figures each.

1. Here are three numbers. What is the first? What the second? What the third?

$$\begin{array}{r} 234 \\ 312 \\ 153 \\ \hline \end{array}$$

How many hundreds in the first? How many in the second? How many in the third? How many hundreds in all? In order to remember it, we will write it under the hundreds, thus

$$\begin{array}{l} 234 \\ 312 \\ 153 \\ \hline 6 \end{array}$$

How many tens in the first number? How many in the second? How many in the third? How many in all? In order to remember it, we will write it under the tens, thus

234
312
153
69

How many units in each of the numbers? How many units in all of them taken together? In order to remember it, we will write it under the units, thus

234
312
153
699

Now we have put all the hundreds of the three numbers together, and all the tens, and all the units. How many do 234, 312, and 153 make when added? What is the sum of 234, 312, and 153?

2 to 6. Find the sum of the following in the same way:

(2)	(3)	(4)	(5)	(6)
322	456	341	1212	41236
426	111	522	3151	32451
111	221	115	4123	15112

7. Will it make any difference, in adding the above numbers, whether you add the hundreds first, and then the tens, and then the units; or add the units and then the tens, etc.; or add the tens first, and proceed in any other order? Try it in various ways.

8 to 11. Add the following, beginning with the units, and then taking the tens, hundreds, etc., in order:

(8)	(9)	(10)	(11)
325673	3214	213	2222
131201	2131	120	3333
421012	2211	101	1111
	1123	241	1212

12. How many units are there in these three numbers? 15 units are how many tens and how many units? We will set the 5 units under units column, in order to remember them, thus

$$\begin{array}{r} 8567 \\ 3425 \\ 4653 \\ \hline 5 \end{array}$$

How many tens are there in the three numbers? The 13 tens which the 5 tens, 2 tens, and 6 tens make, with the *one* ten which the units made, make how many tens? 14 tens are how many hundreds, and how many tens? In order to remember the 4 tens, we will write the 4 under tens column, thus

$$\begin{array}{r} 8567 \\ 3425 \\ 4653 \\ \hline 45 \end{array}$$

Now the 1 hundred which the tens made, and the 6, 4, and 5 hundreds which there are in the three numbers, make how many hundreds? 16 hundreds are how many thousands, and how many hundreds? We will write the 6 hundreds under hundreds column, in order that we do not forget it.

$$\begin{array}{r} 8567 \\ 3425 \\ 4653 \\ \hline 645 \end{array}$$

Finally, the 1 thousand which the hundreds made, and the 4, 3, and 8 thousands which there are in the three numbers, make how many thousands? 16 thousands are how many tens of thousands and how many thousands? Writing these in their proper orders, how many do 8567, 3425, and 4653 make when added?

$$\begin{array}{r} 8567 \\ 3425 \\ 4653 \\ \hline 16645 \end{array}$$

13. Add as above these numbers.

$$\begin{array}{r} 347 \\ 836 \\ 548 \\ 684 \\ \hline \end{array}$$

Suggestions.—How many units are there? How many tens and how many units are 25? Write the units under units column, in order to remember them. The 2 tens which the units of the number made, and the 8, 4, 3, and 4 tens in the numbers, make how many tens? 21 tens are how many hundreds, and how many tens? Write the tens under tens column, in order to remember them. The 2 hundreds which the

tens made, and the 6, 5, 8, and 3 hundreds in the numbers, make how many hundreds? 24 hundreds are how many thousands, and how many hundreds? Writing the 2 thousands and the 4 hundreds each in its own order, we have 2415 as the sum of the numbers.

14 to 17. Add as above the following:

(14)	(15)	(16)	(17)
4256	38765	348	5432764
3847	27543	279	8264852
2105	12030	546	1023457
4328	57083	832	5437208

18. Let us see if it is just as well to begin with hundreds as with units in such examples as these.

Experiment.—The 5, 7, 8, and 9 hundreds make 29 hundreds. Now let us write the 2 thousands and the 9 hundreds in their proper places. Then adding the tens, we find that 6, 5, 8, and 4 tens make 23 tens, or 2 hundreds and 3 tens. But the 2 hundreds must go in with the 9 hundreds, and that will make 11 hundreds, or 1 thousand and 1 hundred. Again, this 1 thousand must go in with the other 2 thousands; *so that when we add the tens column, we shall have to change both the other figures,* and instead of writing 2 thousands, and 9 hundreds, we shall have 3 thousands and one hundred. Correcting the hundreds and thousands figures, and writing the 3 tens in the tens order, we will add the units. 7, 9, 2, and 6 units make 24 units, or 2 tens and 4 units. Now these 2 tens must go in with the other 3 tens, and we shall have *to change the 3 tens to 5 tens.* Doing this, and writing the 4 units in units order, we have the correct sum.

946	946	946
882	882	882
759	759	759
567	567	567
29	313	3154

Thus we see that we *can* begin at hundreds (or the highest order) to add; but that it is not so convenient, for when we add the higher order, we do not know how many are to go into it from the lower.

To Add several Numbers represented by several Figures each.

39. Rule.—*I. Write the numbers one under another, so that all the units shall fall in one column, all the tens in another; all the hundreds in another, etc.*

II. Add the units column first, and having found its sum, consider how many tens and units there are in it. Write the units under the units column, and keep the tens in mind to be added to the tens column.

III. Add the tens which came from the units column, and those in the tens column, and observe how many hundreds and tens the sum consists of. Write the tens under the tens column, and keep the hundreds in mind to be added to the hundreds column.

IV. Continue in this way till all the columns are added. After adding the last column, write the figures of its entire sum in their proper orders.

Reasons.—I. We write the numbers so that units shall fall in one column, tens in another, etc., in order that we may more readily see what units there are in all the numbers, and then what tens there are, etc.; as we want to add the units first, then the tens, etc.

II. We begin to add with the units, or lowest order, and proceed regularly through the orders, so that when we have added any one order, we may know whether there are any from the lower order to add in with the higher one which we are to add next.

III. When we have gone through all the orders in this way we have the sum of the several numbers, since we have one number which is made up of all the others put together.

40. Addition is the process of finding the sum of two or more numbers by means of a knowledge of the *Addition Table.*

There are two ways of finding the sum of numbers, viz., by *counting* and by *adding*. The Addition Table gives the sum of each possible pair of the nine digits, and only when the pupil knows all these sums and uses this knowledge in the process, do we admit that he *adds.*

Examples for Practice in Adding.

1. Add 427, 342, 856, and 728.

2. Add 5648, 3726, 8972, and 6025.

3. Add 38450, 64008, 56027, 37124, 73500, and 23478.

4. Add 642, 35827, 25, 634851, 3206, and 15432.

Suggestion.—When written for addition, these numbers stand as in the margin. First add a column from bottom to top, and then from top to bottom, repeating the process till the results agree.

642
35827
25
634851
3206
15432

41. *When adding a column of figures, do not name each figure, but only name the sums.* Thus in adding the units column in this example, do not say "2 and 6 are 8, and 1 are nine, and 5 are 14," etc.; but say, or *think*, 2, 8, 9, 14, etc.

5. What is the sum of 50802, 345, 289764, 30726, 29, 8, and 712800?

6. What is the sum of 8, 29, 347, 5284, 70504, 2536475, 976421, 24351, 5002, 437, 50, and 5?

7. Find the sum of 78206, 843, 964271, 1853, 2679, 570012, 8206143, 77899.

8. Add 749831, 8632, 54317, 48, and 432613. The following are the figures in the answer: 1445421. How should they be arranged?

9. 181 + 24 + 897156 + 881 + 71512 gives the following figures: 456799. How should they be arranged?

10. The sum of the following numbers is an important series of figures: 48, 5627, 82160, 3475, 426, 654136, 2485796, 92541643, 743260041, and 395534538.

11. What is the sum of 3241, 476, 84324, 18472, 31421, and 62066?

12. Add 18243, 32341, 7147, 165, 2342, and 50772.

13. 156890 + 34875 + 217006 + 1000 + 2005 + 406 + 342 = how many ?

14. 2000 + 11001 + 801 + 5000 + 88 + 5764 + 872 + 99 + 447 = how many ?

15. 88888 + 7777 + 6666 + 55555 + 4444 + 33333 + 222 + 11111 = how many ?

16. 576843 + 5891476 + 438275 + 789642 + 12384 + 987640 = how many ?

17. 870095 + 984573 + 642785 + 998877 + 679488 + 1257 = how many?

18. 576443 + 203 + 4703 + 56428 + 121 + 2546 + 70058 + 46 + 343 = how many?

19. 8192735742 + 2407643728 + 544337126 + 98724603 + 825473281 + 88116457263 = how many?

20. What is the sum of the following numbers: three thousand four hundred sixty-five, two thousand fifty-four, nine hundred six thousand two hundred forty-seven ?

Ans., 911766.

21. What is the sum of the numbers, one hundred sixty-seven thousand, three hundred sixty-seven thousand, nine hundred six thousand, two hundred forty-seven thousand, ten thousand, seven hundred thousand, nine hundred seventy-six thousand, one hundred ninety-five thousand, ninety-seven thousand ? *Ans.*, 3665000.

Applications.*

1. Harry had 7 books, and his father gave him 4 more. How many books had he then?

* In such exercises as the following, the main purpose is not to give practice in adding, but to develop the ability to see *when* and *why* we must add, *i. e.*, to notice what operations the conditions of a problem require. Let this be remembered in class explanation, and a good form of solution be always required.

Solution.—If Harry had 7 books, and his father gave him 4 more, he then had the sum of 7 books and 4 books, which is 11 books.

2. If James rode 8 miles and walked 3, how far did he travel in all?

3. A merchant sold from a piece of cloth 2 yards at one time, 3 yards at another, and 6 yards at another. How much did he sell from the piece in all?

4. Jane is 7 years old, and her brother, George, is 6 years older than she is. How old is George?

Solution.—If Jane is 7 years old, and George is 6 years older than she, George's age is 7 + 6, or 13, years.

Note.—Avoid stereotyped forms of solution.

5. A farmer had 13 head of cattle and bought 8 more. How many had he then?

6. Mr. A. had 37 sheep; Mr. B. had 9 more than Mr. A. How many had Mr. B.?

7. Henry earned 37 cents on Monday, 28 cents on Tuesday, 56 cents on Wednesday, 48 cents on Thursday, was idle Friday, and earned nineteen cents on Saturday. How much did he earn during the week?

8. Mr. Jones bought a horse for 250 dollars, and sold it for 37 dollars more than he gave for it. For how much did he sell it?

9. James bought an orange for 8 cents, and a melon for 5 cents more than the orange cost. How much did he pay for both? Do not use the slate for this.*

10. There are 6 boys and 3 girls in one class, and 7 girls and 4 boys in another class. How many pupils are there in both classes?

---

* In all cases where the numbers are small and the combinations few, the work should be wholly mental.

11. John bought a knife for 23 cents and sold it for 3 cents more than he gave for it. How much did he receive for it?

12. There are thirty days in June, and 31 each in July and August. How many days in these three summer months?

13. Henry gave 85 cents for a sled, and 126 cents more for a pair of shoes than for the sled. How much did both cost him? *Ans.*, 296 cents.

14. If it is 2 feet from the ground to the top of the foundation of my house, and the first story is 11 feet, the second 10, the ridge of the roof 8 feet above the upper ceiling, the chimney top 4 feet above the ridge, and the lightning rod extends 3 feet above the chimney, how high is the top of the lightning rod above the ground?

15. A farmer has 47 acres in wheat, 36 in corn, 52 meadow land, 18 in oats, his house and barn yards and garden contain 2 acres; he has 43 acres of pasture, 17 acres occupied as an orchard and with vegetables, and 81 acres of woodland. How many acres in his farm?

16. A farmer's stock consists of 27 cattle, 126 sheep, 7 horses, and 15 hogs. How many animals has he in all?

17. It is 30 miles from Detroit to Ypsilanti, 8 miles from Ypsilanti to Ann Arbor, 38 miles from Ann Arbor to Jackson, 20 miles from Jackson to Albion, 12 miles from Albion to Marshall, 13 miles from Marshall to Battle Creek, 23 miles from Battle Creek to Kalamazoo, 47 miles from Kalamazoo to Niles, 37 miles from Niles to Michigan City, and 56 miles from Michigan City to Chicago. These places occurring in order along the Michigan Central Railroad, how far is it from Detroit to Chicago?

How far from Ann Arbor to Kalamazoo?

How far from Ypsilanti to Jackson?

How far from Jackson to Niles?

18. In a certain house there were 2 parlors, one of which required 37 yards of carpeting and the other 42 yards; a sitting-room requiring 28 yards, 2 bed-rooms requiring 16 yards each, and 2 other bed-rooms requiring 12 yards each. How much carpeting was required for all these rooms?

19. A drover paid 17428 dollars for 530 head of cattle, 7689 dollars for 125 head, and 63850 for 1225 head. How much did he pay for all? How many cattle did he buy?

20. A gentleman is 15 years older than his wife, and she is 20 years older than their eldest son, who is 29 years of age. Required the gentleman's age, and the age of his wife?

Ans., The gentleman's age is 64 years; his wife's, 49.

21. A farmer bought three plantations for 3750 dollars each, and sold them again so as to make 1000 dollars on the whole. For what sum did he sell the three?

Ans., 12250 dollars.

22. Several persons contributed towards the establishment of a library. A gave 200 dollars, and B 50 dollars more than A; C gave 300 dollars, and D 25 dollars more than C. What was the whole amount contributed?

Ans., 1075 dollars.

23. At the battle of Moscow there were 13000 Russians killed, 5000 taken prisoners, about 27000 wounded, and 40 generals either killed, wounded, or taken prisoners; 2500 of Napoleon's army were killed, 7500 wounded, and 15 generals either killed or wounded. What was the total loss? *Ans.* 55055.

24. At the battle of Waterloo the French lost 40000 men; the Prussians 38000; the Belgians and Dutch 8000;

the Hanoverians 3500; and the English about 12000; how many men were killed in all?

25. At the battle of Gettysburg the loss in the Union army was 2,834 men killed and 13,790 wounded, and in the Confederate army 4,500 killed and 26,500 wounded. What was the whole loss in each army? What was the whole number of men killed? What was the whole number wounded? What was the whole loss in both armies?

26. At the battle of Antietam the Union army numbered about 87,000 men, and the Confederate about 60,000. The loss of the former was reported at 2,010 killed, and 10,459 wounded and missing. If the Confederate loss was equally great, how many were killed in all?

27. A merchant bought cloth for 375 dollars, and silk for 95 dollars. In selling, he gained 50 dollars on the cloth, and 45 dollars on the silk; for what sum did he sell the whole? *Ans.*, 565 dollars.

28. A provision dealer bought a load of potatoes in the morning containing 48 bushels, out of which he sold 12 bushels to one man and 5 to another. He then bought a load of 37 bushels, and put them in with what were left of the former load. After this he sold to one man 7 bushels, to another 16, and to another 8. Finally, he bought 63 bushels, and put them into the same bin with the others. How many potatoes did he buy? How many did he sell?

29. The mariner's compass was invented in China 1120 years before Christ; America was discovered by Columbus 1492 years after Christ; and steam was first applied by Fulton to propelling boats 315 years after the discovery of America. How many years after the invention of the mariner's compass was steam first applied to propelling boats?

SECTION V.

SUBTRACTION.

FIRST STEP.

To Ascertain the Remainder when any Number represented by One Figure is taken from any Number not less than itself, but less than itself + 10.

1. How many cherries are there on the branch which the boy is holding out to the girl in the picture?

How many bunches of cherries are there?

How many in each bunch?

What is the girl doing?

If she picks 4 of the 9 cherries, how many will remain?

4 taken from (*i. e.*, out of) 9 leaves how many?

2. Five and 3 make what?

Then if 3 be taken from 8, how many will be left?

If 5 be taken from 8, how many will be left?

42. Subtraction is a process of taking one number from (*i. e.* out of) another.

The number to be subtracted is called the **Subtrahend.**

The number from which the Subtrahend is to be taken is called the **Minuend.**

What is left of the *Minuend* after the Subtrahend has been taken out is called the **Remainder.**

3. Six and 5 are how many?

Five and what make 11?

If 5 are taken from 11, how many remain?

In this example what is the 11 called?

What is the 5 called which is to be taken from the 11? What is the 6 called which is left of the 11 after 5 has been taken from it?

4. Four and 8 make how many?

8 and what make 12?

8 taken from 12 leave how many?

What is the minuend in this example? What is the subtrahend? What the remainder?

5. 7 and what = 9?

If 7 and 2 make 9, what is left of 9 after 7 is taken out?

What is left of 9 after 2 is taken out?

To Ascertain the Remainder when any Number represented by One Figure is taken from any Number not less than itself, but less than itself + 10.

43. Rule.—*Consider, from your knowledge of Addition, what number together with the given subtrahend makes the minuend. This other number is the remainder sought.*

6. What is the remainder when 7 is taken from 13?

Suggestion.—Since 7 and 6 make 13, if 6 is taken from 13, 7 is left.

7. If 8 is taken from 14, what is the remainder?

44. The sign $-$ is called *minus*, and indicates that the number after it is to be subtracted from the number before it; thus $11 - 6$ means that 6 is to be subtracted from 11.

This sign is read *minus*, which means less, so that $11-6$ is read "11 minus 6."

8. What is $9 - 4$?

What number with 4 makes 9?

9. What is $7 - 5$? Why?

Ans., $7 - 5$ is 2, because $5 + 2 = 7$.

10. What is $15 - 8$? Why? What is $16 - 7$? Why?

11. Copy the following on your slate, and fill the blanks:

8 + what = 11?	Then 11 − 8 = what?
4 + —— = 7?	Then 7 − 4 = ——?
5 + —— = 14?	Then 14 − 5 = ——?
7 + —— = 12?	Then 12 − 7 = ——?
9 + —— = 17?	Then 17 − 9 = ——?
3 + —— = 9?	Then 9 − 3 = ——?

12. If 6 is one of two parts of 8, what is the other? $8 - 6 =$ what?

13. If 3 is one of two parts of 11, what is the other? $11 - 3 =$ what?

14. If 7 is one of two parts of 15, what is the other? $15 - 7 =$ what?

Three dots placed thus $\therefore$ are read "therefore."

15. Copy on your slate and fill out the table on the following page:

This table must be so thoroughly learned that any remainder can be told instantly. But if addition has been well learned, this will cost only a little labor.

Counting backward from 100 by 1's, 2's, 3's, etc., forms an excellent drill in learning the Subtraction Table.

SUBTRACTION TABLE.

When 1 is one part.

$1 + 1 = \text{—}. \therefore \{ 2 - 1 = \text{—}.$

$2 + 1 = \text{—}. \therefore \left\{ \begin{array}{l} 3 - 1 = \text{—}. \\ 3 - 2 = \text{—}. \end{array} \right.$

$3 + 1 = \text{—}. \therefore \left\{ \begin{array}{l} 4 - 1 = \text{—}. \\ 4 - 3 = \text{—}. \end{array} \right.$

$4 + 1 = \text{—}. \therefore \left\{ \begin{array}{l} 5 - 1 = \text{—}. \\ 5 - 4 = \text{—}. \end{array} \right.$

$5 + 1 = \text{—}. \therefore \left\{ \begin{array}{l} 6 - 1 = \text{—}. \\ 6 - 5 = \text{—}. \end{array} \right.$

$6 + 1 = \text{—}. \therefore \left\{ \begin{array}{l} 7 - 1 = \text{—}. \\ 7 - 6 = \text{—}. \end{array} \right.$

$7 + 1 = \text{—}. \therefore \left\{ \begin{array}{l} 8 - 1 = \text{—}. \\ 8 - 7 = \text{—}. \end{array} \right.$

$8 + 1 = \text{—}. \therefore \left\{ \begin{array}{l} 9 - 1 = \text{—}. \\ 9 - 8 = \text{—}. \end{array} \right.$

$9 + 1 = \text{—}. \therefore \left\{ \begin{array}{l} 10 - 1 = \text{—}. \\ 10 - 9 = \text{—}. \end{array} \right.$

When 2 is one part.

$2 + 2 = \text{—}. \therefore \{ 4 - 2 = \text{—}.$

$3 + 2 = \text{—}. \therefore \left\{ \begin{array}{l} 5 - 2 = \text{—}. \\ 5 - 3 = \text{—}. \end{array} \right.$

$4 + 2 = \text{—}. \therefore \left\{ \begin{array}{l} 6 - 2 = \text{—}. \\ 6 - 4 = \text{—}. \end{array} \right.$

$5 + 2 = \text{—}. \therefore \left\{ \begin{array}{l} 7 - 2 = \text{—}. \\ 7 - 5 = \text{—}. \end{array} \right.$

$6 + 2 = \text{—}. \therefore \left\{ \begin{array}{l} 8 - 2 = \text{—}. \\ 8 - 6 = \text{—}. \end{array} \right.$

$8 + 2 = \text{—}. \therefore \left\{ \begin{array}{l} 10 - 2 = \text{—}. \\ 10 - 8 = \text{—}. \end{array} \right.$

$9 + 2 = \text{—}. \therefore \left\{ \begin{array}{l} 11 - 2 = \text{—}. \\ 11 - 9 = \text{—}. \end{array} \right.$

When 3 is one part.

$3 + 3 = \text{—}. \therefore \{ 6 - 3 = \text{—}.$

$4 + 3 = \text{—}. \therefore \left\{ \begin{array}{l} 7 - 3 = \text{—}. \\ 7 - 4 = \text{—}. \end{array} \right.$

$5 + 3 = \text{—}. \therefore \left\{ \begin{array}{l} 8 - 3 = \text{—}. \\ 8 - 5 = \text{—}. \end{array} \right.$

$6 + 3 = \text{—}. \therefore \left\{ \begin{array}{l} 9 - 3 = \text{—}. \\ 9 - 6 = \text{—}. \end{array} \right.$

$7 + 3 = \text{—}. \therefore \left\{ \begin{array}{l} 10 - 3 = \text{—}. \\ 10 - 7 = \text{—}. \end{array} \right.$

$8 + 3 = \text{—}. \therefore \left\{ \begin{array}{l} 11 - 3 = \text{—}. \\ 11 - 8 = \text{—}. \end{array} \right.$

$9 + 3 = \text{—}. \therefore \left\{ \begin{array}{l} 12 - 3 = \text{—}. \\ 12 - 9 = \text{—}. \end{array} \right.$

When 4 is one part.

$4 + 4 = \text{—}. \therefore \{ 8 - 4 = \text{—}.$

$5 + 4 = \text{—}. \therefore \left\{ \begin{array}{l} 9 - 4 = \text{—}. \\ 9 - 5 = \text{—}. \end{array} \right.$

$6 + 4 = \text{—}. \therefore \left\{ \begin{array}{l} 10 - 4 = \text{—}. \\ 10 - 6 = \text{—}. \end{array} \right.$

$7 + 4 = \text{—}. \therefore \left\{ \begin{array}{l} 11 - 4 = \text{—}. \\ 11 - 7 = \text{—}. \end{array} \right.$

$8 + 4 = \text{—}. \therefore \left\{ \begin{array}{l} 12 - 4 = \text{—}. \\ 12 - 8 = \text{—}. \end{array} \right.$

$9 + 4 = \text{—}. \therefore \left\{ \begin{array}{l} 13 - 4 = \text{—}. \\ 13 - 9 = \text{—}. \end{array} \right.$

When 5 is one part.

$5 + 5 = \text{—}. \therefore \{ 10 - 5 = \text{—}.$

$6 + 5 = \text{—}. \therefore \left\{ \begin{array}{l} 11 - 5 = \text{—}. \\ 11 - 6 = \text{—}. \end{array} \right.$

$7 + 5 = \text{—}. \therefore \left\{ \begin{array}{l} 12 - 5 = \text{—}. \\ 12 - 7 = \text{—}. \end{array} \right.$

$8 + 5 = \text{—}. \therefore \left\{ \begin{array}{l} 13 - 5 = \text{—}. \\ 13 - 8 = \text{—}. \end{array} \right.$

$9 + 5 = \text{—}. \therefore \left\{ \begin{array}{l} 14 - 5 = \text{—}. \\ 14 - 9 = \text{—}. \end{array} \right.$

When 6 is one part.

$6 + 6 = \text{—}. \therefore \{ 12 - 6 = \text{—}.$

$7 + 6 = \text{—}. \therefore \left\{ \begin{array}{l} 13 - 6 = \text{—}. \\ 13 - 7 = \text{—}. \end{array} \right.$

$8 + 6 = \text{—}. \therefore \left\{ \begin{array}{l} 14 - 6 = \text{—}. \\ 14 - 8 = \text{—}. \end{array} \right.$

$9 + 6 = \text{—}. \therefore \left\{ \begin{array}{l} 15 - 6 = \text{—}. \\ 15 - 9 = \text{—}. \end{array} \right.$

When 7 is one part.

$7 + 7 = \text{—}. \therefore \{ 14 - 7 = \text{—}.$

$8 + 7 = \text{—}. \therefore \left\{ \begin{array}{l} 15 - 7 = \text{—}. \\ 15 - 8 = \text{—}. \end{array} \right.$

$9 + 7 = \text{—}. \therefore \left\{ \begin{array}{l} 16 - 7 = \text{—}. \\ 16 - 9 = \text{—}. \end{array} \right.$

When 8 is one part.

$8 + 8 = \text{—}. \therefore \{ 16 - 8 = \text{—}.$

$9 + 8 = \text{—}. \therefore \left\{ \begin{array}{l} 17 - 8 = \text{—}. \\ 17 - 9 = \text{—}. \end{array} \right.$

$9 + 9 = \text{—}. \therefore \{ 18 - 9 = \text{—}.$

SECOND STEP.

To Perform Subtraction when Minuend and Subtrahend are represented by several Figures each, and no Figure in the Subtrahend exceeds in value the Figure in the same order in the Minuend.

1. The packages on the table are our old familiar packages of counters. The large ones are 100's; the next in size are 10's, and the others are single counters. How many counters are there on the table? *Ans.*, 357.

Ellen wishes to take off 234 counters.

How many packages of hundreds will she take off? Then how many hundreds will remain?

How many packages of tens will she take off? Then how many 10's will remain?

How many single counters will she take off? How many single counters will remain?

If therefore Ellen takes 234 counters from the 357 which are on the table, how many of the 357 will remain?

2. Take 425 from 768.

Suggestion.—As we want to take units from units, tens from tens, and hundreds from hundreds, *if we can*, it will be convenient to write the numbers so that the orders of the subtrahend shall fall under the corresponding orders of the minuend.

768
425

Will it make any difference in this case whether we subtract the 4 hundreds first, or the 2 tens, or the 5 units?

5 units from 8 units leave how many units?

2 tens from 6 tens leave how many tens?

4 hundreds from 7 hundreds leave how many hundreds?

Then 425 from 768 leaves how many?

3. From 856 take 324. From 743 take 531.

To Perform Subtraction when Minuend and Subtrahend are represented by several Figures each, and no Figure in the Subtrahend exceeds in value the Figure in the same order in the Minuend.

45. Rule.—*I. Write the subtrahend under the minuend, so that each figure shall fall under one of like order in the minuend.*

II. Subtract each figure in the subtrahend from the one of like order in the minuend, and write the remainder underneath, bringing down any figures in the minuend which have no figures under them in the subtrahend.

4. From 3739 take 516. From 7892 take 5321.

3739	*Min.*
516	*Sub.*
3223	*Rem.*

Suggestions.—Taking the 6 units of the subtrahend from the 9 of the minuend, there are 3 in the remainder. Taking the 1 ten of the subtrahend from the 3 tens of the minuend, there are 2 tens in the remainder. Taking the 5 hundreds of the subtrahend from the 7 hundreds of the minuend, there are 2 hundreds in the remainder. As there are no thousands in the subtrahend, all the thousands of the minuend will remain.

5. From 82564 subtract 303.

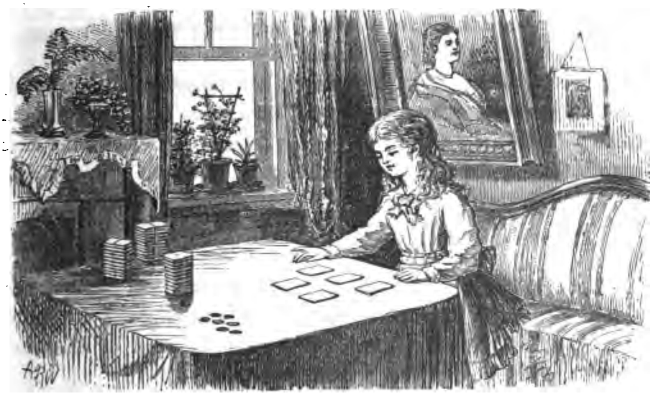
How many tens are to be taken from the minuend? Then how many tens will remain?

6. If 15264 is the minuend and 3150 the subtrahend, what is the remainder? What is 312506 – 11406?

7. If 20056 is the subtrahend and 872086 the minuend, what is the remainder? What is 50003 – 5001?

THIRD STEP.

To Perform Subtraction when Minuend and Subtrahend are represented by several Figures each, and there is a Figure in the Subtrahend whose value exceeds that of the corresponding Figure in the Minuend.



1. Here is Ellen with the counters again.
How many 100's packages are there?
How many 10's? How many single counters?
How many counters in all?
Suppose, now, that Ellen wishes to take 179 counters off

the table. How is she going to get the 9 single counters? If she unwraps one of the 10's packages, how many single counters will there then be? Can she then take out the 9 single counters? How many single counters will remain?* How many 10's packages will remain? Now how is she to get 7 tens out? If she unwraps a 100's package, how many 10's will she find in it? How many 10's will there then be? (14.) If she now takes out the 7 tens, how many tens will remain? How many 100's are left? What more did she wish to take out?

Having done this, how many counters remain? How many 100's? How many 10's? How many ones?

2. From 562 take 237.

Suggestions.—Write the numbers as heretofore. How can we take the 7 units from the 562? How many units are there in the units order of 562? If we take one of the 6 tens, how many units will it make together with the 2 units? Now taking 7 units from 12, how many remain? Again, how many tens were we to subtract? How many have we left from which to take the 3 tens? Why only 5 tens? 3 tens from 5 tens leave how many tens? What remains to be taken out? 2 hundreds from 5 hundreds leave how many?

$$\begin{array}{r} 562 \\ 237 \\ \hline 325 \end{array}$$

3. From 4327 take 1563.

To Perform Subtraction when Minuend and Subtrahend are represented by several Figures each, and there is a Figure in the Subtrahend whose value exceeds that of the corresponding Figure in the Minuend.

46. Rule.—*I. Write the subtrahend under the minuend,*

* The method of adding 10 to the upper figure and compensating by increasing the next figure of the subtrahend by 1, is illustrated by supposing Ellen to put 10 more *single* counters on the table, and to take off 1 of the packages of 10's, *i. e.*, subtract 8 instead of 7; etc. But the method given in the text is the more natural, and in accord with the practice of computers, as in taking complements, etc.

so that each figure shall fall under one of the same order in the minuend.

II. Begin the work of subtraction with units, and proceed through the higher orders in succession.

III. When the figure in the subtrahend is not of greater value than the one over it, subtract the former from the latter, and write the remainder underneath.

IV. When the figure in the subtrahend exceeds in value the one over it, add 10 to the upper, and then subtract. Having done this, consider the next digit * *in the minuend as diminished by 1, observing to call each 0 which intervenes in the minuend 9, and then proceed with the subtraction.*

Reasons.—The reason for writing units under units, tens under tens, etc., is that we can take units from units only, tens from tens, etc., and by placing them thus we can see at a glance how many there are in the minuend of the particular order which we are subtracting. There is no reason, except custom, for putting the subtrahend *under* the minuend. It is often more convenient to put it above.

The reason for beginning the subtraction with the units and proceeding in succession through the higher orders, is that we may have to go over our work but once. Thus in taking 158 from 473, if we take out the 1 hundred and the 5 tens before we do the 8 units, we shall have 3 hundreds and 2 tens left; and having written these in their places, we would have to take 1 of the 2 tens to put with the 3 units, in order to be able to get the 8 units out. This would oblige us to change the 2 tens to 1 ten.

$$\begin{array}{r} 473 \\ \underline{158} \\ 3\cancel{2} \\ \underline{15} \\ 315 \end{array}$$

When a figure in the subtrahend exceeds in value the one over it, we have to take 1 of the next higher order of which there are any in the minuend, in order that we may be able to take out the number required. The reason for calling 0's which intervene 9's, will be best seen from an example. (See next example.)

* A figure which is not 0.

4. From 5800036 take 2835654.

Explanation.—Taking 4 units from 6, 2 units remain. As there are not tens enough in the minuend so that we can take out 5 tens, and as there are no hundreds, thousands, or ten thousands, we have to take 1 of the 8 hundred thousands. This makes 10 ten thousands and leaves 7 hundred thousands. Taking 1 of these 10 ten thousands, it makes 10 thousands and leaves 9 ten thousands. Again taking 1 of these thousands, it makes 10 hundreds and leaves 9 thousands. Finally taking 1 of these hundreds, it makes, with the tens, 13 tens, and leaves 9 hundreds. We can then proceed with the higher orders, and observe that there are 9's where there were 0's.

$$
\begin{array}{ccccccc}
 & 17 & 9 & 9 & 9 & & \\
4 & \cancel{7} & \cancel{10} & \cancel{10} & \cancel{10} & 13 & \\
5 & \cancel{8} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{3} & 6 \\
2 & 8 & 3 & 5 & 6 & 5 & 4 \\
\hline
2 & 9 & 6 & 4 & 3 & 8 & 2
\end{array}
$$

Examples for Practice.

1. What is 856432 – 648571? 32564 – 1768?
2. From 205643 take 32589. From 70542 take 30256.
3. From 72382 take 165. From 46537 take 2006.
4. From 523654 take 82465. From 10125 take 908.
5. From 132406 take 65348. From 2007 take 407.
6. From 34652 take 15836. From 548300 take 83.
7. From 82654 take 34271. From 8001 take 750.
8. From 10000 take 546. From 1000 take 999.
9. Minuend 78206, subtrahend 35264; what is the remainder?
10. Subtrahend 10956, minuend 235043; what is the remainder?
11. What is 564023 – 234560? 782005 – 12758?
12. What is 30012 – 15461? 65430 – 718?
13. Subtract 5056 from 100000. From 601 take 307.
14. Take 3 from 1000. Take 208 from 10000.
15. From 7003 subtract 1815. From 8111 subtract 88.

Applications.

1. Harry had 8 rabbits, and sold 5 of them. How many had he left?

If Harry had 8 rabbits and sold 5 of them, he had remaining 8 -- 5, or 3 rabbits.

2. There were 11 eggs in the nest and Mary took out 7. How many remained?

3. A farmer raised 347 bushels of potatoes, and selling 259 bushels, reserved the remainder for his own use. How many bushels did he reserve?

4. If I borrow 17 dollars, and afterward pay 9 dollars on the debt, how much do I still owe.*

5. A well was sunk through sand and clay to the depth of 30 feet. 6 feet was sand. How much was clay?

6. A drover started from Texas with 2782 head of cattle. On the way to Chicago 547 of them died. How many remained?

7. A person who undertook a journey of 735 miles, has travelled 93 miles of the distance. What distance has he yet to travel?

8. From a farm which contained 2350 acres, 1234 acres were sold. How many acres remained of the original farm?

9. A young man received from his father 5325 dollars, of which he paid 2500 dollars for a house. How many dollars of the first sum had he remaining?

10. A merchant deposited 5800 dollars in bank, but afterwards made a draft upon it for 3270 dollars. What sum remained?

11. Suppose a farmer who has 4000 bushels of wheat in

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* When the numbers are small the slate should not be used.

his granary, should take out 2100 bushels to be sent to market; how many bushels would remain?

12. Here are two bunches of grapes. In one bunch there are 22 grápes and in the other 15. If you were to pick as many from the larger bunch as there are in the smaller, how many would remain? How many more are there in the larger bunch than in the smaller?

What is the difference between the number in the larger and that in the smaller?

47. *The Difference between two numbers is what is left of the larger after the smaller is subtracted.** *If the numbers are equal there is no difference, or the difference is 0.*

13. John is 15 years old, and Mary is 9. What is the difference between their ages.

14. A merchant sold 25726 dollars worth of goods one year, and 34718 dollars worth the next. What was the difference between the sales of the two years?

15. America was discovered in 1492, and the British Colonies declared their independence in 1776. How long after the discovery was the Declaration of Independence?

16. Sir Isaac Newton was born in 1642, and died in 1727. How old was he at his death?

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* In a more enlarged sense, the Difference between two numbers is the number of units which lie between them. Thus the difference between 25 degrees north latitude and 10 degrees south latitude is 35 degrees. But the conception given in the text is as broad as is consistent with our present purpose.

17. The telescope was invented in 1608. How many years since that time?

18. Benjamin Franklin died in 1798, and was 84 years old at his death. When was he born?

19. The art of printing was invented in 1449. How many years since its invention?

20. The area of the Chinese Empire is 4695334 square miles, and the area of the United States is 3578392 square miles; how much greater is the Chinese Empire than the United States?

21. If I buy a horse for 137 dollars, and sell it for 225 dollars, how much more do I get for it than I paid?

48. When we sell an article for more than we paid, what we get for it more than what we paid is called *Gain.* If we sell it for less than we paid, what the selling price lacks of being as much as we paid is called *Loss.*

22. John bought a sleigh for 87 cents, and sold it for 65 cents. Did he gain or lose? How much?

23. Bought a cow for 58 dollars, and lost 17 dollars in selling her. How much did I sell her for?

24. Bought a horse for 139 dollars, and sold it for 250 dollars. What was my gain?

25. Bought cloth for a coat which cost 17 dollars, and handed the salesman a 50-dollar bill. How much change must he give me?

49. The character $ signifies dollars, and is placed before the number. Thus $17 is read "17 dollars."

26. If I buy a horse for $175 and sell it for $225, do I gain or lose? How much?

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27. Bought 2 pieces of cloth for $112 each, and sold them for $320. Did I gain or lose? How much?

28. Bought 2 yards of cloth for $7 a yard, and gave the salesman a $20 bill. How much change must he give me?

29. How old is a man in 1875 who was born in 1832? One who was born in 1827?

30. How long is it since the Declaration of Independence by the United States? (See Ex. 15.)

31. How old are you? From this how do you find in what year you were born?

32. It is now 8 o'clock in the morning. How long is it to noon? How long since 3 o'clock this morning?

33. Borrowed of my neighbor at one time $175, at another $340, and at another $520. Having paid him $685, what balance have I yet to pay? *Ans.*, $350.

34. Put in store at one time 500 pounds of hemp, at another time 3800 pounds, and at another 2005 pounds. Having withdrawn 3473 pounds, what quantity remains in store? *Ans.*, 2832 pounds.

35. A merchant bought flour at one time for $325, and at another time for $460. Having become damaged, the whole was sold at a loss of $184; for what sum was it sold?

36. A man's annual income is $3700. His family expenses are $2500, and he bestows for benevolence $370 a year, and invests the remainder. How much does he invest?

37. A man owed $2000, on which he made 3 payments of $375, $580, and $260, respectively. How much remained unpaid?

38. William has 75 cents more than James, and 125 cents less than Henry, who has 420. What is the number of cents which they all have? *Ans.*, 935.

39. If one part of 13 is 7, what is the other part?

40. A man has a farm of 400 acres. Part is woodland and part is cultivated. The former part is 125 acres; how much is the latter?

41. If the remainder is what is left of the minuend after taking the subtrahend out, what do the remainder and subtrahend when added together make?

As the remainder is one part of the minuend and the subtrahend the other, what will you obtain by taking the remainder out of the minuend?

50. Any device by which we may test the accuracy of an operation in arithmetic by some other operation is called a **Proof** of the work.

Proof of Subtraction.—Subtraction may be proved by adding the remainder to the subtrahend; and if the work is right, the sum will equal the minuend.

Or, we may prove subtraction by taking the remainder from the minuend; and if the work is right, what is left will be equal to the subtrahend.

[The pupil should give the reasons.]

42 to 45. Solve the following examples, and prove them in both the above ways:

(42)	(43)	(44)	(45)
528643	500608	182005	517000
216805	23471	37050	134056

46. The sum of 3 numbers is 5208. Two of the numbers are 1250, and 2340. What is the third?

47. A man paid $5347 for a farm, implements, and stock. The implements cost $500, and the stock $2100. What was the cost of the land?

48. The difference between two numbers is 117, and the less number is 375. What is the greater?

SECTION VI.

MULTIPLICATION.

FIRST STEP.

To Ascertain the Product of any Number not greater than 12, Multiplied by any Number not greater than itself.

1. What is the man in the picture doing? How many eggs has he in his hand? If he takes 3 eggs at a time, how many will he have in his dish when he has taken 4 times? $3 + 3 + 3 + 3 =$ how many?

2. If the man should use both hands and take out 5 eggs at a time, how many would he have when he had taken 3 times? $5 + 5 + 5 =$ how many?

4 times 3 are how many?

3 times 5 are how many?

3. $2 + 2 + 2 + 2 =$ how many?

$2 + 2 + 2 + 2 =$ how many times 2?

4 times 2 are how many?

In this example how many times has 2 been taken?

What number has been taken 4 times?

What number does 2 taken 4 times make?

51. The number which is taken a certain number of times is called the **Multiplicand.**

The number which tells how many times the Multiplicand is to be taken is called the **Multiplier.**

The number which tells how many a certain number of times a given number makes is called the **Product.**

Thus when I say "3 times 5 are 15," 5 is the *Multiplicand,* 3 is the *Multiplier,* and 15 is the *Product.*

52. The sign $\times$ is called the **Sign of Multiplication,** and is read "*Times.*" Thus 4×3 is read "4 times 3." 3×5 is read "3 times 5."

4. $6 + 6 + 6 =$ how many?

$6 + 6 + 6$ is how many times 6?

3 times 6 are how many? $3 \times 6 =$ what?

What is the multiplicand in this example? What the multiplier? What the product?

5. When you know that 4 times 7 are 28, how can you find how many 5 times 7 are? How can you find 6 times 7 when you know 5 times 7?

6. If you have added 6 and 6, and found what 2 times 6 are, how many times will you have to add 6 to this to make 3 times 6?

7. To *Multiply* 4 by 6 is to *take* 6 times 4. What is the product of 7 multiplied by 3? By 4? By 5?

8. What is meant when we speak of multiplying 4 by 5?

9. If you multiply 8 by 6, what is the product?

To Ascertain the Product of any Number not greater than 12, Multiplied by any Number not greater than itself.

53. Rule.—*Add the multiplicand to itself until it is taken as many times as are indicated by the multiplier.*

10. How many are 5 times 3; that is, how many are 3 + 3 + 3 + 3 + 3?

11. In this manner fill out the following table after having copied it on your slate:

MULTIPLICATION TABLE.

1 × 1* = —			
1 × 2 = —	2 × 2 = —		
1 × 3 = —	2 × 3 = —	3 × 3 = —	
1 × 4 = —	2 × 4 = —	3 × 4 = —	4 × 4 = —
1 × 5 = —	2 × 5 = —	3 × 5 = —	4 × 5 = —
1 × 6 = —	2 × 6 = —	3 × 6 = —	4 × 6 = —
1 × 7 = —	2 × 7 = —	3 × 7 = —	4 × 7 = —
1 × 8 = —	2 × 8 = —	3 × 8 = —	4 × 8 = —
1 × 9 = —	2 × 9 = —	3 × 9 = —	4 × 9 = —
1 × 10 = —	2 × 10 = —	3 × 10 = —	4 × 10 = —
1 × 11 = —	2 × 11 = —	3 × 11 = —	4 × 11 = —
1 × 12 = —	2 × 12 = —	3 × 12 = —	4 × 12 = —
5 × 5 = —			
5 × 6 = —	6 × 6 = —		
5 × 7 = —	6 × 7 = —	7 × 7 = —	
5 × 8 = —	6 × 8 = —	7 × 8 = —	8 × 8 = —
5 × 9 = —	6 × 9 = —	7 × 9 = —	8 × 9 = —
5 × 10 = —	6 × 10 = —	7 × 10 = —	8 × 10 = —
5 × 11 = —	6 × 11 = —	7 × 11 = —	8 × 11 = —
5 × 12 = —	6 × 12 = —	7 × 12 = —	8 × 12 = —
9 × 9 = —			
9 × 10 = —	10 × 10 = —		
9 × 11 = —	10 × 11 = —	11 × 11 = —	
9 × 12 = —	10 × 12 = —	11 × 12 = —	12 × 12 = —

12. Commit this table thoroughly to memory.

* Read these, "once 1 is 1," "2 times 2 are 4," "2 times 3 are 6," etc.

13. Test your knowledge of the preceding table by seeing if you can answer the following, *at once, without adding:*

5×7? 7×8? 3×4? 2×7? 8×9? 9×11?
8×12? 4×7? 5×8? 2×12? 6×9? 7×12?
6×9? 3×11? 8×10? 11×11? 5×5? 4×4?
7×10? 6×7? 7×11? 8×12? 12×12? 4×8?

[For drill exercises and other expedients, see TEACHER'S HAND-BOOK.]

SECOND STEP.

To Ascertain the Product of any Number not greater than 12, Multiplied by any Number greater than itself, and not greater than 12.

1. How many stars are there in one of these *columns?* How many columns are there? If there are 7 stars in one column, how many are there in 6 columns? 6 times 7 are how many?

```
*  *  *  *  *  *
*  *  *  *  *  *
*  *  *  *  *  *
*  *  *  *  *  *
*  *  *  *  *  *
*  *  *  *  *  *
*  *  *  *  *  *
```

How many stars are there in one of the *rows* from left to right? How many such rows? If there are 6 stars in a row and 7 rows, how many stars in all? Then 7 times 6 are how many?

What is the difference between 6 times 7 and 7 times 6?

2. 3 times 4 are how many? Make 3 columns with 4 stars in each, and show it.

Then 4 times 3 are how many? How can you show from your 3 columns of stars, with 4 in a column, that 3×4 is the same as 4×3?

3. 3×8 are how many? Then 8×3 are how many?

To Ascertain the Product of any Number not greater than 12, Multiplied by any Number greater than itself and not greater than 12.

54. Rule.—*Observe, from your knowledge of the Multiplication Table, what the product of the greater of the two numbers by the less is, and then remember that this is the same as the product of the less by the greater.*

4. How many are 7 times 4?

Suggestion.—Think how many 4 times 7 are.*

5. How many are 8 times 3? 5×4? 7×6? 4×2? 6×4? 11×6? 12×8? 12×7?

6. Copy on your slates and fill out the following:

$7 \times 5 =$ —	$3 \times 2 =$ —	$9 \times 8 =$ —	$9 \times 4 =$ —
$6 \times 3 =$ —	$2 \times 1 =$ —	$9 \times 5 =$ —	$9 \times 1 =$ —
$5 \times 3 =$ —	$4 \times 1 =$ —	$9 \times 3 =$ —	$10 \times 5 =$ —
$8 \times 6 =$ —	$5 \times 3 =$ —	$7 \times 2 =$ —	$12 \times 1 =$ —
$8 \times 4 =$ —	$6 \times 1 =$ —	$8 \times 2 =$ —	$10 \times 10 =$ —
$9 \times 6 =$ —	$9 \times 7 =$ —	$6 \times 2 =$ —	$11 \times 9 =$ —
$12 \times 8 =$ —	$11 \times 5 =$ —	$12 \times 10 =$ —	$8 \times 5 =$ —
$12 \times 11 =$ —	$10 \times 6 =$ —	$12 \times 12 =$ —	$11 \times 3 =$ —
$10 \times 8 =$ —	$12 \times 9 =$ —	$12 \times 6 =$ —	$12 \times 2 =$ —
$11 \times 7 =$ —	$12 \times 7 =$ —	$12 \times 4 =$ —	$12 \times 3 =$ —
$3 \times 1 =$ —	$4 \times 3 =$ —	$4 \times 2 =$ —	$5 \times 4 =$ —
$5 \times 1 =$ —	$5 \times 2 =$ —	$6 \times 4 =$ —	$7 \times 4 =$ —
$6 \times 5 =$ —	$7 \times 1 =$ —	$7 \times 3 =$ —	$8 \times 5 =$ —
$7 \times 6 =$ —	$8 \times 1 =$ —	$8 \times 3 =$ —	$10 \times 1 =$ —
$8 \times 7 =$ —	$9 \times 2 =$ —	$10 \times 2 =$ —	$10 \times 7 =$ —
$10 \times 4 =$ —	$10 \times 3 =$ —	$10 \times 5 =$ —	$11 \times 4 =$ —
$10 \times 9 =$ —	$11 \times 1 =$ —	$11 \times 2 =$ —	$12 \times 5 =$ —
$11 \times 5 =$ —	$12 \times 8 =$ —	$12 \times 6 =$ —	$12 \times 10 =$ —

[This will afford a drill exercise, the teacher repeating what is written, and the pupils, individually or in concert, filling in the blank. Thus the teacher says "7 times 5,"—the pupil says "35;" teacher, "6 times 3," pupil, "18," etc.]

* Of course the pupil needs to know the product of a less number by a greater as a fact of memory; but this will be attained without special effort.

7. If Jane goes to the table 4 times and gets 0 books each time, how many books will she get in the 4 times? 4 times 0 = how many?

8. How many are 3 times 0? 8×0? 12×0? Any number of times 0?

9. Copy the following on your slates, and write the product under each:

4	5	6	8	3	9	10	11	7	7	4	5
3	7	2	3	8	9	10	11	7	6	8	7
9	7	6	7	3	6	8	6	7	9	4	8
8	9	8	4	7	9	9	12	12	12	7	8

55. The Factors of a number are those numbers which multiplied together produce it. Thus 2 and 3 are the factors of 6, because $2 \times 3 = 6$.

10. Of what number are 5 and 3 the factors? 7 and 2? 3 and 4?

11. What are the factors of 8? Of 10? Of 12? Of 9? Of 6? Of 18?

12. If 3 is one of the factors of 12, what is the other? 3 times what number makes 12?

13. If 4 is one of the factors of 28, what is the other? 4 times what number makes 28?

THIRD STEP.

Four Principles on which the General Problem in Multiplication is based.

1. How many groups of 4 stars each are there in the first row? How many in the second row? How many 4's are 3 4's and 5 4's? 3 times 4, and 5 times 4 are how many times 4?

```
    **    **    **
    **    **    **

**    **    **    **    **
**    **    **    **    **
```

3 times 4 and 5 times 4 are how many?

2. Four times 6 and 3 times 6 are how many times 6? How many are 4 times 6? How many are 3 times 6? 24 and 18 are how many?

Then how in this way can you get 7 times 6?

3. Find out how many 7 times 8 are by taking 2 times 8 and 5 times 8.

4. If you have 6 times any number, how many more times that number must you take to make 9 times the number? Find out how many 9 times 7 are by taking 6 times 7 and 3 times 7.

Principle I.

56. *One number may be multiplied by another by multiplying the multiplicand by the parts of the multiplier and adding the products.*

5. Find how many 11 times 8 are by first taking 5 times 8, and then 6 times 8.

6. If you take 5 times a number, how many more times must you take it to make 8 times the number? Find by this principle 9 times 12.

7. Find 8 times 7 by first taking 3 times 7.

8. Find 12 times 9 by first taking 4 times 9.

9. Find 12 times 7 by first taking 10 times 7.

10. Find 12 times 8 by first taking 2 times 8.

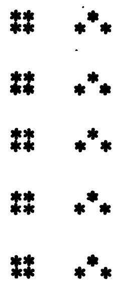
11. Find 13 times 9 by first taking 3 times 9.

12. Find 9 times 8 by taking 4 times 8, 3 times 8, and 2 times 8.

13. Find 12 times 7 by taking 4 times 7, 5 times 7, and 3 times 7.

14. If you take 3 times 7, and 2 times 7, and 8 times 7, and add the products, how many times 7 have you?

15. Find 14 times 9 by taking 3 times 9, 4 times 9, 5 times 9, and 2 times 9.



1. Four stars and 3 stars are how many stars? If we have 5 4's and 5 3's, how many 7's have we? 5 times 4 are how many? 5 times 3 are how many?

Then from this how many are 5 times 7?

2. Since 5 is 3 and 2, if we take 4 times 3 and 4 times 2, what number shall we have taken 4 times? Find 4 times 5 in this way.

3. Find 8 times 11 by taking 8 times 6 and 8 times 5.

4. Find 7 times 9 by taking 7 times 5 and 7 times 4. Also by taking 7 times 6 and 7 times 3.

Principle II.

57. *One number may be multiplied by another by multiplying the parts of the multiplicand by the multiplier, and adding the products.*

5. Find how many 8 times 13 are by taking 8 times 3 and 8 times 10.

6. Find how many 7 times 17 are by taking 7 times 7 and 7 times 10. By taking 7 times 8 and 7 times 9.

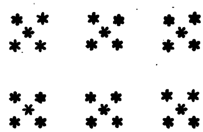
7. Find how many 9 times 11 are by taking 9 times 2, 9 times 3, and 9 times 6.

8. Find how many 8 times 9 are by taking 8 times 2, 8 times 3, and 8 times 4.

9. Find how many 7 times 16 are by taking 7 times 6 and 7 times 10.

10. Find how many 6 times 18 are by taking 6 times 8 and 6 times 10.

1. How many times 5 stars are there in the first row? How many times 3 times 5 stars are there in both rows? 2 times 3 times 5 stars make how many times 5 stars?



2 times 5 are how many? 3 times 10 are how many? Then how many are 6 times 10?

Two times 3 times a number are how many times the number?

2. If you picked 4 quarts of currants in the forenoon and 4 quarts in the afternoon, how many would you pick in a day? How many in 5 days?

How many times 4 quarts would you pick in 5 days?

5 times 2 times 4 make how many times 4?

3. Four times 2 times 6 make how many times 6? 2 times 6 are how many? 4 times 12 are how many? Then from this how many are 8 times 6?

Principle III.

58. *One number may be multiplied by another by multiplying successively by all the factors of the multiplier; that is, by multiplying the multiplicand by one of the factors, and this product by another, and so on.*

4. Multiply 2 by 12 by multiplying successively by 3 and 4. By multiplying successively by 2 and 6.

5. Multiply 3 by 21 by multiplying successively by 3 and 7.

6. Multiply 4 by 9 by multiplying successively by 3 and 3.

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1. If we write 0 at the right of 4, what does it make? How many times 4 is 40?

2. If we write 0 at the right of 5, what does it make?

How many times 5 is 50? What does it do to a number to write a 0 at the right of it?

3. To multiply 7 by 10 what have we to do?

4. If we write *two* 0's at the right of 4, what does it make? How many times 4 is 400?

5. If we write *two* 0's at the right of 3, what does it make? 300 is how many times 3? What then does it do to a number to write two 0's at the right of it?

6. In 23 what order is the 3? What the 2? If now I annex two 0's, thus, 2300, what is the 3? How many times what it was before? What is the 2? How many times what it was before?

7. If I annex *three* 0's to 4, what does the 4 then represent? 4000 is how many times 4?

Principle IV.

59. *To multiply by* 10, 100, 1000, *or* 1 *with any number of* 0's *annexed, annex as many* 0's *to the multiplicand as there are in the multiplier.*

8. In this way multiply 7 by 10. By 100. By 1000. By 10000.

9. In the same way multiply 37 by 10. By 100. By 1000. By 10000. By 1000000.

FOURTH STEP.

To Multiply a Number represented by several Figures by another represented by One Figure.

1. 2 hundreds and 3 tens, and 4 units are the parts of what number?

Then by what principle will 3 times 2 hundreds, and 3 tens, and 2 units be 3 times 234?

3 times 2 hundreds, and 3 tens, and 4 units are how many hundreds, tens, and units?

Then 3 times 234 are how many?

2. 2 hundreds, 3 tens, and 4 units are the parts of what number?

Then by the **2d Principle**, how many are 2 times 234?

2 times 2 hundred are how many hundreds? 2 times 3 tens are how many tens? 2 times 4 units are how many units?

3. In like manner find how many 3 times 322 are?

Does it make any difference whether we take 3 times the 3 hundreds, then the tens, and then the units, or reverse the order? Try it both ways.

4. How many are 4 times 212?

Write the multiplier under the multiplicand as in the margin, and write the product underneath, thus:

$$\begin{array}{r} 212 \\ 4 \\ \hline 848 \end{array}$$

5. In like manner perform the following:

$$\begin{array}{r} 122 \\ 3 \\ \hline \end{array} \qquad \begin{array}{r} 241 \\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 123 \\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 121 \\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 321 \\ 2 \\ \hline \end{array}$$

6. Multiply 347 by 6.

$$\begin{array}{r} 347 \\ 6 \\ \hline \end{array}$$

Suggestions.—Writing the multiplier under the multiplicand, so that we may see them both at the same time conveniently, we observe that 6 times the 3 parts of which the multiplicand is composed make 18 hundreds, 24 tens, and 42 units. If now we were to take 6 times the 3 hundreds *first*, and write down this product, 18 hundreds, we should be obliged to change the figures when we came to add in 6 times the 4 tens, or 24 tens, for this makes 2 hundreds and 4 tens, so that we should have 20 hundreds instead of 18.

$$\begin{array}{r} 347 \\ 6 \\ \hline 18 \end{array}$$

If, however, we begin with the units part of the multiplicand, and take the parts in succession from the lowest order to the

highest, and if the product arising from multiplying any lower order makes any of the next higher order, we shall know it, and be able to add it as we go along, without the necessity of changing the figures after we have once written them. Thus 6 times 7 units are 42 units, or 4 tens and 2 units. Writing down the 2 units so as to remember them, we keep the 4 tens in mind to be added to the 6 times 4 tens. Now 6 times 4 tens are 24 tens, and adding the 4 tens which came from 6 times the 7 units, we have 28 tens, or 2 hundreds and 8 tens. As before, writing down the 8 tens and holding the 2 hundreds in mind, we take 6 times the 3 hundreds, and finding that it makes 18 hundreds, add the 2 hundreds which arose from the tens, and have 20 hundreds, or 2 thousands and 0 hundreds. Writing these down, we have the product of 6 times 347, since we have multiplied the parts of 347 by 6, and have added the results.

$$\begin{array}{r} 347 \\ 6 \\ \hline 2 \end{array} \qquad \begin{array}{r} 347 \\ 6 \\ \hline 82 \end{array} \qquad \begin{array}{r} 347 \\ 6 \\ \hline 2082 \end{array}$$

7. Multiply 587 by 4. *Prod.*, 2348.
8. Multiply 326 by 9.

To Multiply a Number represented by several Figures by another represented by One Figure.

60. Rule.—*Write the multiplier under the units figure of the multiplicand, and beginning with the units, multiply each figure of the multiplicand successively by the multiplier, observing that when any order is multiplied, only that part of the product which is of this order is to be written down, while the other part is to be held in mind to be added to the product arising from multiplying the next figure.*

Reasons.—There is no reason except custom for writing the multiplier *under* the multiplicand. It would be just as well to write it *over*. (Try it.) We do, however, want them so near each other that we can see them both at a glance, and so it is convenient to write one of them under the other. It is not necessary that the multiplier be written under the units of the multiplicand. The

process of multiplying gives the correct product, because we multiply all the parts of the multiplicand by the multiplier, and add the resulting products. Principle II. We begin at units to multiply, because by multiplying the lower orders first, we can discern how many the product of any lower order by the multiplier will make of the next higher order, and thus add it in as we go along, and not have to change our work.

9. Multiply 756 by 3.

10. Multiply 8027 by 5.

Perform the following:

11.	12.	13.	14.	15.	16.
382	5876	8546	502	246025	512704
6	4	7	9	5	8

17.	18.	19.	20.	21.	22.
648	1082	5050	73046	10708	980789
7	9	4	3	2	9

FIFTH STEP.

To Multiply a Number represented by several Figures by a Number represented by several Digits.

1. If we take a number 4 times, and 3 tens (30) times, and 2 hundreds times, and add the results, how many times shall we have taken the number? By what principle?

2. Since 3 tens, or 30, is 10×3, if we multiply a number by 3 and then this product by 10, how many times the number shall we have? By what *principle?* 10 times 3 times anything makes how many times that thing?

How do you multiply a number by 10? 4th *Principle.*

3. Since 2 hundreds, or 200, is 100×2, if we multiply

any number by 2 and then this product by 100, how many times the number shall we have? By what *Principle?* 100 times 2 times anything makes how many times that thing?

How do you multiply by 100? 4th *Principle.*

4. Let us now multiply 576 by 234.

Explanation.—As we have just seen, we want to take 4 times 576, 10 times 3 times 576, and 100 times 2 times 576, and add the results.

	576
	234
4 times 576 is	2304
3 times 576 is 1728, and 10 times this is . .	17280
2 times 576 is 1152, and 100 times this is . .	115200
Hence 234 times 576 is	134784

5. Multiply 497 by 342.

Explanation. 342 is 300 + 40 + 2. Hence we are to take 2 times 497, 40 times 497, and 300 times 497. To get 2 times 497, we multiply by 2. To get 40 times 497, we take 10 times 4 times 497. To get 300 times 497, we take 100 times 3 times 497.

	497
	342
2 times 497 is	994
4 times 497 is 1988, and 10 times this is	1988
3 times 497 is 1491, and 100 times this is	1491
Hence 342 times 497 is	169974

In this work we did not write the units figure of 19880, but wrote its tens, hundreds, thousands, and ten thousands in their proper places without the 0. So we neglected the two 0's of 149100, observing to write the digits in their proper orders without these 0's.

To Multiply a Number represented by several Figures by a Number represented by several Digits.

61. Rule.—*I. Write the multiplier under the multiplicand, so that the orders in the multiplier shall fall under like orders in the multiplicand.*

II. Beginning with the units of the multiplier, multiply the multiplicand by each of the figures of the multiplier in succession, observing to write the first figure in each product under the one by which you are multiplying.

III. Add these products.

Reasons.—We write the multiplier *under* the multiplicand as a matter of custom; it would do just as well to write it *above*. But we want both multiplier and multiplicand where we can see them at a glance.

We multiply first by the units also because it is customary. It is about as convenient to use the highest order in the multiplier first.

When we multiply by the tens figure, we get as many times the multiplicand as this figure indicates, and then by moving this product *one* place to the right, we multiply it by 10; thus we multiply the multiplicand successively by the factors of this part of the multiplier.

In like manner we multiply by the factors of the hundreds part of the multiplier, etc., according to Principles III. and IV.

Finally, having multiplied the multiplicand by the parts of the multiplier, we add these products together, and so have a number which is as many times the multiplicand as are indicated by the multiplier, by **Proposition I.**

5. Multiply 3587 by 5462.

These are often called partial products.

```
    3587
    5462
    ----
    7174
   21522
  14348
 17935
--------
19592194
```

62. Multiplication is the process of finding the product of two numbers by means of a knowledge of the *Multiplication Table.*

It is to be observed that there are *three* ways of finding how many a certain number of times a given number makes, viz., by counting, by adding, and by the process we call Multiplication. The Multiplication Table gives the product of each possible pair of the nine digits, and it is using these facts so as to find the product of any *two numbers* that we call Multiplication.

Examples for Practice.

1. Multiply 3456 by 74. *Prod.*, 255744.
2. Multiply 345 by 34.
3. Multiply 234 by 26.
4. Multiply 345 by 789. *Prod.*, 272205.
5. Multiply 1357908 by 144. *Prod.*, 195538752.
6. Multiply 543 by 254. *Prod.*, 137922.
7. Multiply 3407 by 682. *Prod.*, 2323574.
8. Multiply 781 by 23.
9. Multiply 5807 by 19. *Prod.*, 110333.
10. Multiply 7005 by 37. *Prod.*, 259185.
11. Multiply 850407 by 3789. *Prod.*, 3222192123.
12. Multiply 23456789 by 14. *Prod.*, 328395046.
13. Multiply 65432 by 15.
14. Multiply 8429638 by 7294. *Prod.*, 61485779572.
15. Multiply 7364951 by 888. *Prod.*, 6540076488.
16. Multiply 653842 by 18. *Prod.*, 11769156.
17. Multiply 603040 by 19.
18. Multiply 364812 by 71. *Prod.*, 25901652.
19. Multiply 482436 by 81.
20. Multiply 2468 by 91. *Prod.*, 224588.
21. Multiply 6739542 by 346. *Prod.*, 2331881532.
22. Multiply 72926495 by 4567.
23. Multiply 123456 by 16. *Prod.*, 1975296.
24. Multiply 437426 by 17. *Prod.*, 7436242.
25. Multiply 89764267 by 999. *Prod.*, 89674502733.
26. Multiply 46371674 by 49684. *Prod.*, 2303930251016.
27. Multiply 4364369 by 51. *Prod.*, 222582819.
28. Multiply 6937845 by 61. *Prod.*, 423208545.
29. Multiply 36598674 by 432. *Prod.*, 15810627168.
30. Multiply 46354897816 by 56843.

31. Multiply 6847 by 207.

Suggestion. 207 is 200 and 7. Hence we are to take 7 times 6847, which is 47929, and 100 times 2 times 6847; that is, 13694 multiplied by 100, or written two places to the left. Hence we see that the 0 in the multiplier makes no exception to the rule that we are to write the first figure of each partial product under the figure in the multiplier by which we multiply to produce it. As we have *no* tens in this example, we skip that order in multiplying.

$$\begin{array}{r} 6847 \\ \underline{207} \\ 47929 \\ \underline{13694} \\ 1417329 \end{array}$$

32. Multiply 30257 by 2305. *Prod.*, 69742385.
33. Multiply 587640 by 4008. *Prod.*, 2355261120.
34. Multiply 380900 by 301. *Prod.*, 114650900.
35. Multiply 4008 by 4008. *Prod.*, 16064064.
36. Multiply 808058 by 808058. *Prod.*, 652957731364.

SIXTH STEP.

When there are 0's at the right of either Multiplier or Multiplicand, or of both.

1. 2 times 4 hundreds are how many hundreds? 10 times 8 hundreds are how many hundreds?

2. Multiply 300 by 40.

Suggestions. 4 times 3 hundreds are 12 hundreds; and 10 times 12 hundreds are 120 hundreds, which is written 12000. Hence $40 \times 300 = 12000$.

To Multiply when there are 0's at the right of either the Multiplier or Multiplicand, or of both.

63. Rule.—*Neglect the 0's at first, and multiply as though there were none. To the product thus obtained*

annex as many 0's as there are at the right of both multiplier and multiplicand.

3. Multiply 38400 by 260.

Explanation. 38400 may be considered as 384 hundreds, and 260 as 10 times 26. Hence we may first take 260 times 384. This we can do by taking 26 times 384, which is 9984, and then 10 times this product. This gives 99840, which is 260 times 384 hundreds, or 99840 hundreds. This is written 9984000.

$$\begin{array}{r} 38400 \\ \underline{260} \\ 2304 \\ \underline{768} \\ 9984000 \end{array}$$

Queries.—Why could we neglect the two 0's in 38400? *Answer* Because we could remember that the 384 was hundreds without them. Why could we neglect the 0 at the right of 260? *Answer.* Because as we wished to multiply 384 hundreds by 260, we could do it by multiplying successively by the factors 26 and 10, and neglecting the 0 gives us the factor 26 to multiply by. Why do we annex three 0's to the product of 384 by 26? *Answer.* We annex one 0 to multiply by the factor 10 of 260; and the other two 0's because the result is hundreds.

4. Multiply 75800 by 5600.

5. Multiply 308000 by 1280.

6. Multiply 5864 by 300.

7. Multiply 280050000 by 80.

8. Multiply 5000 by 700.

Do this and the following without slate and pencil (mentally), and explain the operation.

9. Multiply 1200 by 40.

10. Multiply 11000 by 600.

11. Multiply 80 by 90. 60 by 50.

Explanation. 60 is 6 tens. We then wish to multiply 6 tens by 50. This can be done by multiplying successively by the factors 5 and 10. Thus 5 times 6 tens are 30 tens, and 10 times 30 tens are 300 tens. But 300 tens are 3000. Hence $50 \times 60 = 3000$.

Applications.

1. George bought 7 lemons at 6 cents each. How much did they cost?

If each lemon cost 6 cents, 7 lemons cost 7 times 6 cents, which is 42 cents.

2. If I pay $7 a cord for wood, how much will 3 cords cost me?

3. If I pay $6 a cord for wood, how much will 12 cords cost me? 23 cords?

If each cord costs $6, 23 cords will cost 23 times $6. But since 6 times 23 is the same as 23 times 6, I will use the 6 as the multiplier, it being more convenient.

Or, at $1 a cord 23 cords cost $23; but at $6 a cord they cost 6 times as much, or 6 times $23, which is $138.

4. At $8 a barrel, what do 5 barrels of flour cost? 7 barrels? 10 barrels? 30 barrels? 256 barrels? 3428 barrels? [Use the slate for the last two only.]

5. How much will 14 cords of wood cost at 550 cents a cord?

6. How much will 7 acres of land cost at $128 an acre? How much will 37 acres cost at the same rate?

7. A boy lived 4 miles from the village, and used to go there and back every day in the week except Sunday. How far did he travel in so doing in 1 week?

Ans., 48 miles.

8. Mary bought 13 yards of calico at 17 cents a yard, and 5 spools of thread at 6 cents a spool. How much did she pay for both? *Ans.*, 251 cents.

9. How many hundreds in 251? A dollar is 100 cents. How many dollars in 251 cents? How many cents besides?

2 dollars and 51 cents are written $2.51. As the $2 is 2 hundred cents, we can write $2.51, 251 cents. What is $5.31? How many cents? $15.45 are how many cents? 2347 cents are how many dollars and cents? How written? What is $341.20?

Since dollars are just *hundreds* of cents, we can multiply numbers representing dollars and cents the same as other numbers, and the tens and units of the product will be cents, and the other figures dollars.

10. At $5.37 a barrel, how much will 24 barrels of flour cost?

We do this thus:

$$\begin{array}{r} \$5.37 \\ 24 \\ \hline 2148 \\ 1074 \\ \hline \$128.88 \end{array}$$

11. At $3.75 a cord, how much will 58 cords of wood cost? 20 cords? 200 cords? *Answer to the last*, $750.

Use the slate for the first of these only.

12. If it costs on an average $2527 a mile to build a certain railroad, how much will it cost to build 288 miles?

Ans., $727776.

13. At $5.25 a yard, how much will 11 yards of cloth cost? 13 yards? 5 yards? 250 yards?

Answer to last, $1312.50.

14. I bought a work in 4 volumes, which cost $8.64 a volume. How much did the work cost?

15. There are 24 hours in one day, *i. e.*, a day and night. How many hours in a week? How many in 4 weeks? How many hours in a month of 30 days? A month of 31 days?

16. There are 60 minutes in an hour. How many minutes in a day (24 hours)?

17. There are 12 inches in a foot. How many inches in a yard, a yard being 3 feet?

18. There are 2 pints in a quart, and 8 quarts in a peck.

How many pints in a peck? There are 4 pecks in a bushel. How many pints in a bushel?

Answer to last, 64 pints.

19. At 15 cents a quart, how much does a bushel of strawberries cost?

20. At 13 cents a pound, how much do 3 pounds of beefsteak cost? 5 pounds? 7 pounds? 2 pounds?

---

21. A drover bought 256 horses at an average of $125 each, 347 oxen at $87 each, and 250 sheep at $3 each. How much did all cost him? *Ans.,* $62939.

22. A has 395 acres of land, worth $27 an acre; and B has 493 acres, worth $19 an acre. What is the value of both of their farms? *Ans.,* $20032.

23. A merchant bought 12 boxes of linen, each containing 25 pieces, and each piece containing 36 yards, at 65 cents a yard. How many pieces and how many yards did he buy, and how many dollars did it all cost him?

Ans., $7020.

24. A merchant bought 25 pieces of broadcloth, each piece containing 48 yards, at 9 dollars a yard. How much did he pay for the whole?

25. If a steamship can sail 18 miles in 1 hour, how far can she sail in 34 days of 24 hours each?

26. John bought 3 pencils at 7 cents each, and handed the salesman 25 cents. How much change must he receive?

27. Bought 3 oranges at 8 cents each, and a melon at 12 cents, and handed the grocer 50 cents. How much change must he give me back?

28. A woman brought 27 dozen eggs and 37 pounds of

butter to market. She sold her eggs at 17 cents a dozen, and her butter at 32 cents a pound. She bought 1 pound of tea at \$1.25, 5 pounds of sugar at 15 cents a pound, and 8 pounds of coffee at 39 cents. How much more did she get for her eggs and butter than her tea, sugar, and coffee cost her? *Ans.*, \$11.31.

29. There is an orchard consisting of 126 rows of trees, and in each row are 109 trees. How many apples in the orchard, allowing an average of 1007 on a tree?

Ans., 13830138 apples.

30. A certain state contains 50 counties; each county, 35 towns; each town, 300 houses, and each house, 8 persons. What is the population of the state?

Ans., 4200000.

31. A man bought a farm of 450 acres for \$75 an acre, and spent \$2500 in improvements. He then sold it at \$80 an acre. Did he gain or lose? How much?

Ans., He lost \$250.

32. A farmer purchased a farm of 325 acres, at 45 dollars per acre, and made a payment of 875 dollars. In order to make another payment, he sold 75 acres at 58 dollars per acre. How much did he owe on his farm after the second payment was made? *Ans.*, 9400 dollars.

33. If a young man's salary is 600 dollars per year, of which he takes 45 dollars to purchase books, and 300 dollars for board and other expenses, how much money will he have at the end of 7 years? *Ans.*, 1785 dollars.

34. At 55 dollars per ton, what will the rails for a railroad cost, if the road is 75 miles long, and it takes 112 tons per mile? *Ans.*, 462000 dollars.

35. It has been found by experiment that a sheep which is fed in the open air, consumes 1912 pounds of turnips

from November 18 to March 9, and that a sheep of the same size, fed under a shed, consumes, during the same time, 1394 pounds of turnips. How many pounds of turnips would a farmer save in a single winter by feeding 345 sheep under a shed, instead of in the open air?

Ans., 178710 pounds.

36. A man owned 4376 acres of land; he sold 468 acres at $19 per acre. What is the remainder worth at $20 per acre? *Ans.*, $78160.

37. Henry earned $46 at one time and $27 at another time; his father then gave him three times as much as he had earned. How much did his father give him?

Ans., $219.

38. A hardware merchant bought at one time 250 pounds of tin, at another 345 pounds, and at another 562 pounds, and paid for it all $462.80. He afterward sold it all at 45 cents a pound. How much did he receive for it? *Ans.*, $520.65.

39. A merchant bought 275 yards of cloth, at 6 dollars per yard, of which he has sold 133 yards, at 10 dollars per yard. What would he gain on the whole by selling the remainder at 9 dollars per yard? *Ans.*, 958 dollars.

40. If a wagon cost $48, a yoke of oxen 3 times as much, lacking $54, and a span of horses as much as the wagon and oxen together, what was the cost of the oxen and horses respectively, and of all?

[The TEACHER'S HAND-BOOK contains nearly 200 additional exercises in Multiplication, from which the Teacher can draw for class drill.]

SECTION VII.

DIVISION.

FIRST STEP.

Definitions, and how to make the Division Table.

1. How many apples are there on the table in the picture? How many groups are there? How many in each group? How many 3's in 12?

2. 3 times 2 are how many? 3 2's make how many? How many 2's in 6?

3. How many 5's in 15? This is the same as asking how many times 5 it takes to make 15.

4. How many 9's in 36? Why?

Answer. There are 4 9's in 36, because 4 times 9 are 36.

5. If 12 apples are separated into 4 equal groups, how many are there in each group?

6. If 15 is separated into 3 equal parts, what is one of the parts? Do 3 2's, 3 3's, 3 4's, or 3 5's make 15?

7. If 18 is separated into 6 equal parts, how many are there in each part? 6 times what number makes 18?

64. Division is a process of finding how many times one number is contained in another.

Division enables us also to separate a number into any number of equal parts, and find how many there are in one of these parts.

65. Separating a number into equal parts, or finding how many times another number is contained in it, is called *Dividing* it by the number expressing the number of equal parts, or by the number which is contained in it.

66. The number to be divided is called the **Dividend.**

The number by which the dividend is divided is called the **Divisor.**

The number which tells how many times the divisor is contained in the dividend, or what is one of the required equal parts of the dividend, is called the **Quotient.**

8. How many times is 7 contained in 21? That is, how many 7's make 21?

Which of these numbers is the *Dividend?*

Which the *Divisor?*

What is the *Quotient?*

9. What is one of the 5 equal parts into which 30 can be divided? That is, 5 times what number makes 30?

Which number is the *Dividend?*

Which the *Divisor?*

What is the *Quotient?*

10. Divide 27 by 9; that is, find how many 9's there are in 27.

11. How many times is 6 contained in 42? That is, how many 6's make 42?

12. How many times is 4 contained in 24? In 32? *In 36?* In 16? In 8?

To Find what the Quotient is when neither Divisor nor Quotient exceeds 9.

67. Rule.—*Consider, from your knowledge of the Multiplication Table, how many times the given divisor must be taken in order to make the given dividend.*

68. The sign ÷ is the sign of division, and is read "divided by." Thus 24 ÷ 6 is read "24 divided by 6." The dividend is written before the sign and the divisor after it.

13. Copy on your slate the following and fill the blanks:

7 times —— = 28; therefore 28 ÷ 7 = ——.
4 times —— = 20; therefore 20 ÷ 4 = ——.
3 times —— = 18; therefore 18 ÷ 3 = ——.
6 times —— = 18; therefore 18 ÷ 6 = ——.

14. Copy on your slate and fill out the following:

$3 \times 5 =$ —; therefore $15 \div 3 =$ —, and $15 \div 5 =$ —.
$7 \times 8 =$ —; therefore $56 \div 7 =$ —, and $56 \div 8 =$ —.
$6 \times 9 =$ —; therefore $54 \div 6 =$ —, and $54 \div 9 =$ —.
$7 \times 5 =$ —; therefore $35 \div 5 =$ —, and $35 \div 7 =$ —.
$6 \times 7 =$ —; therefore $42 \div 6 =$ —, and $42 \div 7 =$ —.

15. How many times is 1 contained in 6? That is, how many 1's in 6?

16. What is 3 ÷ 1? 7 ÷ 1? 8 ÷ 1? 1 ÷ 1? 2 ÷ 1? 9 ÷ 1? 4 ÷ 1?

69. As multiplication is finding the product when the two factors are given, so division is finding one of the factors when the product and the other factor are given. The Dividend is the Product of the Quotient and Divisor.*

17. If one of the factors of 24 is 4, what is the other? What is 24 ÷ 6?

If one of the factors of 24 is 4, what is the other? What is 24 ÷ 4?

* This statement is absolutely correct when the division is complete, and it is not thought advisable to confuse the pupil by introducing the idea of a remainder at this stage of his progress.

18. Copy on your slate and fill out the following

DIVISION TABLE.

When 2 is one factor.

$2 \times 2 = \text{—}.\ \therefore \{\ 4 \div 2 = \text{—}.$

$2 \times 3 = \text{—}.\ \therefore \begin{cases} 6 \div 2 = \text{—}. \\ 6 \div 3 = \text{—}. \end{cases}$

$2 \times 4 = \text{—}.\ \therefore \begin{cases} 8 \div 2 = \text{—}. \\ 8 \div 4 = \text{—}. \end{cases}$

$2 \times 5 = \text{—}.\ \therefore \begin{cases} 10 \div 2 = \text{—}. \\ 10 \div 5 = \text{—}. \end{cases}$

$2 \times 6 = \text{—}.\ \therefore \begin{cases} 12 \div 2 = \text{—}. \\ 12 \div 6 = \text{—}. \end{cases}$

$2 \times 7 = \text{—}.\ \therefore \begin{cases} 14 \div 2 = \text{—}. \\ 14 \div 7 = \text{—}. \end{cases}$

$2 \times 8 = \text{—}.\ \therefore \begin{cases} 16 \div 2 = \text{—}. \\ 16 \div 8 = \text{—}. \end{cases}$

$2 \times 9 = \text{—}.\ \therefore \begin{cases} 18 \div 2 = \text{—}. \\ 18 \div 9 = \text{—}. \end{cases}$

When 3 is one factor.

$3 \times 3 = \text{—}.\ \therefore \{\ 9 \div 3 = \text{—}.$

$3 \times 4 = \text{—}.\ \therefore \begin{cases} 12 \div 3 = \text{—}. \\ 12 \div 4 = \text{—}. \end{cases}$

$3 \times 5 = \text{—}.\ \therefore \begin{cases} 15 \div 3 = \text{—}. \\ 15 \div 5 = \text{—}. \end{cases}$

$3 \times 6 = \text{—}.\ \therefore \begin{cases} 18 \div 3 = \text{—}. \\ 18 \div 6 = \text{—}. \end{cases}$

$3 \times 7 = \text{—}.\ \therefore \begin{cases} 21 \div 3 = \text{—}. \\ 21 \div 7 = \text{—}. \end{cases}$

$3 \times 8 = \text{—}.\ \therefore \begin{cases} 24 \div 3 = \text{—}. \\ 24 \div 8 = \text{—}. \end{cases}$

$3 \times 9 = \text{—}.\ \therefore \begin{cases} 27 \div 3 = \text{—}. \\ 27 \div 9 = \text{—}. \end{cases}$

When 4 is one factor.

$4 \times 4 = \text{—}.\ \therefore \{\ 16 \div 4 = \text{—}.$

$4 \times 5 = \text{—}.\ \therefore \begin{cases} 20 \div 4 = \text{—}. \\ 20 \div 5 = \text{—}. \end{cases}$

$4 \times 6 = \text{—}.\ \therefore \begin{cases} 24 \div 4 = \text{—}. \\ 24 \div 6 = \text{—}. \end{cases}$

$4 \times 7 = \text{—}.\ \therefore \begin{cases} 28 \div 4 = \text{—}. \\ 28 \div 7 = \text{—}. \end{cases}$

$4 \times 8 = \text{—}.\ \therefore \begin{cases} 32 \div 4 = \text{—}. \\ 32 \div 8 = \text{—}. \end{cases}$

$4 \times 9 = \text{—}.\ \therefore \begin{cases} 36 \div 4 = \text{—}. \\ 36 \div 9 = \text{—}. \end{cases}$

When 5 is one factor.

$5 \times 5 = \text{—}.\ \therefore \{\ 25 \div 5 = \text{—}.$

$5 \times 6 = \text{—}.\ \therefore \begin{cases} 30 \div 5 = \text{—}. \\ 30 \div 6 = \text{—}. \end{cases}$

$5 \times 7 = \text{—}.\ \therefore \begin{cases} 35 \div 5 = \text{—}. \\ 35 \div 7 = \text{—}. \end{cases}$

$5 \times 8 = \text{—}.\ \therefore \begin{cases} 40 \div 5 = \text{—}. \\ 40 \div 8 = \text{—}. \end{cases}$

$5 \times 9 = \text{—}.\ \therefore \begin{cases} 45 \div 5 = \text{—}. \\ 45 \div 9 = \text{—}. \end{cases}$

When 6 is one factor.

$6 \times 6 = \text{—}.\ \therefore \{\ 36 \div 6 = \text{—}.$

$6 \times 7 = \text{—}.\ \therefore \begin{cases} 42 \div 6 = \text{—}. \\ 42 \div 7 = \text{—}. \end{cases}$

$6 \times 8 = \text{—}.\ \therefore \begin{cases} 48 \div 6 = \text{—}. \\ 48 \div 8 = \text{—}. \end{cases}$

$6 \times 9 = \text{—}.\ \therefore \begin{cases} 54 \div 6 = \text{—}. \\ 54 \div 9 = \text{—}. \end{cases}$

When 7 is one factor.

$7 \times 7 = \text{—}.\ \therefore \{\ 49 \div 7 = \text{—}.$

$7 \times 8 = \text{—}.\ \therefore \begin{cases} 56 \div 7 = \text{—}. \\ 56 \div 8 = \text{—}. \end{cases}$

$7 \times 9 = \text{—}.\ \therefore \begin{cases} 63 \div 7 = \text{—}. \\ 63 \div 9 = \text{—}. \end{cases}$

When 8 is one factor.

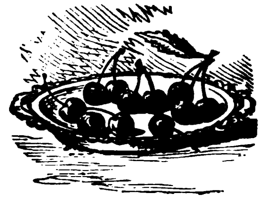
$8 \times 8 = \text{—}.\ \therefore \{\ 64 \div 8 = \text{—}.$

$8 \times 9 = \text{—}.\ \therefore \begin{cases} 72 \div 8 = \text{—}. \\ 72 \div 9 = \text{—}. \end{cases}$

When 9 is one factor.

$9 \times 9 = \text{—}.\ \therefore \{\ 81 \div 9 = \text{—}.$

This table must be so well learned that any quotient can be given at once. But if multiplication has been well learned, this will cost little labor.



1. How many cherries in this dish? If you take out 3 at a time, how many times can you take? How many will remain after you have taken out 3 3's?

If you take 4 at a time, how many times can you take? How many will remain?

How many 4's in 11, and how many over?

How many 5's in 11, and how many remain? How many 2's?

2. How many 3's in 8, and how many over? 3 is contained in 8 how many times, and leaves what remainder?

3. $15 \div 4 =$ how many, and what remainder? Does it take all of 15 to contain 4 3 times?

70. When the entire divisor has been taken out of the dividend as many times as possible, if anything is left it is called the **Remainder.***

If there is no remainder, the division is said to be *exact.*

4. How many times is 3 contained in 17, and what is the remainder?

5. Fill the blanks in the following:

$23 \div 4 =$ ——, and —— remainder.
$19 \div 6 =$ ——, and —— remainder.
$47 \div 7 =$ ——, and —— remainder.
$50 \div 8 =$ ——, and —— remainder.
$43 \div 5 =$ ——, and —— remainder.
$28 \div 4 =$ ——, and —— remainder.

* If thought desirable, the teacher can explain the significance of the term *Remainder* when division is conceived to be a separation of a number into equal parts. It is not thought best to cumber the definition with both ideas.

SECOND STEP.

Three Fundamental Principles.

1. How many 3's are there in 12 ?

Then how many 3's are there in 2 12's, or 24 ?

How many in 3 12's, or 36 ?

How many in 10 12's, or 120 ?

2. How many 4's are there in 20 ?

Then how many 4's are there in 40 ? In 60 ?

How many 4's in 10 20's, or 200 ?

3. How many times is 2 contained in 6 ?

Then how many times is 2 contained in twice 6, or 12 ? In 4 times 6 ? In 10 times 6 ?

4. How many times is 3 contained in 9 ?

Then how many times is 3 contained in 900, which is 100 times 9 ? In 9000, which is 1000 times 9 ?

Principle I.

71. *Whatever number of times a given divisor is contained in a given dividend, this divisor is contained in twice this dividend, twice as many times; in 3 times this dividend, 3 times as many times; in 10 times this dividend, 10 times as many times, etc.*

5. How many times is 4 contained in 80 ? In 800 ? In 8000 ?

First, 4 is contained in 8 2 times. Hence it is contained in 80 10 times 2, or 20 times, since 80 is 10 times 8. In 800 4 is contained 100 times 2, or 200 times, since 800 is 100 times 8, etc.

6. How many times is 5 contained in 1500 ? In 15000 ? In 150000 ?

7. How many times is 6 contained in 42 ? Then in 420 ? In 4200 ? In 42000 ? In 42000000 ?

1. How many 2's are there in 4? How many in 8? Then how many in 4 and 8 together, or 12?

2. How many 2's in 6? How many in 8? How many in 12? Then how many in 6 + 8 + 12, or 26?

3. How many times is 4 contained in 20? In 8? Then in 28?

4. 6, 9, and 12 are the parts of what number? How many times is 3 contained in each of these parts? Then how many times in all the parts together?

Principle II.

72. *We may find how many times a given divisor is contained in a given dividend, by finding how many times it is contained in each of the parts of the dividend and adding the results together.*

5. 10 and 8 are the parts of what number? 2 is contained in 10 how many times? In 8 how many times? Then in 10 + 8, or 18?

6. 846 are 800, 40, and 6. Find how many times 2 is contained in 846, according to the two principles just explained.

Suggestion. 2 is contained in 8 4 times, and in 800 100 times 4 times, or 400 times, *Principle I.*

2 is contained in 4 2 times, and in 40 10 times 2, or 20 times, *Principle I.*

2 is contained in 6 3 times.

Hence 2 is contained in 846 400 + 20 + 3, or 423 times.

7. Find, as in the last, how many times 3 is contained in 1296, that is, in 1200, 90, and 6.

8. How many times is 4 contained in 1284? In 844? In 888? In 444? In 1688?

1. Here are 4 bunches of grapes, with 11 grapes in each bunch. If you take 3 grapes from a bunch as many times as you can, how many times will it be, and how many will be left? If you do so with each bunch, how many times will you have taken 3 grapes, and how many will be left in all?

3 is contained in 11 3 times, and 2 remainder. Then in 4 times 11, it is contained 4 times as many times, with 4 such remainders.

2. How many times is 5 contained in 23, and how many remain? Then how many times is 5 contained in 7 times 23, and how many remain?

Principle III.

73. *If a given divisor is contained in any dividend a certain number of times with a certain remainder, it is contained in* 2 *times that dividend* 2 *times as many times with* 2 *times as great a remainder, in* 3 *times that dividend* 3 *times as many times with* 3 *times as great a remainder, in* 10 *times as great a dividend* 10 *times as many times with* 10 *times as great a remainder, etc.**

3. How many times is 7 contained in 25, and with what remainder? In 6 times 25, with what remainder? In 10 times 25, etc.? In 100 times 25, etc.?

Answer to last, 3 hundred times and 4 hundred remainder.

---

* Special pains should be taken to secure a clear conception of this principle. It is *absolutely essential* as a foundation for intelligent work in division.

4. $37 \div 7 = 5$ and 2 remainder. Then how many times is 7 contained in 37 tens, and with what remainder? In 37 hundreds? In 37 thousands?

THIRD STEP.

Short Division.

1. Divide 537 by 2.

$$\begin{array}{r} 2)\underline{537}-1 \\ 268 \end{array}$$

Explanation.—In order that we may see both at once, conveniently, we write the divisor on the left of the dividend.

2 is contained in 5 2 times, with 1 remainder. Then by *Principle III.* it is contained in 5 hundreds, 2 hundreds times, with 1 hundred remainder. The 1 hundred remainder is 10 tens, which with the 3 tens make 13 tens. 2 is contained in 13 6 times, with 1 remainder. Then by *Principle III.* it is contained in 13 tens 6 tens times, with 1 ten remainder. The 1 ten remaining is 10 units, which with the 7 units make 17 units. 2 is contained in 17 units 8 times, and 1 remainder.*

To Divide by the Method called Short Division.

74. Rule.—*I. Write the divisor on the left of the dividend, and beginning with the highest order, or orders, which regarded as units will contain the divisor, divide and write the quotient figure underneath the lowest order thus used, remembering the remainder.*

II. Proceeding to the next lower order, divide it together with what was left of the higher, if there was a remainder, and write the quotient thus arising under this last order used. Proceed in this manner until all the orders in the dividend have been divided.

---

* It may be profitable to illustrate this operation by means of the packages of counters. It makes a very pretty exercise, and puts the truth in concrete form.

III. If at any time after the first quotient figure has been written, the divisor is not contained in the next lower order, together with what is brought to it from the higher, write 0 in the quotient, and proceed to the next lower order.

Reasons.—The divisor is written at the left of the dividend, simply that we may be able to see both at once conveniently.

We begin at the highest order to divide, because by so doing we can put what remains after each division into the next lower order and divide it; and thus we get all there is of any order in the quotient as we go along.*

We write the quotient figures under the orders from whose division they arise, because they are of the same orders.

The process ascertains how many times the divisor is contained in the dividend by finding how many times it is contained in the parts of the dividend and adding the results, *Principle II.* This can be readily illustrated by an example. For this purpose let us divide 1547 by 4.

ANALYSIS OF OPERATION.

1547 =	12 hds.	In this 4 is contained 3 hds, or	300 times.
	32 tens.	In this 4 is contained 8 tens, or	80 times.
	24 } units.	In this 4 is contained 6 units, or	6 times.
	and 3 } units.	In this 4 is contained *no* times.	

∴ In 1547 4 is contained . . . . 386 times, with a remainder 3.

2. Divide 17834 by 7.

Operation.

$$7\,)\,\underline{17834}-5$$
$$2547$$

---

* On page 89, *Ex.* 13, is an example solved by beginning with the lowest order. Such illustrations might be introduced here by the teacher.

Perform the following, explaining the process:

3.	4.	5.	6.	7.
2) 1349	3) 736	5) 2483	6) 3486	7) 268905

8. Divide 4238 by 7.

$$7\,)\,\underline{4238} - 3$$
$$605$$

Suggestion.—We observe that 7 is contained in 42 (hundreds) 6 (hundreds) times, with no remainder. Now 7 is not contained in 3 (tens) any (*tens*) times; so we write a 0 in the tens place in the quotient to mark the vacant order, and unite the 3 tens with the 8 units, making 38 units. In this 7 is contained 5 times, with a remainder 3.

Perform the following, explaining the process:

9.	10.	11.	12.
5) 25200	9) 723856 — 4	8) 5462	8) 1768162
5040	80428		

Examples for Practice.

1. 5462 ÷ 2; by 5; by 7.
2. 1256 ÷ 3; by 4; by 9.
3. 10702 ÷ 5; by 6; by 8.
4. 78520 ÷ 7; by 9; by 4.
5. 653426 ÷ 2; by 3; by 5.
6. 827001 ÷ 4; by 6; by 7.
7. 10000 ÷ 2; by 4; by 6; by 8.
8. 10000 ÷ 3; by 5; by 7; by 9.
9. 20003 ÷ 5; by 8; by 6; by 2.
10. 102504 ÷ 3; by 7; by 8; by 9.
11. 101010 ÷ 8; by 4; by 2; by 6.
12. 202020 ÷ 4; by 5; by 7; by 3.

13. Divide 1547 by 4, dividing the *lower orders first,* all the time, and see how much advantage there is in the method we have learned; that is, in beginning with the highest order.

Operation.

	4) 1547	111	*1st Quot.*
1st Rem.,	1103	200	*2d Quot.*
2d Rem.,	303	70	*3d Quot.*
3d Rem.,	23	5	*4th Quot.*
True Rem.,	3	386	*Entire Quotient.*

Observe that the principles are the same as in the common method.

FOURTH STEP.

Two Ways of Disposing of the Remainder.

1. Mr. Mason has 15 apples, which he wishes to divide equally among 4 boys. How can he do it?

Answer.—First, he can give each boy 3 apples, which will take 12 apples; and then he will have 3 apples left to be divided. He can now cut each of these three apples into 4 equal parts, called fourths, or quarters. Then he can give to each of the boys one-fourth of each of the 3 remaining apples, or 3-fourths of an apple. Thus each boy will have 3 apples and 3-fourths of an apple.

2. When any thing is divided into *two* equal parts, what is *one* of the parts called? When into 3 equal parts? Four? Five? Six?

Ans., 1-half; 1-third; 1-fourth; 1-fifth; 1-sixth.

3. What do you call 3 of the 7 equal parts into which an apple may be divided? 8 of the 11 equal parts? 11 of the 20 equal parts?

75. One half is written $\frac{1}{2}$; 1-third $\frac{1}{3}$; 1-fourth $\frac{1}{4}$; 1-seventh $\frac{1}{7}$; 1-eleventh $\frac{1}{11}$; 1-twenty-first $\frac{1}{21}$, etc.

Two-thirds are written $\frac{2}{3}$; 2-fourths $\frac{2}{4}$; 3-fifths $\frac{3}{5}$; 5-elevenths $\frac{5}{11}$, etc.

76. Numbers which are written in this way, and which represent one or more of the equal parts into which a unit, or some number taken as a whole, is conceived to be divided, are called **Fractions.** We shall have much to learn about them by and by.

4. Tell what each of the following expressions means: $\frac{3}{7}$; $\frac{4}{5}$; $\frac{9}{11}$; $\frac{16}{17}$; $\frac{4}{8}$; $\frac{1}{10}$; $\frac{5}{10}$; $\frac{23}{46}$.

77. In such expressions the number below the line, which tells into how many equal parts the division is made, is called the **Denominator.** The number above

the line, which tells how many of the equal parts are represented by the expression, is called the **Numerator.**

78. Such an expression as $4\frac{3}{5}$ is read "4 and $\frac{3}{5}$." $12\frac{7}{8}$ is read "12 and $\frac{7}{8}$." $143\frac{11}{15}$ is read "143 and $\frac{11}{15}$." $13\frac{2}{3}$ means 13 and $\frac{2}{3}$, etc.

5. Divide 38 by 5.

Explanation. 38 divided by 5 is $7\frac{3}{5}$; for 5 is contained in 38 7 times, with a remainder 3. Now to divide 3 into 5 equal parts, we can consider each one of the 3 as divided into 5 parts, and then 1 taken from each. Thus we shall have $\frac{3}{5}$.

$$5\overline{)38}$$
$$7\frac{3}{5}$$

Or, we may explain the dividing of 3 by 5 thus: 1 divided into 5 equal parts, gives $\frac{1}{5}$ as one of the parts, or the quotient, and 3 gives 3 times $\frac{1}{5}$, or $\frac{3}{5}$.

6. What is 2 divided by 3? 5 divided by 7? 13 divided by 17? Explain as above.

79. *In dividing when the division is not exact, we may write the divisor under the remainder, thus forming a fraction, and annex this to the integral quotient.*

Perform the following, disposing of the remainder as above, and giving the explanation:

7. $39 \div 4 = 9\frac{3}{4}$.	11. $417 \div 5$.	15. $1000 \div 7$.
8. $52 \div 6 =$?	12. $3428 \div 6$.	16. $2015 \div 4$.
9. $18 \div 7 =$?	13. $5000 \div 3$.	17. $18276 \div 9$.
10. $143 \div 2 = 71\frac{1}{2}$.	14. $20 \div 6$.	18. $41287 \div 8$.

We will now give another way of disposing of the remainder, which is much used.

1. Divide 347 by 8.

Explanation.—Having divided 347 by 8, we find a remainder 3. Now if we divide each of these 3 into 10 equal parts, or tenths, there will be 30 of them. 30

$$8\overline{)347}$$
$$43.875$$

tenths divided by 8 gives 3 tenths, and 6 tenths remaining. Again, dividing each of the 6 tenths into 10 equal parts, the parts will be hundredths, and there will be 60 of them. Dividing 60 hundredths by 8, we have 7 hundredths, and 4 hundredths remaining. 4 hundredths make 40 thousandths; and 40 thousandths divided by 8 gives 5 thousandths. These tenths, hundredths, and thousandths we separate from the integral part of the quotient by a dot (.) called a *Decimal Point.*

In this process we multiply each remainder by 10, and as this is done by annexing a 0 (**60**), we have the following rule:

80. *When the division is not exact, we can place a point after the integral part of the quotient, annex a* 0 *to the remainder, and divide this result. To this remainder annex a* 0, *and divide again, till the work terminates, or till we have gone as far as we wish. The orders thus obtained at the right of units are tenths, hundredths, thousandths, etc.*

81. It will be seen that these numbers are a kind of *Fractions.* They are called **Decimal Fractions,** or simply *Decimals.* We shall have more about them by and by.

82. .3 is 3-tenths; .4 is 4-tenths; .05 is 5-hundredths; .006 is 6-thousandths, etc. .37 is 37-hundredths, since it is 3-tenths and 7-hundredths, and 3-tenths make 30 hundredths. .375 is 375-thousandths, because 3-tenths and 7-hundredths make 37-hundredths, and 37-hundredths are 370-thousandths, which, with the 5-thousandths, make 375-thousandths.

2. Read the following and tell what they mean: .3; .05; .38; .072; .135; 4.2; 5.23; 42.26; 12.346.

Perform the following, carrying out the division to three places of decimals, if it does not terminate before:

3. $26 \div 8 = 3.25$	9. $1342 \div 4$.
4. $132 \div 8 = 16.5$	10. $521 \div 5$.
5. $346 \div 7 = 49.428 +$ *	11. $1376 \div 9$.
6. $3471 \div 2 =$?	12. $8156 \div 8$.
7. $581 \div 6 =$?	13. $417 \div 2$.
8. $5312 \div 3 =$?	14. $1826 \div 3$.

FIFTH STEP.

To Find how many times a Large Divisor is contained in a Dividend less than 10 times itself.

1. How many times are 2 tens contained in 6 tens; that is, 20 in 60?

2. How many times are 3 hundreds contained in 12 hundreds? What is $1200 \div 300$?

3. How many times is 400 contained in 2400? In 1200? In 3200?

4. How many times is 40 contained in 80? that is, 4 tens in 8 tens? 40 in 120? 60 in 180? 70 in 210?

5. How many times is 23 contained in 74, and how many remain?

2 tens are contained in 7 tens how many times?

3 times 23 are how many?

$74 - 69 =$ how many?

6. How many times is 346 contained in 872, and how many remain?

How many *hundreds* in this divisor?

How many *hundreds* in this dividend?

How many times are 3 hundreds contained in 8 hundreds?

---

* This + sign means that the division does not terminate here.

How many are 2 times 346? Then is 346 contained in 872 2 times? What remainder?

7. How many times is 257 contained in 1143?

How many hundreds in the divisor? How many in the dividend? $11 \div 2 =$ how many?

Is 257 contained in 1143 5 times? How many are 5 times 257? How many are 4 times 257?

83. To ascertain how many times one number is contained in a greater number represented by the same number of figures, divide the highest order in the dividend by the highest order in the divisor. Then multiply the divisor by this quotient, and see if the product is greater than the dividend. If it is, try the next lower number, multiplying the divisor by it; and so continue to try lower and lower numbers till one is found which, when multiplied into the divisor, does not give a product greater than the dividend.

When the divisor is not contained in an equal number of figures from the left of the dividend, and the dividend has one more figure than the divisor, divide the first *two* left hand figures in the dividend by the first *one* in the divisor, and then test this quotient by multiplying the divisor by it, as in the former case.*

8. How many times is 342 contained in 927?

Suggestion. 3 is contained in 9 3 times; hence we try 3. 3 times 342 are 1026. Thus we see that 3 is too large. Then we try 2. 2 times 342 are 684. So we learn that 342 is contained in 927 2 times; and by multiplying 342 by 2, and

$$\begin{array}{r} 927 \\ 684 \\ \hline 243 \end{array}$$

* This is one of the greatest obstacles the young learner encounters in division, and he should be made *practically* familiar with this process before attempting *Long Division*. It may not be best to require him to memorize this article; but ***he must know the process.***

subtracting the product, 684, from 927, we find the remainder to be 243.

We also see, from this remainder, that 342 will not go another time in 927, for the 243 is less than 342.

9. How many times is 278 contained in 956, and with what remainder?

You will naturally do this thus: Finding that 2 is contained in 9 4 times, you will try 4, by multiplying 278 by 4. But you find this makes 1112, which is greater than 956. Hence 278 is not contained in 956 4 times. You would then *erase* the 4 and the 1112, and try 3. This would give you the work in the margin.

```
278) 956 (4
    1112
    ----

278) 956 (3
     834
     ---
     122
```

10. How many times is 468 contained in 983, and what remainder?

11. How many times is 782 contained in 2436, and what remainder?

Can a final remainder be as great as the divisor? Why not?

12. Find the quotients and remainders in the following:

78 ÷ 23;	375 ÷ 87;	1141 ÷ 248;
56 ÷ 16;	1248 ÷ 581;	2248 ÷ 240;
91 ÷ 37;	3853 ÷ 826;	5670 ÷ 683;
178 ÷ 41;	7048 ÷ 6341;	12457 ÷ 2846;
563 ÷ 87;	1124 ÷ 357;	58582 ÷ 9341;
782 ÷ 93;	8888 ÷ 999;	6512 ÷ 814;
341 ÷ 84;	7156 ÷ 734;	6624 ÷ 736;
1206 ÷ 75;	1542 ÷ 178;	26999 ÷ 3857;
847 ÷ 87;	2159 ÷ 261;	10458 ÷ 1743;
7643 ÷ 785;	1471 ÷ 493;	20006 ÷ 2858;
6387 ÷ 692;	562 ÷ 58;	7580 ÷ 879.

SIXTH STEP.

Long Division.

1. Divide 73482 by 214.

Explanation.—The principles used in this process are just the same as those in *Short Division*, the only difference being that the divisor is so large that it is not convenient to find out the several remainders without writing down the product of the divisor by each quotient figure as we obtain it, and performing the subtraction. Hence we write the quotient at the right of the dividend, that it may not be in our way.

$$\begin{array}{r l}
214\,)\,73482\,(\,343 & \textit{Quot.} \\
642 & \\
\hline
928 & \\
856 & \\
\hline
722 & \\
642 & \\
\hline
80 & \textit{Rem.}
\end{array}$$

214 is contained in 734 3 times, with a remainder 92; hence it is contained in 734 *hundreds*, 3 *hundreds* times, with a remainder 92 *hundreds*. (*Principle I.*)

Again, 92 hundreds and 8 tens make 928 tens. 214 is contained in 928 4 times, with a remainder 72; hence it is contained in 928 *tens* 4 *tens* times, with a remainder 72 *tens*.

Finally, 72 tens and 2 units make 722 units; and 214 is contained in 722 3 times, with a remainder 80.

Thus we have found how many times 214 is contained in 73482, by finding how many times it is contained in the parts of 73482, and adding the quotients thus obtained. (*Principle II.*) This may be exhibited at one view, thus:

$$73482 = \begin{cases} 642 \ \textit{hds.} & \text{In this 214 is contained 3 } \textit{hds.}\text{, or 300 times.} \\ 856 \ \textit{tens.} & \text{In this 214 is contained 4 } \textit{tens}\text{, or 40 times.} \\ 642 \ \textit{units.} & \text{In this 214 is contained 3 } \textit{units}\text{, or 3 times.} \\ 80 \ \textit{units.} & \text{In this 214 is contained } \textit{no} \text{ times.} \end{cases}$$

$\therefore$ In 73482 . 214 is contained . 343 times, with a remainder 80.

To Divide by the Method called Long Division.

84. Rule.—*I. Write the divisor at the left of the dividend, and the quotient as it is obtained at the right.*

II. Seek how many times the divisor is contained in the fewest of the left hand figures of the dividend which will contain it, and write the number of times as the highest order in the quotient.

III. Multiply the divisor by this quotient figure, and writing the product under the part of the dividend used, subtract it therefrom. Annex to this remainder the figure of the next lower order of the dividend. Divide the number so formed by the divisor, writing the number of times it is contained as the second figure in the quotient.

IV. Multiply the divisor by this quotient figure, and subtract the product from the part of the dividend used. To this remainder annex the next figure of the dividend, and proceed as before. Continue to repeat the process till the dividend is exhausted, or until the remainder will not contain the divisor.

V. If at any time a remainder, with the next figure of the dividend annexed, will not contain the divisor, write 0 *in the quotient, and bring down the next figure of the dividend.*

Reasons.—The reasons for this rule are the same as for the rule for Short Division.

Questions.—1. Why do we write the divisor on the left and the quotient on the right of the dividend? * 2. Why do we begin to divide with the highest order or orders, and proceed through the lower orders in succession? 3. How do we find out how many times the divisor is contained each time? (**73.**) 4. On what principle do we determine what the order of any quotient figure

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* Were it customary it would be better to write the divisor on the right of the dividend and the quotient underneath the divisor; or, writing the divisor at the left, write the quotient above the dividend, putting its first figure over the units figure arising from multiplying the divisor by this figure. See 2d and 3d forms of operation, Ex. 4, page 98, and (**156**, *a*).

is? (71.) 5. On what principle are we able to take part of the dividend at a time? (72.) 6. Finally, how does it appear that this process determines how many times the divisor is contained in the dividend?

Examples for Practice.

1. Divide 82756 by 234, and afterward put the work in form to exhibit the principles at one view, as on page 100. *Rem.*, 154.

2. Divide 7854 by 96, and explain as directed in the last. *Rem.*, 78.

3. Divide 346827 by 271, and then multiply the quotient by the divisor, and to this product add the remainder. What ought the result to be?

85. Division may be proved (**50**) by multiplying divisor and quotient together, and to the product adding the remainder. The result should be the dividend. Why?

4. Divide 17856 by 39, and prove the process as above.

Operation.	**Second Form.**		**Third Form.**	**Proof.**
			457	
39) 17856 (457	17856	39	39) 17856	457
156	156	457	156	39
225	225		225	4113
195	195		195	1371
306	306		306	17823
273	273		273	33
33 *Rem.*	33 *Rem.*		33 *Rem.*	17856

Query.—Why is it that if you add the remainder and several subtrahends just as they stand in the work, their sum will make the dividend? Try it.

This then is another method of proof.

Perform the following divisions, and prove the process *in each case*:

5. 2592 ÷ 63.
6. 7776 ÷ 108.
7. 6750 ÷ 15.
8. 437639 ÷ 42.
9. 1893312 ÷ 2076.
10. 84764367 ÷ 431.
11. 4683579 ÷ 234.
12. 2686211248 ÷ 296.
13. 99424788962 ÷ 978.
14. 347628 ÷ 84.
15. 57432168 ÷ 8762.
16. 134007502 ÷ 34007.
17. 88888888 ÷ 9999.
18. 100000001 ÷ 785.
19. 2000000 ÷ 691.
20. 4360000 ÷ 436.
21. 84200 ÷ 421.
22. 13230000 ÷ 735.

SEVENTH STEP.

To Divide by 10, 100, 1000, etc.

1. How many 10's in 36, and how many units remain? Then 36 ÷ 10 = how many, with what remainder?

2. How many 10's in 75, and how many units remain? 75 ÷ 10 = how many, with what remainder?

3. If I had 2 of our hundreds packages of counters, 4 of the tens, and 7 single counters, how many tens would I have in all? How many tens are there in 2 hundreds? Then 247 ÷ 10 = 24 and 7 remainder.

4. Divide 347 by 100; that is, find how many hundreds there are in 347. Of course all we have to do is to read the hundreds. There are 3 hundreds, and 47 remainder.

5. Divide 2346 by 100. How many hundreds in 2 thousands and 3 hundreds?

6. Divide 34568 by 100. By 1000.

To Divide any Number by 10, 100, 1000, or 1 with any Number of Zeros annexed.

86. Rule.—*Cut off from the right of the dividend as many figures as there are 0's in the divisor. The remain-*

ing figures at the left represent the quotient, and those cut off represent the remainder.

The reason for this is that all at the left of any particular order, including that order, can be read as so much of that order (**21**).

7. Divide 7856 by 1000; that is, tell how many thousands there are in it, and how many over. In like manner, divide 17541 by 1000.

8 to 15. Speak the quotients and remainders in the following: 58 ÷ 10; 736 ÷ 10; 34568 ÷ 10; 12057 ÷ 1000; 19027 ÷ 100; 34567 ÷ 1000; 30456 ÷ 100; 750263 ÷ 100000.

EIGHTH STEP.

To Divide by a Composite Number.

87. In arithmetic a **Composite Number** is the product of two or more integral factors each greater than 1.

Thus 6 is a composite number, for it is the product of 2 and 3. 15 is a composite number, for it is the product of 3 and 5.

1. Point out the composite numbers in the following, and tell what their factors are: 21; 12; 24; 36; 7; 8; 11; 13; 28; 23; 20; 30; 130; 300; 230; 1200.

2. Can there be a number whose right hand figure is 0, which is not composite?

88. Numbers which are not *composite* are called **Prime Numbers.**

3. Point out *all* the prime numbers between 1 and 100. Make a list of them on your slate.

4. Of what 3 factors is 12 composed? 30? 66? 60? 8? 27?

5. May the factors of a composite number ever be all alike? What are the 3 equal factors of 64? The 5 equal factors of 32?

---

1. If you take 42 counters and put them up in packages of 3 each, how many packages will there be?

If now you take these 14 packages of 3's and put them up into packages of 2 3's in each, how many such packages can you make?

The last packages of 2 3's each contain how many counters? Then how many 6's in 42?

2. Find how many 15's there are in 5265 by first finding how many 3's there are, and then how many 5's containing 3 each there are in this result.

Suggestion.—Dividing by 3, we find that there are 1755 *threes* in 5265. Now, since 5 *threes* make 15, there are as many 15's in 5265, as there are 5's in 1755, which is 351. Hence there are 351 15's in 5265.

$$\begin{array}{r l}
3\,)\,\underline{5265} & \\
5\,)\,\underline{1755} & \text{Number of 3's in 5265.} \\
351 & \text{Number of 5's containing 3 each, or of 15's in 5265.}
\end{array}$$

3. Divide 5796 by 21, by using the factors of 21.

To Divide by a Composite Number by using its Factors.

89. Rule.—*Divide the given dividend by one of the factors and the quotient thus arising by the other.**

Reasons.—To divide a number by 5 shows how many 5's there are in it. Then as every 7 of these 5's make 35, if we find how many 7's there are in this first quotient, the result will show

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* Of course this process is capable of extension to cases of more than two factors; but we have no occasion to use such cases, and hence give only that of two factors.

how many 7's containing 5 each, or how many 35's there are in the given number.

This reasoning applies to dividing by any composite number.

Perform each of the following by resolving the divisor into two factors, and using these factors as divisors. Give the explanation as above in each case:

4. 1530 ÷ 6;	6. 9576 ÷ 28;	8. 4704 ÷ 84;
5. 8748 ÷ 12;	7. 7740 ÷ 45;	9. 2670 ÷ 30.

90. If there are remainders, it is easy to tell how many of the original dividend remain, by noticing what the remainders are. The number of the original dividend which remains undivided is called the **True Remainder.**

10. Divide 5276 by 15, using the factors of 15, and find the true remainder.

$$\begin{array}{r} 3\,)\,5276 - 2 \\ 5\,)\,\overline{1758 - 3} \\ \overline{351} \end{array}$$

Suggestion.—Dividing by 3, we find 2 of the 5276 remaining. Now the 1758 are 3's of the 5276, that is, every 1 of the 1758 corresponds to 3 of the 5276. But when we divide the 1758 by 5, we find 3 of it remaining. Hence, as each one of these 3 corresponds to 3 of the 5276, the 3 corresponds to 9 of that number. The remainder of the 5276 is, therefore, 9 + 2, or 11.

11. Perform the following, using the factors of the divisors, and find the true remainders as above:

578 ÷ 21; 3412 ÷ 15; 12047 ÷ 28; 1879 ÷ 30.

---

1. Divide 3528 by 300.

$$\begin{array}{r} 3|00\,)\,35|28 \\ \overline{11;\ \text{Rem. } 228.} \end{array}$$

Suggestion.—The divisor, 300, is composed of the two factors 100 and 3. Now 3528 divided by 100 gives 35 and 28 remainder (86). Then dividing 35 by the other factor, 3, we have 11 and 2 remainder. But this 2 is hundreds, hence the entire remainder is 2 hundreds and 28, or 228.

2. As above, divide 5208 by 70. By 500.

To Perform Division when there are 0's at the Right in the Divisor.

91. Rule.—*Cut off the* 0's *from the divisor, and a like number of figures from the right of the dividend. Divide the remaining figures in the left of the dividend by the significant * figures of the divisor. The remainder after this division, prefixed to the figures of the dividend cut off, is the entire remainder.*

Reasons.—Cutting off the 0's from the divisor may be considered as pointing out its factors, and cutting off the figures at the right in the dividend is dividing by 10, 100, 1000, or by 1 with as many 0's annexed as there are figures cut off (**86**). Dividing by the significant figures, is dividing the first quotient by the other factor of the divisor, according to (**89**). The remainder from the last division being merely of the next higher orders than the first remainder, needs simply to be prefixed according to the principles of notation.

3 to 12. Perform the following according to this rule:

(3.) $6285 \div 70$;
(4.) $5786 \div 200$;
(5.) $23478 \div 700$;
(6.) $3005080 \div 12000$;
(7.) $5200700 \div 2300$; †
(8.) $58276432 \div 13600$;
(9.) $20075006 \div 3510$;
(10.) $50707032 \div 25700$;
(11.) $500504000 \div 32000$;
(12.) $8264 \div 230$.

Operation.

```
23|0)826|4(35 Quotient.
     69
     ---
     136
     115
     ---
      214 Entire remainder.
```

* That is, the digits. See (**7**.)

† Divide 52007 by 23 by *Long Division*. See operation below.

Applications.

92. Since $\frac{1}{2}$ of anything is one of the *two* equal parts into which it may be divided, we obtain $\frac{1}{2}$ of a number by dividing the number by 2. In like manner, $\frac{1}{3}$ of a number is found by dividing the number by 3; $\frac{1}{5}$ by dividing by 5; $\frac{1}{6}$ by dividing by 6, etc. (**75—79**).

1. If it require 3 yards of cloth to make a pair of pantaloons, how many pairs can be made from a piece containing 27 yards? From 18 yards? From 30 yards?

Solution.—As every 3 yards will make 1 pair, 27 yards will make as many pairs as there are 3's in 27. 27 ÷ 3 = 9. Hence 27 yards will make 9 pairs, if 3 yards make one pair.*

2. If 9 pairs of pantaloons of equal size are made from 27 yards of cloth, how many yards does it take for one pair?

Solution.—If 27 yards make 9 pairs, one pair requires $\frac{1}{9}$ of 27 yards. $\frac{1}{9}$ of a number is found by dividing it by 9. 27 ÷ 9 = 3. Hence 1 pair requires 3 yards, if 9 pairs require 27 yards.

[Notice the difference between this solution and the preceding.]

3. If apples cost $3 per barrel, how many barrels can be bought for $12? For $15? For $30? For $327? For $18756?

[When the numbers are small the computation should be without writing—mentally, as it is called.]

4. If 21 barrels of apples are bought for $63, how much is that per barrel? If 21 barrels cost $84, how much is it per barrel?

5. At $7 per yard, how much cloth can be bought for $84? For $357?

*** Some such *form of solution* should be insisted on in the class; for let it be remembered that in the "*Applications*" the important thing is to tell *why we divide, multiply, add, or subtract.* These exercises are not designed to teach division—that is, *how to divide*—but to teach the *uses of division.***

6. If 6 bushels of potatoes cost $4.50 (450 cents), what is the price per bushel?

7. A drover bought 7 horses for $1274, paying the same price for each. What did one horse cost him?

8. If 5 oranges cost 35 cents, what is that per orange? If they cost 45 cents? If 40 cents?

9. If 46 pounds of sugar cost $5.98, what is that per pound?

10. What is the price of coal per ton when 6 tons cost $66?

11. John's board bill for a term of 13 weeks was $78. What was that per week?

12. A laborer received $39.00 for a month's work of 26 days. How much was that per day?

13. How many stoves, at $47 each, can be bought for $893?

14. If 19 stoves are bought for $893, what is the price of one stove?

15. If one stove cost $47, what will 19 stoves cost?

16. A man going to buy sheep took with him $1256. His expenses were $150. He bought 217 sheep at a uniform price, and had $21 left. How much did one sheep cost? *Ans.*, $5.

17. If a man on horseback ride 53 miles each day, how much will he lack of 1000 miles in 16 days?

Ans., 152 miles.

18. If a man pay $75 each for oxen, how many oxen can he buy for $2000, and how much money will he have left?

19. A young man attending college had $200 at the beginning of a term of 18 weeks. He spent $35 for books, and $23 for other purposes besides board, and had $34 left

at the end of the term. How much did his board cost him per week?

Suggestion.—How much of the $200 did he *not* spend for board? How much *did* he spend for board? If he spent so much for 18 weeks' board, how much was that for one week? *Ans.*, $6.

20. If a man had an income of $3742 a year (52 weeks), and spent $1500 for his family expenses, gave to charitable purposes $370, and saved the rest, how much did he save per week? *Ans.*, $36.

21. A has 340 head of cattle worth $9860, and B has 760 acres of land worth $32680. Required the value of A's cattle per head, and of B's land per acre?

Ans., $29, and $43.

22. Having a tract of land containing 540 acres, I wish to divide it into fields containing 45 acres each. What number of fields will it make? *Ans.*, 12 fields.

23. A planter has 2280 dollars to lay out for mules and oxen, and wishes to purchase the same number of each. If he pay 65 dollars a head for mules, and 30 for oxen, how many of each can he buy?

Suggestion.—How much do 1 ox and 1 mule together cost? Then how many times can he buy 1 ox and 1 mule? *Ans.*, 24.

24. How many yards of cloth at 7 dollars a yard, 8 dollars a yard, and 9 dollars a yard—the quantity of each kind to be the same—can a merchant buy for 1800 dollars? *Ans.*, 75 yards of each kind.

25. A cistern, the capacity of which is 10000 gallons, is to be filled with water by 3 pipes discharging into it. The first pipe discharges 200 gallons per hour, the second and third each 150 gallons per hour. In what time will the cistern be filled by the three pipes running together?

Ans., 20 hours.

26. A farmer wishes to fill three kinds of sacks, containing 3 bushels, 4 bushels, and 5 bushels, and the same number of each kind, with 1728 bushels of corn. How many sacks can he fill? *Ans.*, 144 of each kind.

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27. If 6 yards of calico can be bought for $1, how much will 12 yards cost? 24 yards? 48 yards? 60 yards?

28. If 8 melons cost $1, how many dollars will 3744 melons cost?

29. When 6 books are bought for $1, how many dollars must be paid for 1743 books?

30. If 7 days make 1 week, how many weeks in 21 days? 42 days? 365 days?

31. If there are 12 inches in 1 foot, how many feet in 578 inches, and how many inches over?

32. How many cents make $1? Then how many dollars in 5682 cents? In 586 cents? In 1280 cents? How many cents over in each case? How do you divide by 100?

33. Shingles are put up in bunches of 500 each. How many bunches does it take to make 1000? How many bunches will it take to shingle a roof which requires 8000 shingles?

34. If 3 pounds of coffee cost $1, how much will 27 pounds cost?

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35. If 3 gallons of molasses cost $2, how much will 6 gallons cost? How many times $2?

36. If 6 oranges can be had for 35 cents, how much will 36 oranges cost? How many times 35 cents?

37. If 7 lemons can be bought for 25 cents, how many

can be had for $1, which is 100 cents? How many times as many can be had for 100 cents as for 25 cents?

38. If you can buy 5 slate pencils for 3 cents, how many can you get for 18 cents? How much will 35 pencils cost?

39. If 2 horses can be bought for $350, how many can be bought for $8400, at the same rate?

Suggestion.—Observe that such problems can be solved in two ways. 1. How many times the price of 2 horses is $8400? Then how many times 2 horses can be bought? 2. How much does 1 horse cost? Then how many can be bought for $8400?

Sometimes one method involves fractions, while the other does not. In such cases take the latter.

40. If 7 sheep can be bought for $24.50, how much will a flock of 133 cost at the same rate?

41. If 4 bushels of apples can be bought for $3, how many can be bought for $51 at the same rate?

42. If 6 oranges can be bought for 42 cents, how many can be bought for 56 cents?

---

43. If you divide 7 apples equally between 2 boys, how many will each boy have? If 9 apples among 4 boys? 13 apples among 3 boys? 27 apples among 5 boys?

Answer to last, $5\frac{2}{5}$ apples.

44. If a pound of sugar cost 8 cents, how much at that rate can be bought for 17 cents? For 25 cents?

45. At $7 a yard, how much cloth can be had for $45? For $105? For $23? For $63? For $5829?

46. If a man with a reaper can cut 8 acres of wheat in a day, how long will it take him to cut 25 acres? 30 acres? 270 acres? 375 acres? *Answer to last,* $46\frac{7}{8}$ days.

47. If a man with a reaper can cut 7 acres of wheat in a day, in how many days can 3 men with reapers cut 250 acres? How much will they cut in one day? Then how long will it take them to cut 250 acres?

48. There being 1760 yards in a mile, find how many miles there are in 65129 yards. *Ans.*, $37\frac{9}{1760}$ miles.

49. How many days would 21 horses subsist on an amount of food which would suffice one horse 300 days? *Ans.*, $14\frac{6}{21}$ days.

50. Allowing a steamboat to run 275 miles in a day, in what time would she make a trip of 5349 miles? *Ans.*, $19\frac{124}{275}$ days.

---

51. A flour merchant sold 108 barrels of flour for $9 a barrel, and gained on it $216. What price then must he have paid for it by the barrel? *Ans.*, $7.

52. A farmer sold a grocer 10 bushels of corn at $1 a bushel; 12 barrels of cider at $2 a barrel; he received in payment 3 barrels of flour at $7 a barrel, and the balance in cash. How much money did he receive? *Ans.*, $13.

53. A country woman brought to market 5 dozen eggs, for which she got 28 cents a dozen, and 28 pounds of butter, for which she got 32 cents a pound. She bought 12 yards of calico at 15 cents a yard, a shawl for $7, and took the rest in sugar at 9 cents a pound. How much sugar did she get? *Ans.*, $17\frac{1}{3}$ pounds.

$\frac{3}{9} = \frac{1}{3}$, since if a thing is divided into 9 equal parts, 3 of those parts make $\frac{1}{3}$ of it.

54. What part of anything is $\frac{2}{8}$ of it? $\frac{4}{8}$?

If flour is $8 a barrel, how much can a man buy for $4? For $2? How many pounds for $4? (196 pounds make a barrel.) How many pounds for $2?

55. If the divisor is 43, the quotient 135, and the remainder 21, what is the dividend?

56. If the sum of two numbers is 782, and one of them is 243, what is the other?

57. If 428 is the subtrahend and 356 the remainder, what is the minuend?

58. If the sum of 3 numbers is 726, and two of them are 116 and 325, what is $\frac{1}{3}$ of the other?

59. $\frac{1}{3}$ of A's property is worth \$2500, $\frac{1}{5}$ of B's \$1200, and $\frac{1}{4}$ of C's \$875. How much is $\frac{1}{2}$ of all their property?

60. If 5 bushels of wheat yield a barrel of flour, and a barrel of flour is 196 pounds, how many bushels of wheat will be required to make 3332 pounds of flour?

To Resolve a Number into its Prime Factors.

93. Rule.—*Divide the number by the least number which will divide it exactly. Treat this quotient in the same way, and continue the process until there is no exact divisor of the quotient last obtained, other than itself and* 1.

The several divisors and the last quotient are the prime factors sought.

1. Resolve 60 into its prime factors.

Operation.

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

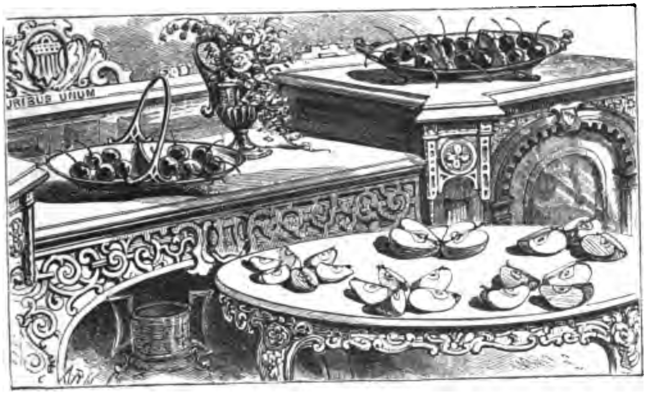
Therefore 2, 2, 3, and 5 are the prime factors of 60, for they are all prime numbers and $2 \times 2 \times 3 \times 5 = 60$.

1. Resolve 108 into its prime factors. So also 56, 14, 27, 135, 42, 75, 45, 128, 63, 59, 43, 48, 160.

[The TEACHER'S HAND-BOOK contains 250 exercises for class drill in Division, a large number of which are for oral or mental work.]

CHAPTER II.

COMMON FRACTIONS.



SECTION I.

DEFINITIONS AND FUNDAMENTAL PRINCIPLES.

*What a Fraction is.**

1. If an apple is divided into 2 equal parts, what is one of the parts called?

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* This step and the succeeding one are but reviews. See page 94. Also at this point a slight change in style will be noticed. The pupil is supposed to have gained strength as well as knowledge. The sub-heads "*First Step*," "*Second Step*," etc., will be omitted. The habit of observing these for one's self ought to be cultivated.

If into 3 equal parts, what is one called?

If into 4? 5? 6? 7? 12? 20? 23? 156?

2. If you have 10 cherries and separate them into 2 equal groups, what part of the cherries is one group?

If you have 12 cherries and separate them into 3 equal groups, what part of the whole is one group?

If you have 30 things and separate them into 6 equal groups, what part of the 30 is one of the groups? If into 10 equal groups, what part is one group? If into 15?

3. If you divide a melon into 11 equal parts, and give John 1 of them, what part of the melon has he? If you give him 2 of the equal parts, what part has he? If 3? If 4? If 5? If 6?

4. What is one-third of 12? Two-thirds?

5. What is one-fifth of 15? Two-fifths? Three-fifths? Four-fifths?

94. A Fraction is a number representing one or more of the equal parts into which a unit, or some number taken as a whole, is conceived to be divided.

6. Why is two-thirds a fraction? One-seventh? Five-ninths? Three-tenths? One-twelfth? Eleven-fifty-sixths?

95. The number which indicates into how many equal parts the number is to be divided is called the **Denominator.**

Denominator means *namer;* and it will be seen that the number of equal parts into which a thing is divided determines the *name* of one of those parts: thus, if we divide a thing into 3 equal parts, the parts are called (denominated) *thirds;* if into 4, *fourths;* if into 13, thirteenths, etc.

96. The number which indicates how many of the

equal parts are represented by the expression is called the **Numerator.**

Numerator means *numberer*, and hence the appropriateness of the name. If 2-thirds are taken, 2 tells the *number* of thirds, and hence is the *numerator*, or numberer; if 5-sevenths are taken, 5 is the *numerator*, or numberer, etc.

97. The Terms of a fraction are the Numerator and Denominator.

How a Common Fraction is Written.

98. A Common Fraction is written in figures by writing the numerator above the denominator with a line between them.

1. What does $\frac{2}{3}$ signify? Into how many equal parts is the division to be made? What are such parts called? How many of them are indicated?

2. Explain as above the meaning of $\frac{3}{5}$, $\frac{1}{7}$, $\frac{3}{7}$, $\frac{4}{9}$, $\frac{3}{11}$, $\frac{2}{10}$, $\frac{3}{156}$, $\frac{127}{342}$.

3. Write in figures three-sevenths; four-ninths, one-thirteenth; 5-twelfths; 7-eighths; 11-seventeenths; 123-three hundred and fifty-sixths; two-tenths; 203-thousandths; one-half.

4. If John has $\frac{2}{3}$ of a pie and James has the rest of it, how much has James?

5. If Mary has $\frac{3}{5}$ of an orange and Jane has the rest of it, what part has Jane? How many fifths in the whole? How many sevenths? How many thirds? How many elevenths?

6. Henry, James, and Philip divided some marbles; Henry took $\frac{3}{12}$, James $\frac{5}{12}$, and Philip the rest. How many did Philip have?

Kinds of Common Fractions.

99. An Integer is a *Whole Number* in distinction from a Fraction.

Whole numbers are also called *Entire Numbers.*

100. A Mixed Number is an *Integer* and a *Fraction* written in connection, thus $4\frac{2}{3}$, and signifies that the two are to be taken together.

A mixed number is read by naming the whole number and then the fraction; thus $4\frac{2}{3}$ is read "four and two-thirds;" $136\frac{5}{6}$ is read "136 and $\frac{5}{6}$," etc.

101. A Proper Fraction is a fraction whose numerator is less than its denominator.

102. An Improper Fraction is a fraction whose numerator is equal to or greater than its denominator.

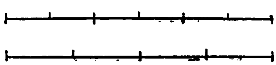
1. Point out the Integers, Proper Fractions, Improper Fractions, and Mixed Numbers in the following: $\frac{15}{7}$, $4\frac{1}{2}$, $\frac{3}{5}$, 224, $\frac{1}{4}$, $\frac{2}{2}$, $\frac{11}{5}$, $\frac{17}{17}$, $42\frac{3}{4}$, 180.

2. Write the following numbers and tell to which of the above kinds they belong, and why: five-sixths; six-sixths; 4 and two-sevenths; twenty-eight; one twenty-eighth; seven and one-third; four-thirds; 12-fifths.

103. The Value of a fraction is the amount which it represents. Thus two fractions are said to have the same value when they represent the same amount of anything.

3. What is the difference in value of $\frac{4}{4}$ and $\frac{2}{2}$? Of $\frac{2}{2}$ and 1? Of $\frac{3}{3}$ and $\frac{7}{7}$?

4. What is the difference in value of $\frac{2}{3}$ and $\frac{4}{6}$? Have $\frac{1}{2}$ and $\frac{2}{4}$ the same value? Why?



5. Have $\frac{2}{3}$ and $\frac{3}{4}$ the same value? Why not?

Fundamental Principles.

1. A melon is divided into 7 equal parts, and James takes 2 of them; how much has he? If John takes 2 times as many parts, how many has he? 2 times $\frac{2}{7}$ are how many sevenths?

2. What are the things represented by $\frac{3}{8}$? Are they halves, thirds, sevenths, or eighths? How many are represented? How many would 2 times as many be? 2 times $\frac{3}{8}$ are how much?

3. $\frac{4}{9}$ are how many times $\frac{2}{9}$? How many times $\frac{1}{9}$?

4. 2 times $\frac{5}{12}$ are how many? 3 times $\frac{5}{12}$?

Principle I.

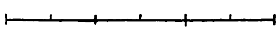
104. *Multiplying the numerator of a fraction, while the denominator remains the same, multiplies the value of the fraction.*

Demonstration.*—The reason for this is that as the numerator tells how many parts are represented, multiplying it multiplies the *number* of parts represented; and as the denominator is not changed, the *size* of the parts remains the same.

5. Multiply $\frac{3}{7}$ by 2, and give the reason for the process.

6. Multiply $\frac{5}{9}$ by 4, and give the reasons.

So also find 4 times $\frac{3}{10}$; 7 times $\frac{5}{12}$; 128 times $\frac{13}{179}$; 2145 times $\frac{314}{4371}$. What kind of fractions are these results? Why?



1. Into how many parts is this line divided by the marks on its upper side? Into what part of this number of parts is it divided by the marks on its

* Let the teacher explain that a Demonstration is a condensed and formal

lower side? How does dividing the line into only half as many parts as at first affect the size of the parts?

2. Which is the more, $\frac{2}{6}$ or $\frac{2}{3}$? What is the *number* of parts in each case? Which are the larger, sixths or thirds? How much?

3. Which is the more, $\frac{5}{8}$ or $\frac{5}{4}$? Which are the larger, eighths or fourths? How much?

4. If you divide the denominator of $\frac{2}{6}$ by 2, and let the numerator remain the same, how do you affect the value of the fraction? $\frac{2}{3}$ are how many times $\frac{2}{6}$? Why?

Principle II.

105. *Dividing the denominator of a fraction, while the numerator remains the same, multiplies the value of the fraction.*

Demonstration.—The reason for this is that it multiplies the *size* of the parts represented, while the number represented remains the same.

5. Show that $\frac{4}{6}$ is 2 times $\frac{4}{12}$.

6. Show that $\frac{3}{5}$ is 2 times $\frac{3}{10}$.

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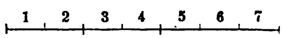
7. Into how many equal parts is this line divided? What is one of these parts called? What are 4 of them called? How much of the line is one-half of $\frac{4}{7}$ of it?



How much is $\frac{1}{2}$ of $\frac{6}{7}$ of this line?

8. How many parts are represented by $\frac{8}{11}$? What are the parts? How many parts are $\frac{1}{4}$ of 8 parts? What is $\frac{8}{11} \div 4$?

What is $\frac{8}{11} \div 2$? $\frac{6}{7} \div 3$?

---

presentation of the reasons for a statement. It is the argument which proves the statement true.

Principle III.

106. *Dividing the numerator of a fraction, while the denominator remains the same, divides the value of the fraction.*

Demonstration.—The reason for this is that it divides the *number* of parts represented, while the size of the parts remains the same.

9. Divide $\frac{10}{11}$ by 2. By 5. Give the reasons.

10. Why is $\frac{4}{7}$ $\frac{1}{2}$ of $\frac{8}{7}$? What part of $\frac{12}{7}$ is $\frac{4}{7}$?

11. Divide $\frac{6}{8}$ by 3. By 2. By 6.

---

1. Into how many parts is this line divided by the marks on its upper side? If we divide the line into 2 times as many parts, how does it affect their size? Thirds are how many times as large as sixths? Halves are how many times as large as fourths? How do fifths compare with fifteenths?

2. What will $\frac{2}{3}$ become if you multiply its denominator by 2? Does it become more or less? Why? What part of $\frac{2}{3}$ is $\frac{2}{6}$?

3. What will $\frac{2}{5}$ become if you multiply its denominator by 3? Does it become more or less? What part of $\frac{2}{5}$ is $\frac{2}{15}$? Why?

4. What effect does it have on a fraction to multiply its denominator by 2? By 3? By 5?

Principle IV.

107. *Multiplying the denominator of a fraction, while the numerator remains the same, divides the value of the fraction.*

Demonstration.—The reason for this is that multiplying the denominator divides the *size* of the parts, while the number remains the same.

5. Divide $\frac{3}{5}$ by 2. By 3. By 4. Give the reasons.

6. Divide $\frac{5}{7}$ by 4. By 6. By 12.

---

1. What effect will it have upon 3 to multiply it by 4, and then divide the result by 4? What effect will it have on 7 to multiply it by 5, and then divide the result by 5?

2. What is $\frac{2}{3}$ multiplied by 5? What is $\frac{10}{3}$ divided by 5 according to (**107**)? Which is the more, $\frac{2}{3}$ or $\frac{10}{15}$? Why?

3. How do the divisions of this line show that $\frac{2}{3} = \frac{4}{6}$?

4. Show that $\frac{3}{4} = \frac{6}{8}$. Also that $\frac{3}{4} = \frac{9}{12}$.

Suggestion.—How many times as large as twelfths are fourths? Then how many times as many twelfths as fourths must we take to make the same part of a thing?

5. What effect does it have on the value of a fraction to multiply both numerator and denominator by the same number? Why? Why does $\frac{3}{5} = \frac{12}{20}$?

6. What effect does it have on the value of a fraction to divide both numerator and denominator by the same number? Why? Why is $\frac{8}{24} = \frac{1}{3}$?

Principle V.

108. *The value of a fraction is not altered by multiplying both terms of the fraction by the same number, or by dividing both terms by the same number.*

Demonstration.—Multiplying both terms of a fraction by the same number does not alter the value of the fraction; because

multiplying the numerator *multiplies* the *number* of parts, while multiplying the denominator *divides* the *size* of the parts. Thus, if we multiply the terms by 3, the new fraction represents 3 times as many parts, but the parts are only one-third as large.

If we divide both terms, we *divide* the *number* of *parts* and multiply the *size*, and hence do not change the value of the fraction. Thus, if we divide both terms by 5, the new fraction represents only one-fifth as many parts, but they are 5 times as large.

7. Why is $\frac{2}{7} = \frac{8}{28}$? Why is $\frac{10}{12} = \frac{5}{6}$? Why is $\frac{3}{5} = \frac{12}{20}$? Why is $\frac{18}{24} = \frac{3}{4}$?

[See TEACHER'S HAND-BOOK for exercises for class drill on all subjects in Fractions. It contains over 600 such exercises.]

SECTION II.

REDUCTION OF FRACTIONS.

109. Reduction is changing the form of an expression without altering its value. Thus changing $\frac{4}{8}$ into $\frac{1}{2}$ is a reduction.

To Lower or Lowest Terms.

110. A fraction is said to be in its lowest terms when there is no number greater than 1 which will exactly divide both terms of it.

111. A Common Divisor of two or more numbers is a common integral factor; that is, a whole number which exactly divides each of the numbers.

Thus 3 is a common divisor of 12 and 18. 5 is a common divisor of 10, 15, and 20. Why?

Note.—Although in the higher mathematics it is not well to try to avoid calling 1 and the number itself factors or divisors, for the purposes of Elementary Arithmetic some prefer to exclude them.

112. The Greatest Common Divisor of two or more numbers is the greatest whole number which will exactly divide each of them, and is the product of all their common prime factors.

Thus 8 is the greatest common divisor of 16, 24, and 32. Why? Again, any factor of a number, or the product of any number of its factors, is a divisor of it. Thus, as $60 = 2 \times 2 \times 3 \times 5$, 2 is contained in 60 $2 \times 3 \times 5$, and $2 \times 2 \times 3$ is contained 5 times, etc. Now, as no number which is not a factor of a number can divide it, the Greatest *Common* Divisor of two or more numbers is the product of all their common prime factors.

1. What is a common divisor of 6 and 4? Of 10 and 15? Of 8 and 12? 27 and 42? 128 and 364?

2. How many common divisors of 8 and 12 can you name? What is the greatest common divisor of 8 and 12?

3. What is the greatest common divisor of 16 and 40? Of 42 and 63? 45 and 360? 72 and 90?

4. What common divisor have the terms of the fraction $\frac{6}{8}$? If you divide both terms of this fraction by 2, what does it become? Does this change its value? Why not?

5. Is $\frac{3}{4}$ in its lowest terms? Why?

6. Is $\frac{12}{20}$ in its lowest terms? Why not?

What is $\frac{12}{20}$ when put in its lowest terms?

Why is $\frac{3}{5}$ equal to $\frac{12}{20}$?

To Reduce a Fraction to its Lowest Terms.

113. Rule.—*Reject all common factors greater than* 1 *from both terms; or divide both terms by their greatest common divisor.*

Demonstration.—To reject a factor is the same as to divide by that factor. Hence rejecting the common factors does not change the value of the fraction by *Principle V.* And when all the common factors are rejected, the fraction is in its lowest terms by **(110)**.

Reduce the following to their lowest terms mentally, *giving the* reason in each case:

7. $\frac{5}{10}$.	10. $\frac{9}{12}$.	13. $\frac{36}{64}$.	16. $\frac{37}{74}$.
8. $\frac{6}{15}$.	11. $\frac{27}{45}$.	14. $\frac{72}{81}$.	17. $\frac{17}{85}$.
9. $\frac{8}{14}$.	12. $\frac{13}{26}$.	15. $\frac{15}{45}$.	18. $\frac{12}{15}$.

19. Reduce $\frac{294}{378}$ to its lowest terms.*

Suggestion.—As it is not easy to see what is the greatest common divisor of the terms of this fraction, we proceed thus:

$$^{2)}\frac{294}{378} = {}^{7)}\frac{147}{189} = {}^{3)}\frac{21}{27} = \frac{7}{9}.$$

This process consists in dividing the terms successively by any number which will divide them both. These operations do not change the value of the fraction by (108), and when there is no number which will exactly divide the terms, the fraction is in its lowest terms by (110).

Show the truth of the following:

20. $\frac{25}{75} = \frac{1}{3}$.	24. $\frac{99}{225} = \frac{11}{25}$.	28. $\frac{225}{375} = \frac{3}{5}$.
21. $\frac{216}{360} = \frac{3}{5}$.	25. $\frac{180}{468} = \frac{5}{13}$.	29. $\frac{909}{2323} = \frac{9}{23}$.
22. $\frac{125}{150} = \frac{5}{6}$.	26. $\frac{210}{294} = \frac{5}{7}$.	30. $\frac{873}{1067} = \frac{9}{11}$.
23. $\frac{279}{403} = \frac{9}{13}$.	27. $\frac{80}{112} = \frac{5}{7}$.	31. $\frac{182}{196} = \frac{13}{14}$.

Reduce the following to their lowest terms:

32. $\frac{665}{700}$.	34. $\frac{144}{240}$.	36. $\frac{171}{280}$.	38. $\frac{1344}{1536}$.
33. $\frac{195}{299}$.	35. $\frac{352}{364}$.	37. $\frac{221}{765}$.	39. $\frac{133}{152}$.

Improper Fractions to Whole or Mixed Numbers.

1. How many thirds in the whole of a thing? Then how many whole things are 6-thirds equal to? How many whole ones are 12-thirds equal to?

2. How many fourths make a unit? Then how many units in 8 fourths? In $\frac{12}{4}$? In $\frac{20}{4}$?

3. How many units in 12 fifths, and how many fifths

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* The best practical way for the young learner to proceed in such cases is to begin with small numbers as divisors and keep trying.

remain? $\frac{23}{5}$ are how many units and how many fifths? How do you write four and three-fifths in figures?

4. Show that $\frac{39}{4}$ are $9\frac{3}{4}$.

To Reduce an Improper Fraction to a Whole or Mixed Number.

114. Rule.—*Divide the numerator by the denominator.*

Demonstration.—As the denominator shows into how many equal parts a unit is conceived to be divided, it shows how many such parts make a unit. Hence dividing the number of parts represented by the fraction—*i. e.*, the numerator—by the number of parts which it takes to make a unit—*i. e.*, by the denominator—we find how many units are represented by the fraction, and how many parts, if any, remain.

5. Reduce $\frac{125}{13}$ to a whole or mixed number.

Solution.*—Since 13 thirteenths make one unit, 125 thirteenths make as many units as 13 is contained times in 125. $125 \div 13 = 9$ and 8 remainder. Hence $\frac{125}{13} = 9\frac{8}{13}$.

6. Reduce the following without writing, giving the solution as above: $\frac{7}{3}$; $\frac{8}{3}$; $\frac{9}{2}$; $\frac{10}{5}$; $\frac{13}{4}$; $\frac{28}{7}$; $\frac{30}{7}$; $\frac{63}{4}$; $\frac{71}{6}$; $\frac{20}{4}$.

Reduce the following to whole or mixed numbers, giving the explanation as above:

7. $\frac{346}{23} = 15\frac{1}{23}$.	12. $\frac{847}{15}$.	17. $\frac{3482}{412}$.
8. $\frac{747}{61} = 12\frac{15}{61}$.	13. $\frac{1246}{22}$.	18. $\frac{50082}{7185}$.
9. $\frac{456}{79} = 5\frac{61}{79}$.	14. $\frac{1312}{136}$.	19. $\frac{126}{10}$.
10. $\frac{444}{16} = 27\frac{3}{4}$.	15. $\frac{1312}{17}$.	20. $\frac{85}{10}$.
11. $\frac{111}{9} = 12\frac{1}{3}$.	16. $\frac{56789}{21}$.	21. $\frac{9372}{100}$.

* Teacher explain that a Solution is a formal statement of how an example is performed, and *the reasons* for *each step*. They are the *explanations*. These solutions are such as the pupil should be taught to give.

Whole or Mixed Numbers to Improper Fractions.

1. When divided into thirds, how many thirds does 1 make? Then how many do 2 make? 3? 4? 5?

2. How many fourths does 1 make? Then how many do 6 make? 7? 12? 13? 5?

3. How many fifths in 7? Why?

4. How many fifths in $7\frac{3}{5}$? How many in 7? Then 35 fifths and 3 fifths make how many fifths?

5. How many thirds in $5\frac{2}{3}$? Why?

To Reduce a Whole or Mixed Number to an Improper Fraction.

115. Rule.—*Multiply the whole number by the denominator of the proposed fraction, and to this product add the numerator of the given fraction, if any, and write the result over the proposed denominator.*

Let the pupil write out a demonstration.

6. Reduce $7\frac{2}{11}$ to elevenths.

Solution.—As there are 11 elevenths in 1, in 7 there are 7 times 11 elevenths, or 77 elevenths. Now 77 elevenths and 2 elevenths make 79 elevenths. Hence $7\frac{2}{11} = \frac{79}{11}$.

7. Reduce $8\frac{2}{5}$ to fifths?

8. Reduce the following mixed numbers to fractions having the same denominators as the fractional parts, without writing. Give the solution in each case:

$5\frac{2}{3}$; $4\frac{3}{7}$; $9\frac{5}{6}$; $8\frac{5}{6}$; $7\frac{1}{2}$; $5\frac{1}{3}$; $12\frac{2}{7}$; $11\frac{5}{9}$.

9. How many thirteenths in 5? In 8? In 11? In 43? In 27? In 128? Tell why in each case.

Answers to three, 104, 143, 351.

10. Reduce 7 to thirds; 3 to fifths; 6 to halves; 13 to sevenths; 21 to nineteenths; 123 to 374ths; 151 to 765ths.

Reduce the following to improper fractions:

11. $23\frac{3}{4}$.	17. $124\frac{1}{2}$.	23. $1\frac{34}{71}$.
12. $5\frac{2}{7}$.	18. $342\frac{2}{3}$.	24. $2\frac{128}{607}$.
13. $16\frac{3}{8}$.	19. $200\frac{5}{9}$.	25. $3\frac{17}{19}$.
14. $17\frac{5}{12}$.	20. $1256\frac{2}{7}$.	26. $5\frac{2}{10}$.
15. $57\frac{3}{11}$.	21. $48\frac{5}{13}$.	27. $7\frac{3}{100}$.
16. $81\frac{5}{9}$.	22. $2\frac{721}{935}$.	28. $13\frac{21}{1000}$.

Common Multiples.

1. Does 6 contain 3 as a factor?* 12? 8? 10? 21?

2. Does 12 contain 4 as a factor? Does 28? 20? 21? 10? 32? 40? 37?

116. A Multiple of a number is a number which contains that number as a factor.* Hence a *Composite Number* is a multiple of each of its factors.

117. A Common Multiple of two or more numbers is a multiple of each of them.

118. The Least Common Multiple of two or more numbers is the least number which is a multiple of each of them.

3. Is 12 a common multiple of 3 and 4? Why? Is 12 a common multiple of 6 and 5? Is it a multiple of either?

4. Is 36 a common multiple of 4 and 6? Is 24 a common multiple of 4 and 6?. Is 12? Is there any number less than 12 which is a common multiple of 4 and 6?

5. Mention some number which is a multiple of 3 and 7. Of 2, 3, and 5.

6. Mention some number which is a multiple of 5, 7, 3,

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* In speaking of multiples the number itself and 1 are considered as factors, and any number is its own least multiple; but fractions are excluded.

and 2. If you multiply these all together, will their product contain each as a factor? Will it be a common multiple of them all?

119. The Product of two or more numbers is a *Common Multiple* of them all, since it contains each of them as a factor.

7. Does 8 contain the factors of 6? Which of the factors of 6 does it not contain?

8. In division we learned that we could divide by any number by dividing by its factors (**89**). If then 24 is a multiple of 6, must it be exactly divisible by the factors of 6? Is it? Is 24 then a multiple of 6? Why? *Ans.*, Because it contains the factors of 6.

9. Can a number be a multiple of another if it does not contain *all* the factors which make up the latter? Why not? Because, according to (**89**) in division, if a number is exactly divisible by another, it is exactly divisible by its factors.

10. Does 30 contain the factors of 12? The factors of 12 are *two* 2's and 3. Does 30 contain 2 2's as factors? Is 30 a multiple of 12? Why not? What factor does it lack? If we put in this lacking factor and make 60, is 60 a multiple of 12? Why?

11. Is 60 a *common multiple* of 6 and 4? Does it contain any factor which is not a factor of 6 and 4? Is this factor of any use in it as a multiple of 6 and 4? Is 60 the least common multiple of 6 and 4? Why not?

Answer. Because it contains a factor 5, which is unnecessary in it as a multiple of 6 and 4.

12. Why is not 12 a multiple of 8?

Answer. Because 8 has *three* factors 2, and 12 has only 2 such factors.

13. Why is not 30 a common multiple of 9 and 15? Does it contain the factors of 15? Does it contain the factors of 9? It has *one* factor 3; why does it not contain 9?

If we introduce another factor 3 and have 3 times 30, or 90, have we then a common multiple of 9 and 15? Has 90 any factor which neither 9 nor 15 has? What one? Has it more than one such factor?

What is the least common multiple of 9 and 15? Why?

Answer. 45 is the least common multiple of 9 and 15, for it contains the factors which compose each of the numbers and no other factors.

14. Can a multiple of a number be less than the number itself? Why?

Can a common multiple of several numbers be less than the largest of the numbers? Why?

To Find the Least Common Multiple of Two or More Numbers.

120. Rule.—*I. Resolve each of the numbers into its prime factors* **(93)**.

II. Multiply the largest number by all the prime factors found in the next smaller number and not in it.

III. Treat this product and the next smaller number in the same way, and continue the process till all the numbers are used.

Demonstration.—We take the largest number, because the least common multiple must contain it. Then we multiply this by all the prime factors contained in the next smaller but not in the largest, because if there is a component factor of this number lacking in the product, it will not contain this number; and so on of all the numbers.

The product thus obtained is the *least* common multiple, because no factor can be left out of it without preventing its containing some one of the numbers.

15. Find the least common multiple of 27, 42, and 36.

Solution.

$42 = 2 \times 3 \times 7$. $36 = 2 \times 2 \times 3 \times 3$. $27 = 3 \times 3 \times 3$.

$\therefore\ 42 \times 2 \times 3 \times 3 = 756$, the least common multiple.

For as the number must contain 42 as a factor, we write it as one of the factors of the multiple sought. Then as 36 contains *two* factors 2, and *two* factors 3, and 42 has but *one* each of these, we write these factors as factors of the multiple, and have $42 \times 2 \times 3$. Now this product has but *two* factors 3 in it, whereas 27 has 3 such factors. Hence we write another factor 3, and have $42 \times 2 \times 3 \times 3 = 756$ as the least common multiple. It is the *least* common multiple, because no factor can be omitted from it without preventing its containing some one of the numbers.

If we strike out one factor 2 from $42 \times 2 \times 3 \times 3$, which number will the product not contain? If we strike out one factor 3, which number will the product not contain? If the 7 (in the 42)?

16. Find the least common multiple of 3, 8, 9, 12, and 16. *Least common multiple,* 144.

17. Find the least common multiple of 45, 63, and 81. *Least common multiple,* 2835.

18. Find the least common multiple of 8, 36, 9, and 17. *Least common multiple,* 1224.

19. Find the common multiple of 15, 20, 32, and 75.

20. Find the least common multiple of 4, 6, 8, and 10.

21. Find the least common multiple of 9, 3, 12, and 15.

22. Find the least common multiple of 21, 7, 4, and 9.

23. Find the least common multiple of 6, 4, 12, and 20.

24. Find the least common multiple of 8, 7, 10, and 14.

25. Find the least common multiple of 15, 2, 7, and 13.

26. Find the least common multiple of 24, 5, 6, and 10.

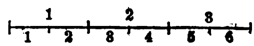
27. Find the least common multiple of 5, 10, 13, and 24.

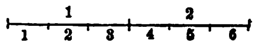
28. Find the least common multiple of 6, 7, 2, and 17.

29. Find the least common multiple of 11, 4, 5, and 19.

30. Find the least number which 1, 2, 3, 4, 5, 6, 7, 8, and 9 will exactly divide.

Common Denominators.





1. How many sixths in $\frac{2}{3}$?

How many sixths in $\frac{1}{2}$?

2. How does it appear from (**108**) that $\frac{2}{3} = \frac{4}{6}$?

How that $\frac{1}{2} = \frac{3}{6}$?

3. Have $\frac{2}{3}$ and $\frac{1}{2}$ a *common* denominator?

Have $\frac{4}{6}$ and $\frac{3}{6}$ a *common* denominator? What is it?

4. Show that $\frac{1}{3} = \frac{4}{12}$; $\frac{3}{4} = \frac{9}{12}$; $\frac{5}{6} = \frac{10}{12}$.

Have $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ a common denominator?

Have $\frac{4}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$ a common denominator? What is it?

5. Can you multiply the denominator of $\frac{2}{3}$ by any whole number that will make it 10? By any one that will make it 24?

If you multiply the denominator by 8, what must you do to the numerator so as not to alter the value of the fraction? $\frac{2}{3}$ = how many twenty-fourths?

Can you reduce $\frac{5}{6}$ to twenty-fourths? By what will you have to multiply both terms of the fraction? $\frac{5}{6}$ = how many 24ths?

$\frac{5}{8}$ = how many 24ths? $\frac{7}{12}$ = how many 24ths?

Fractions are said to have a *Common Denominator* when their denominators are alike.

To Reduce Fractions to Equivalent Ones having a Common Denominator.

121. Rule.—*Multiply both terms of each fraction by the denominators of all the other fractions.*

Demonstration.—This gives a common denominator, because each denominator is the product of all the denominators of the several fractions. The value of any one of the fractions is not changed, because both numerator and denominator are multiplied by the same number (**108**).

6. Reduce $\frac{5}{7}$, $\frac{2}{3}$, and $\frac{3}{5}$ to equivalent fractions having a common denominator.

Solution.—Multiplying both terms of $\frac{5}{7}$ by 3 and 5 we have

$$\frac{5 \times 3 \times 5}{7 \times 3 \times 5} = \frac{75}{105}.$$

In like manner, $$\frac{2 \times 7 \times 5}{3 \times 7 \times 5} = \frac{70}{105}.$$

So also $$\frac{3 \times 7 \times 3}{5 \times 7 \times 3} = \frac{63}{105}.$$

The denominators are all alike, because each is the product of 7, 3, and 5. $\frac{75}{105} = \frac{5}{7}$, because it arises from multiplying both terms of $\frac{5}{7}$ by 15, which does not change the value of the fraction according to (**108**).

[In like manner, show that $\frac{70}{105} = \frac{2}{3}$, and $\frac{63}{105} = \frac{3}{5}$.]

Reduce the following to equivalent sets of fractions having common denominators, and give the explanation as above:

7. $\frac{5}{11}$, $\frac{2}{7}$, $\frac{2}{3}$.
8. $\frac{3}{5}$, $\frac{4}{9}$, $\frac{3}{13}$.
9. $\frac{4}{5}$, $\frac{5}{7}$, $\frac{2}{9}$.
10. $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{11}$.
11. $\frac{2}{13}$, $\frac{1}{12}$, $\frac{1}{7}$.
12. $\frac{5}{11}$, $\frac{1}{15}$, $\frac{2}{3}$.
13. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{7}$.
14. $\frac{12}{27}$, $\frac{10}{19}$.
15. $\frac{103}{257}$, $\frac{53}{125}$.
16. $\frac{1}{131}$, $\frac{1}{147}$.

Perform the following mentally:

17. $\frac{1}{2}$, $\frac{5}{3}$.
18. $\frac{2}{5}$, $\frac{3}{7}$.
19. $\frac{5}{8}$, $\frac{1}{3}$.
20. $\frac{11}{3}$, $\frac{4}{5}$.
21. $\frac{3}{2}$, $\frac{4}{3}$.
22. $\frac{1}{7}$ $\frac{1}{6}$.

To Reduce Fractions to Equivalent Fractions having the Least Common Denominator.

122. Rule.—*Find the Least Common Multiple of all the denominators for the Least Common Denominator. Then multiply both terms of each fraction by the quotient of the Least Common Multiple divided by the denominator of that fraction.*

Demonstration.—The purpose for which we get the least common multiple is that we may know what the *least* number is which can be produced by multiplying each denominator by some number. Then we divide this least common multiple by each of the denominators in turn to find by what both terms of each particular fraction must be multiplied in order to reduce the fraction to one having this least common multiple for its denominator. That the values of the fractions are not changed is evident from (**108**).

N. B.—*This rule presumes all the fractions to be in their lowest terms.*

1. Reduce $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{7}{12}$ to equivalent fractions having the least common denominator.

Solution.—The least common multiple of 6, 8, and 12 is 24 (**118**). Now to make the donominator of $\frac{5}{6}$ 24, we must multiply it by 4. But to preserve the value of the fraction we must multiply the numerator also by 4. Hence we have $\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$.

In like manner, $\frac{3 \times 3}{8 \times 3} = \frac{9}{24}$.

And $\frac{7 \times 2}{12 \times 2} = \frac{14}{24}$.

[The pupil tell why $\frac{5}{6} = \frac{20}{24}$, $\frac{3}{8} = \frac{9}{24}$, and $\frac{7}{12} = \frac{14}{24}$.]

Reduce the following fractions to equivalent sets of fractions having the least common denominators, performing all that you can mentally:

2. $\frac{5}{8}$ $\frac{2}{6}$.	7. $\frac{3}{5}$, $\frac{7}{10}$.	12. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$.
3. $\frac{7}{15}$, $\frac{9}{10}$, $\frac{4}{5}$.	8. $\frac{1}{10}$, $\frac{1}{100}$.	13. $\frac{3}{16}$, $\frac{13}{20}$, $\frac{25}{60}$.
4. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{8}{9}$, $\frac{3}{10}$.	9. $\frac{4}{5}$, $\frac{11}{3}$, $\frac{1}{12}$, $\frac{9}{16}$.	14. $\frac{1}{30}$, $\frac{7}{40}$, $\frac{19}{100}$.
5. $\frac{4}{15}$, $\frac{1}{30}$, $\frac{5}{6}$.	10. $\frac{3}{5}$, $\frac{1}{3}$, $\frac{7}{9}$, $\frac{8}{11}$.	15. $\frac{3}{20}$, $\frac{1}{4}$, $\frac{11}{32}$, $\frac{5}{8}$.
6. $\frac{2}{24}$, $\frac{5}{32}$, $\frac{7}{16}$.	11. $\frac{1}{6}$, $\frac{5}{12}$.	16. $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$.

SECTION III.

ADDITION AND SUBTRACTION OF FRACTIONS.

1. If John has $\frac{5}{8}$ of a melon and James has $\frac{3}{8}$, how many eighths have both? $\frac{5}{8} + \frac{3}{8} =$ how many?

How much more has John than James? $\frac{5}{8} - \frac{3}{8} =$ how many?

2. If John has $\frac{1}{6}$ of a melon and James has $\frac{2}{3}$, how much have both together? The $\frac{2}{3}$ are how many 6ths?

How much more has James than John?

To Add or Subtract Fractions.

123. Rule.—*Reduce the fractions to equivalent ones having a common denominator (if they have not such a denominator at first), and then add or subtract the numerators as any other numbers, writing the result over the common denominator.*

Demonstration.—Reducing the fractions to forms having a common denominator does not change the values of the fractions, and hence does not change their sum or difference. When they have like denominators, their numerators represent parts of the same kind, and hence can be added or subtracted.*

3. What is the sum of $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$?

Solution.—Reducing these fractions to forms having a common denominator, we have $\frac{2}{3} = \frac{8}{12}$, $\frac{5}{6} = \frac{10}{12}$. Hence if we find the sum of $\frac{8}{12}$, $\frac{10}{12}$, and $\frac{7}{12}$, it will be the sum of $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$, since the former are equivalent to the latter.

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* It may be necessary for the teacher to illustrate the fact that only numbers representing *like* things can be added or subtracted.

Now $\frac{8}{12}+\frac{10}{12}+\frac{7}{12}=\frac{25}{12}$, for they represent 8, 10, and 7 things all of the same kind, viz., 12ths. Finally the improper fraction $\frac{25}{12}=2\frac{1}{12}$ by (**114**).

Perform the following additions, reducing the results to mixed or whole numbers when improper fractions arise, and to their lowest terms when they are proper fractions:

4. $\frac{3}{5}+\frac{2}{7}+\frac{4}{15}$. *Sum*, $1\frac{16}{105}$.
5. $\frac{5}{32}+\frac{1}{6}+\frac{1}{24}$. *Sum*, $\frac{35}{96}$.
6. $\frac{3}{10}+\frac{2}{11}+\frac{3}{5}+\frac{3}{4}$. *Sum*, $1\frac{183}{220}$.
7. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$. *Sum*, $2\frac{3}{4}$.
8. $\frac{3}{4}+\frac{5}{6}+\frac{2}{3}+\frac{4}{9}$. *Sum*, $2\frac{25}{36}$.
9. $\frac{5}{8}+\frac{3}{5}+\frac{7}{10}+\frac{1}{6}$. *Sum*, $2\frac{11}{120}$.
10. $\frac{3}{7}+\frac{4}{5}+\frac{9}{20}+\frac{5}{18}$. *Sum*, $1\frac{241}{252}$.
11. $\frac{3}{2}+\frac{12}{5}+\frac{8}{3}+\frac{11}{6}$. *Sum*, $8\frac{2}{5}$.
12. $\frac{9}{4}+\frac{17}{3}+\frac{23}{6}+\frac{41}{2}+\frac{3}{4}$. *Sum*, 33.
13. $\frac{111}{5}+\frac{42}{15}+\frac{105}{3}$. *Sum*, 60.

14. Subtract $\frac{3}{5}$ from $\frac{11}{15}$.

Suggestion. $\frac{3}{5}=\frac{9}{15}$. Hence $\frac{11}{15}-\frac{3}{5}=\frac{11}{15}-\frac{9}{15}=\frac{2}{15}$.

Perform the following subtractions. Most of them should be done without writing:

15. $\frac{5}{6}-\frac{1}{3}$. *Rem.*, $\frac{1}{2}$.
16. $\frac{7}{8}-\frac{1}{2}$. *Rem.*, $\frac{3}{8}$.
17. $\frac{4}{5}-\frac{2}{3}$. *Rem.*, $\frac{2}{15}$.
18. $\frac{11}{12}-\frac{4}{7}$. *Rem.*, $\frac{29}{84}$.
19. $\frac{11}{3}-\frac{2}{5}$.
20. $\frac{14}{9}-\frac{1}{2}$.
21. $\frac{7}{10}-\frac{4}{9}$.
22. $\frac{13}{18}-\frac{9}{10}$.
23. $\frac{15}{4}-\frac{3}{4}=3$.
24. $\frac{17}{3}-\frac{51}{9}=0$.
25. $\frac{222}{4}-\frac{1}{2}=55$.
26. $\frac{1145}{137}-\frac{27}{115}=\frac{127976}{15755}$.

Perform the following mentally:

27. $\frac{1}{2}+\frac{2}{3}$.
28. $\frac{5}{6}-\frac{1}{2}$.
29. $\frac{1}{3}+\frac{1}{4}$.
30. $\frac{3}{4}-\frac{1}{3}$.
31. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$.
32. $\frac{11}{12}-\frac{5}{6}$.
33. $\frac{43}{15}-\frac{2}{3}$.
34. $\frac{2}{3}+\frac{3}{2}+\frac{5}{6}$.
35. $\frac{1}{5}+\frac{1}{7}$.
36. $\frac{1}{5}-\frac{1}{7}$.
37. $\frac{11}{4}-\frac{5}{8}$.
38. $\frac{13}{6}+\frac{21}{48}$.

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39. Add $4\frac{2}{3}$ and $5\frac{2}{3}$.

Suggestion.—Adding the fractions, we have $\frac{2}{3}+\frac{2}{3}=\frac{4}{3}=1\frac{1}{3}$. Writing the $\frac{1}{3}$ as a fraction and adding the integer 1 to the integers 4 and 5, we have $10\frac{1}{3}$.

$$\begin{array}{r} 4\frac{2}{3} \\ 5\frac{2}{3} \\ \hline 10\frac{1}{3} \end{array}$$

40. Add $5\frac{1}{2}$, $11\frac{3}{4}$, and $24\frac{1}{6}$.

What is the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{6}$?

41. Add $214\frac{1}{3}$, $517\frac{7}{48}$, and $145\frac{5}{12}$. *Sum*, $876\frac{43}{48}$.

42. Add $3\frac{2}{3}$, $17\frac{3}{4}$, and $28\frac{5}{12}$. *Sum*, $49\frac{5}{6}$.

43. Add $3\frac{5}{6}$, $17\frac{2}{7}$, and $4\frac{3}{21}$. *Sum*, $25\frac{11}{42}$.

44. Add 126, $13\frac{2}{3}$, $40\frac{5}{6}$, and $7\frac{1}{2}$. *Sum*, 188.

To Add Mixed Numbers or Whole Numbers and Fractions.

124. Rule.—*Add the fractions first, reducing the result, if an improper fraction, to a mixed number, and writing the fraction thus arising, prefix to it the sum of the integers and the integer (if any) arising from adding the fractions.*

Mixed numbers may be reduced to improper fractions and then added; but this is not a desirable method.

45. Add $4\frac{2}{3}$, $5\frac{1}{2}$, and $\frac{3}{4}$ by both the above methods, and see which is better.

46. Add $\frac{3}{5}$, $2\frac{1}{7}$, $10\frac{4}{15}$, $3\frac{1}{3}$, and 8.

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47. From $256\frac{5}{6}$ take $117\frac{1}{3}$.

Suggestion.—Since $\frac{1}{3} = \frac{2}{6}$, we have $256\frac{5}{6} - 117\frac{2}{6}$. $\frac{2}{6}$ from $\frac{5}{6}$ leaves $\frac{3}{6}$ or $\frac{1}{2}$; and 117 from 256 leaves 139. Hence $256\frac{5}{6} - 117\frac{2}{6} = 139\frac{1}{2}$.

$$\begin{array}{r} 256\frac{5}{6} \\ 117\frac{2}{6} \\ \hline 139\frac{1}{2} \end{array}$$

48. From $34\frac{3}{4}$ take $16\frac{1}{2}$.

49. From $83\frac{11}{12}$ take $27\frac{2}{3}$.

50. From $157\frac{2}{5}$ take $68\frac{5}{7}$.

Suggestion.—As $\frac{14}{35}$ is less than $\frac{25}{35}$, we take $1 = \frac{35}{35}$ from the 7, which with the $\frac{14}{35}$ makes $\frac{49}{35}$. Then taking $\frac{25}{35}$ from $\frac{49}{35}$, and 68 from 156, there remains $88\frac{24}{35}$.

$$\begin{array}{rcr} 157\frac{2}{5} & = & 157\frac{14}{35} \\ 68\frac{5}{7} & = & 68\frac{25}{35} \\ \hline & & 88\frac{24}{35} \end{array}$$

51. From $7\frac{2}{3}$ take $3\frac{3}{4}$. *Rem.*, $3\frac{11}{12}$.

52. From 13 take $9\frac{3}{8}$.

Suggestion.—As there is no fraction in the minuend, we take one of the 3 units from which to subtract the $\frac{2}{3}$ in the subtrahend. $\frac{2}{3}$ from 1 ($\frac{3}{3}$) leaves $\frac{1}{3}$, and 9 from 12 leaves 3. Hence the remainder is $3\frac{1}{3}$.

$$\begin{array}{r} 13 \\ 9\frac{2}{3} \\ \hline 3\frac{1}{3} \end{array}$$

53. From 285 take $156\frac{7}{11}$. *Rem.*, $128\frac{4}{11}$.
54. From 11 take $\frac{3}{5}$. From 11 take $3\frac{2}{7}$.
55. From $6\frac{3}{5}$ take 4. *Rem.*, $2\frac{3}{5}$.
56. From $81\frac{5}{7}$ take 43. *Rem.*, $38\frac{5}{7}$.

To Perform Subtraction when both Whole Numbers and Fractions are involved.

125. Rule.—*First take the fraction in the subtrahend from that in the minuend, or if the latter fraction be the smaller, from it increased by 1; and then subtract the remaining integers, prefixing the latter remainder to the former.*

57. From $75\frac{3}{11}$ take $28\frac{5}{7}$.
58. From $1\frac{3}{4}$ take $\frac{5}{6}$.
59. From 2 take $\frac{2}{13}$.
60. From 8 take $\frac{11}{23}$.
61. From $31\frac{15}{16}$ take $28\frac{3}{4}$.
62. From $1285\frac{17}{23}$ take $999\frac{19}{21}$.

SECTION IV.

MULTIPLICATION OF FRACTIONS.

To Multiply a Fraction by an Integer.

126. Rule.—*Multiply the numerator, or divide the denominator by the integer.*

This is the same as Principles I. and II. (**104**, **105**). If necessary review those, and solve the following, giving the explanations as in those articles:

1. $\frac{5}{7} \times 3 =$ what?	5. $\frac{121}{347} \times 264.$	9. $\frac{7}{16} \times 4.$
2. $\frac{3}{8} \times 2 =$ what?	6. $\frac{71}{189} \times 81.$	10. $\frac{28}{125} \times 25.$
3. $\frac{11}{13} \times 7 =$ what?	7. $\frac{134}{182} \times 91.$	11. $\frac{43}{17} \times 12.$
4. $\frac{5}{23} \times 12 =$ what?	8. $\frac{251}{1000} \times 500.$	12. $\frac{1412}{3471} \times 286.$

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13. Multiply $\frac{3}{16}$ by 40, using the factors of 40, 8 and 5.

Suggestion. $\frac{3}{16} \times 8 = \frac{3}{2}$; and $\frac{3}{2} \times 5 = \frac{15}{2} = 7\frac{1}{2}$. Hence $\frac{3}{16} \times 40 = 7\frac{1}{2}$. What principles are used? What in Multiplication? What *two* in fractions?

Solve as above and explain the following:

14. $\frac{9}{56} \times 24.$	17. $\frac{12}{25} \times 15.$	20. $\frac{124}{343} \times 147.$
15. $\frac{11}{81} \times 27.$	18. $\frac{8}{9} \times 6.$	21. $\frac{51}{144} \times 60.$
16. $\frac{37}{125} \times 50.$	19. $\frac{4}{15} \times 21.$	22. $\frac{31}{121} \times \frac{7}{11}.$

[Let the pupils *write* a rule for this case, that is, when the multiplier contains a factor which is also a factor of the denominator of the multiplicand.]

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23. Multiply $12\frac{3}{7}$ by 9.

Suggestion. 9 times $\frac{3}{7}$ are $\frac{27}{7}$, which is 3 and $\frac{6}{7}$. 9 times 12 are 108, to which adding the 3 we find that 9 times $12\frac{3}{7}$ are $111\frac{6}{7}$.

$$\begin{array}{r} 12\frac{3}{7} \\ 9 \\ \hline 111\frac{6}{7} \end{array}$$

Solve as above the following, performing such as you can mentally:

24. $23\frac{2}{3}. \times 11.$	28. $3472\frac{3}{4} \times 7.$	32. $482\frac{3}{7} \times 126.$
25. $137\frac{11}{43} \times 12.$	29. $126\frac{123}{411} \times 23.$	33. $1\frac{2}{5} \times 10.$
26. $2\frac{1}{2} \times 5.$	30. $4\frac{3}{7} \times 8.$	34. $10\frac{1}{2} \times 6.$
27. $5\frac{1}{3} \times 6.$	31. $5\frac{3}{8} \times 4.$	35. $16\frac{1}{2} \times 8.$

N. B.—Let the pupils *write* the rule and state the case.

36. Multiply $\frac{3}{5}$ by 5.

Suggestion.—If we divide the denominator by 5, according to the rule, we have $\frac{3}{1}$, or 3. Or, if we multiply the numerator, we have $\frac{15}{5}$, or 3. Hence we see that

127. *To multiply a fraction by a number equal to its denominator, we simply drop the denominator.*

37. What is $\frac{5}{7} \times 7$? $\frac{2}{3} \times 3$? $\frac{12}{13} \times 13$? $4\frac{1}{2} \times 2$?

To Multiply by a Fraction.

1. If I multiply a number by 12, how many times as great a product do I get as when I multiply by 6, which is $\frac{1}{2}$ of 12? How many times as great as when I multiply by 3? By 4?

2. If I wish the product of 13 when multiplied by 3, how can I get it from 6 times 13, which is 78? How can I get 3 times 13 from 12 times 13, which is 156?

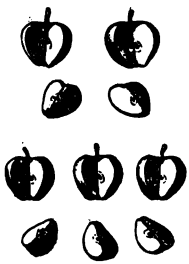
Principle I.

128. *Any multiplicand multiplied by any multiplier gives* 2 *times as great a product as when multiplied by* $\frac{1}{2}$ *that multiplier,* 3 *times as great a product as when multiplied by* $\frac{1}{3}$ *that multiplier,* 4 *times as great as when multiplied by* $\frac{1}{4}$ *that multiplier, etc.*

3. By what must I divide to get 5 times 23 from 230, which is 10 times 23? From 460, which is 20 times 23?

4. Eight times 7 are 56; from this how many are 4 times 7? 2 times 7?

What part of 8 times 7 is 4 times 7? 2 times 7?



1. Here are two apples divided into thirds. $\frac{1}{3}$ of each of the 2 apples makes how much?

What part of 2 is $\frac{2}{3}$?

2. What part of 3 is $\frac{3}{4}$? That is, if 3 things are divided into 4ths, and 1 is taken from each, how much is taken? What part of the 3 is taken?

3. What part of 5 is $\frac{5}{8}$? Analyze * the operation as above.

4. In like manner show that $\frac{7}{8}$ is $\frac{1}{8}$ of 7. That $\frac{11}{13}$ is $\frac{1}{13}$ of 11. That $\frac{4}{5}$ is $\frac{1}{5}$ of 4. That $\frac{13}{7}$ is $\frac{1}{7}$ of 13. That $\frac{141}{8}$ is $\frac{1}{8}$ of 141.

Principle II.

129. *The Numerator of a fraction may be considered as a Dividend and the Denominator as a Divisor of it, the value of the fraction being the quotient.*

5. Show that $\frac{7}{11}$ may be considered as 7 divided by 11.

If we take 7 things and divide each into 11 equal parts, and then take 1 part from each, what part of the 7 have we? How do we get $\frac{1}{11}$ of anything (92)? Then how many 11ths have we when we divide 7 by 11?

6. Show that $\frac{6}{5}$ may be considered as 6 divided by 5.

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1. Multiply 15 by $\frac{3}{5}$.

Solution. If we multiply 15 by 3, we have 45; but according to (**129**) $\frac{3}{5}$ is only $\frac{1}{5}$ of 3; hence, according to (**128**), 45 is 5 times as large as it should be. Therefore $45 \div 5 = 9$ is the correct product.†

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* The teacher may explain that solving a question and giving the reasons for each step is sometimes called analyzing it.

† If thought best the teacher can explain that this is the same as multiplying

2. Multiply 7 by $\frac{2}{5}$.

Solution. 2 times 7 = 14. But since the multiplier is only $\frac{1}{5}$ of 2, this product is 5 times as large as it should be. Hence the true product is $14 \div 5 = 2\frac{4}{5}$.

3. Multiply 8 by $\frac{3}{4}$, and explain as above.

4. Multiply 11 by $\frac{3}{4}$, and explain as above.

5. Multiply 13 by $\frac{2}{3}$; 15 by $\frac{2}{3}$; 14 by $\frac{4}{5}$; 18 by $\frac{7}{9}$.

6. Multiply $\frac{3}{7}$ by $\frac{4}{5}$.

Solution.—Multiplying $\frac{3}{7}$ by 4 (**104**) we have $\frac{12}{7}$. But this product is 5 times as great as it should be by (**128**). Hence we must divide $\frac{12}{7}$ by 5. $\frac{12}{7} \div 5 = \frac{12}{35}$ by (**107**).

7. Multiply $\frac{7}{8}$ by $\frac{2}{5}$.

How do you multiply by 2? How divide by 5?

8. Multiply $\frac{4}{9}$ by $\frac{3}{2}$.

To multiply by 3 divide the denominator; thus $\frac{4}{9} \times 3 = \frac{4}{3}$ (**105**). Now to divide $\frac{4}{3}$ by 2, divide the numerator; thus $\frac{4}{3} \div 2 = \frac{2}{3}$. Hence $\frac{4}{9} \times \frac{3}{2} = \frac{2}{3}$.

9. Multiply $\frac{6}{21}$ by $\frac{7}{3}$, performing both operations by dividing.

To Multiply by a Fraction.

130. Rule.—*Multiply by the numerator and divide by the denominator.*

Why multiply by the numerator? Why divide by the denominator?

Perform the operations by division as far as practicable; and when the multiplicand is an integer which is divisible by the denominator of the multiplier, divide *first*.

Solve the following, performing the operations by division whenever practicable; and when the multipli-

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by the factors of the multiplier in succession, according to (**58**). Thus the factors of $\frac{3}{5}$ are 3 and $\frac{1}{5}$. $15 \times 3 = 45$, and to multiply by $\frac{1}{5}$ is to take $\frac{1}{5}$ of a number or to divide by 5. Hence $45 \times \frac{1}{5} = 9$.

cand is an integer, divide *first* if you can. Do *all* mentally:

10. $12 \times \frac{5}{6}$. *Prod.*, 10.	16. $\frac{5}{8} \times \frac{2}{5}$.	22. $\frac{3}{7} \times \frac{4}{5}$.
11. $13 \times \frac{5}{6}$. *Prod.*, $10\frac{5}{6}$.	17. $\frac{4}{9} \times \frac{3}{5}$.	23. $\frac{8}{11} \times \frac{3}{2}$.
12. $\frac{12}{35} \times \frac{5}{6}$. *Prod.*, $\frac{2}{7}$.	18. $\frac{14}{5} \times \frac{4}{7}$.	24. $\frac{5}{13} \times \frac{4}{5}$.
13. $\frac{11}{13} \times \frac{5}{6}$. *Prod.*, $\frac{55}{78}$.	19. $\frac{35}{12} \times \frac{6}{7}$.	25. $\frac{4}{7} \times \frac{1}{2}$.
14. $28 \times \frac{7}{4}$. *Prod.*, 49.	20. $15 \times \frac{3}{5}$.	26. $20 \times \frac{1}{4}$.
15. $17 \times \frac{2}{3}$. *Prod.*, $11\frac{1}{3}$.	21. $31 \times \frac{2}{7}$.	27. $11 \times \frac{1}{3}$.

To multiply by $\frac{1}{2}$ is to take what part of a number? To multiply by $\frac{1}{3}$? By $\frac{1}{5}$?*

Will multiplying by $\frac{2}{3}$ give a greater or a less product than multiplying by $\frac{1}{3}$? How much?

To multiply by $\frac{3}{5}$ is to take $\frac{3}{5}$ of the multiplicand. How do you get $\frac{1}{5}$? From this result how do you get $\frac{3}{5}$ of the multiplicand?

See if you can explain multiplying by a fraction on these principles.

The rule for multiplying one fraction by another is sometimes given thus: *Multiply the numerators together for the numerator of the product, and the denominators together for the denominator of the product.*

Can you give the reasons for this rule? They are just the same as for the rule (**130**). How does multiplying the numerator of the multiplicand affect the multiplicand? How does multiplying the denominator of the multiplicand affect the multiplicand.

Perform and explain the following:

28. $\frac{8}{13} \times \frac{3}{10}$.	31. $184 \times \frac{2}{3}$.	34. $53 \times \frac{127}{43}$.	37. $\frac{17}{43} \times \frac{11}{57}$.
29. $\frac{10}{11} \times \frac{3}{20}$.	32. $300 \times \frac{5}{3}$.	35. $256 \times \frac{342}{576}$.	38. $\frac{121}{19} \times \frac{7}{183}$.
30. $\frac{5}{7} \times \frac{4}{5}$.	33. $72 \times \frac{2}{9}$.	36. $1000 \times \frac{3}{500}$.	39. $\frac{427}{1025} \times \frac{376}{41}$.

* It may be well to omit these six paragraphs until a review is taken. Do not trouble the beginner with too many methods.

Cancellation.

40. Multiply $\frac{10}{21}$ by $\frac{28}{15}$.

Solution. $\frac{10}{21} \times \frac{28}{15} = \frac{10 \times 28}{21 \times 15}$. Now we can see a factor 5 in the 10 in the numerator and also in the 15 in the denominator. Rejecting this factor from the numerator divides it by 5; and rejecting it also from the denominator divides *it* by 5. Hence if we reject it from both we shall not alter the value of the fraction (**108**). Thus we have $\frac{10 \times 28}{21 \times 15} = \frac{2 \times 28}{21 \times 3}$. Again, we see a factor 7 in the 28 in the numerator, and also a factor 7 in the 21 in the denominator, which can be rejected for a like reason with the 5. (What is the reason?) Thus we have $\frac{2 \times 28}{21 \times 3} = \frac{2 \times 4}{3 \times 3} = \frac{8}{9}$.

131. Rejecting equal factors which appear both as multipliers and divisors in an operation is called **Cancellation.**

Cancelling such equal factors does not change the value of the result, because dropping a factor from a multiplier would divide the result, while dropping it from a divisor would multiply the result. Hence one operation compensates for the other.

41. Multiply together the three fractions $\frac{3}{4}$, $\frac{12}{35}$, and $\frac{7}{24}$.

Operation. $\frac{3}{4} \times \frac{12}{35} \times \frac{7}{24} = \frac{3 \times \cancel{12} \times \cancel{7}}{4 \times \underset{5}{\cancel{35}} \times \underset{2}{\cancel{24}}} = \frac{3}{40}$.

Explanation.—Observing the factor 12 in the numerator and a like factor in the 24 in the denominator, we cancel it, leaving a factor 2 in place of 24 in the denominator. This does not change the value of the result. (Why?) In like manner we cancel the 7 in the numerator and the same factor in 35 in the denominator. (Why?) As there are now no common factors, we perform the indicated multiplications, and have $\frac{3}{40}$ as the product.

42. Perform by cancellation $\frac{10}{11} \times \frac{3}{20} \times \frac{1}{6} \times \frac{5}{7} \times \frac{21}{5}$.

Operation. $\frac{10}{11} \times \frac{3}{20} \times \frac{1}{6} \times \frac{5}{7} \times \frac{21}{5} = \frac{\cancel{10} \times \cancel{3} \times 1 \times \cancel{5} \times \overset{3}{\cancel{21}}}{11 \times \underset{2}{\cancel{20}} \times \underset{2}{\cancel{6}} \times \cancel{7} \times \cancel{5}} = \frac{3}{44}$.

Pupil trace the process and give the reasons.

43. What is $\frac{3}{5}$ of $\frac{4}{7}$?

Observe that to multiply by $\frac{3}{5}$ is to take $\frac{3}{5}$ of a number; or to take $\frac{1}{5}$ 3 times. Hence $\frac{3}{5}$ of $\frac{4}{7}$ is $\frac{4}{7} \times \frac{3}{5}$; or, since in multiplying it is immaterial which of the factors we use as the multiplier, $\frac{3}{5}$ of $\frac{4}{7}$ is the same as $\frac{3}{5} \times \frac{4}{7}$.

***Fractions connected by the word "of" are sometimes called* Compound Fractions. This "of" is simply equivalent to the sign $\times$.**

Perform the following by cancellation:

44. $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{3}{5}$ of $\frac{8}{9} = \frac{8}{27}$.
45. $\frac{3}{4} \times \frac{2}{3} \times \frac{15}{18} \times \frac{8}{21} \times \frac{9}{5}$.
46. $8 \times \frac{13}{6} \times \frac{5}{39} \times \frac{3}{10} \times \frac{8}{3}$.
47. $15 \times \frac{2}{5} \times \frac{5}{3} \times \frac{3}{2} \times \frac{7}{10}$.
48. $\frac{3}{5}$ of $\frac{35}{27}$ of $\frac{6}{21}$ of $\frac{14}{27}$.
49. $\frac{1}{2}$ of $\frac{1}{3}$ of 12.
50. $\frac{1}{3}$ of $\frac{1}{4}$; $\frac{1}{5}$ of $\frac{1}{6}$; $\frac{1}{2}$ of $\frac{1}{7}$.
51. $30 \times \frac{2}{5} \times \frac{1}{6} \times \frac{1}{12}$.

Perform the following, first reducing the mixed numbers to improper fractions:

52. $5\frac{3}{8} \times 4\frac{3}{4} = 25\frac{17}{32}$.
53. $8 \times 2\frac{3}{5} \times \frac{5}{39} \times \frac{3}{10} \times 2\frac{2}{3} = 2\frac{2}{15}$.
54. $8\frac{2}{3} \times 2\frac{3}{13} \times 3\frac{3}{4} \times 16\frac{2}{3} = 1208\frac{1}{3}$.
55. $4\frac{2}{7} \times 5\frac{3}{5} = 24$.
56. $2\frac{3}{5} \times 3\frac{3}{4} = 9\frac{3}{4}$.
57. $7\frac{1}{2} \times 4\frac{1}{3}$.

Two mixed numbers may be multiplied as in the margin. See if you can trace the process.

$$\begin{array}{r} 4\frac{2}{7} \\ 5\frac{3}{5} \\ \hline 20 + \frac{10}{7} + \frac{12}{5} + \frac{6}{35} = 24 \end{array}$$

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58. Multiply $\frac{3}{35}$ by $\frac{7}{3}$, by cancellation.

Suggestion.—We have $\frac{\cancel{3} \times \cancel{7}}{\underset{5}{\cancel{35}} \times \cancel{3}}$. Now as both the 3 and 7 are cancelled in the numerator, what remains? Is the numerator of the product 0? No; if we remember that cancellation is only *dividing* the terms of the fraction by the same number, we shall see that really there is a factor 1 to be *understood* when we cancel 7, and also when we cancel 3, so that the result would be $\frac{1 \times 1}{5 \times 1} = \frac{1}{5}$.

59. Perform the following: $\frac{2}{3} \times \frac{15}{16} \times \frac{2}{5} \times \frac{4}{9}$; $\frac{3}{7} \times \frac{21}{32} \times \frac{8}{9} \times 4$; $\frac{1}{5} \times \frac{1}{7} \times 35$; $\frac{12}{7} \times \frac{63}{8} \times 10 \times \frac{1}{9}$.

SECTION V.

DIVISION OF FRACTIONS.

To Divide a Fraction by an Integer.

132. Rule.—*Divide the numerator, or multiply the denominator by the integer.*

This is the same as Principles III. and IV. (**106, 107**). If necessary review those, and solve the following, giving the explanations as under those articles :

1. $\frac{2}{7} \div 5$ = what ?	6. $\frac{128}{137} \div 7$.	11. $\frac{50}{7} \div 10$.
2. $\frac{11}{3} \div 4$ = what ?	7. $\frac{35}{73} \div 7$.	12. $\frac{39}{87} \div 13$.
3. $\frac{5}{6} \div 3$ = what ?	8. $\frac{343}{782} \div 151$.	13. $\frac{143}{782} \div 111$.
4. $\frac{4}{13} \div 2$ = what ?	9. $\frac{38}{163} \div 19$.	14. $\frac{279}{466} \div 93$.
5. $\frac{15}{13} \div 5$ = what ?	10. $\frac{1}{5} \div 5$.	15. $\frac{171}{500} \div 20$.

To Divide by a Fraction.

1. How many sixths are there in 1? Then how many times is $\frac{1}{6}$ contained in 1?

2. What is $1 \div \frac{1}{4}$? $1 \div \frac{1}{3}$? $1 \div \frac{1}{5}$? $1 \div \frac{1}{8}$? Tell why in each case.

3. How many times is $\frac{2}{6}$ contained in 1?

Solution.—Since there are 6 sixths in 1, $\frac{1}{6}$ is contained 6 times in 1. And as $\frac{2}{6}$ is 2 times $\frac{1}{6}$, $\frac{2}{6}$ is contained in 1 only $\frac{1}{2}$ as many times as $\frac{1}{6}$ is. Hence $\frac{2}{6}$ is contained $\frac{1}{2}$ of 6, or 3 times in 1.*

* If thought expedient the teacher may show that this is but dividing successively by the factors of the divisor according to (**89**). Thus to divide 1 by $\frac{2}{3}$ we divide successively by $\frac{1}{3}$ and 2. $1 \div \frac{1}{3} = 3$, and $3 \div 2 = \frac{3}{2}$.

4. How many times is $\frac{2}{3}$ contained in 1?

Solution.—Since there are 3 thirds in 1, $\frac{1}{3}$ is contained 3 times in 1. And as $\frac{2}{3}$ is 2 times $\frac{1}{3}$, $\frac{2}{3}$ is contained in 1 only $\frac{1}{2}$ as many times as $\frac{1}{3}$ is. Hence $\frac{2}{3}$ is contained $\frac{1}{2}$ of 3, or $\frac{3}{2}$ times in 1.*

5. In like manner find how many times $\frac{3}{5}$ is contained in 1; $\frac{2}{7}$; $\frac{3}{4}$; $\frac{6}{11}$; $\frac{7}{9}$.

Principle.

133. *A* **Fraction** *inverted shows how many times that fraction is contained in* 1.

134. The **Reciprocal** of a number is 1 divided by that number; hence the *Reciprocal of a fraction* is the fraction inverted.

6. Find how many times each of the following is contained in 1, and give the solution in each case: $\frac{3}{5}$; $\frac{5}{7}$; $\frac{4}{13}$; $\frac{5}{4}$; $\frac{7}{2}$; $\frac{3}{2}$; $\frac{8}{3}$; $\frac{11}{13}$; $\frac{121}{237}$.

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1. How many times is $\frac{2}{3}$ contained in 5?

Solution. $\frac{2}{3}$ is contained in 1 $\frac{3}{2}$ times: and since 5 is 5 times 1, $\frac{2}{3}$ is contained 5 times as many times in 5 as in 1. Hence $\frac{2}{3}$ is contained 5 times $\frac{3}{2}$, or $\frac{15}{2} = 7\frac{1}{2}$ times in 5.

2. How many times is $\frac{3}{7}$ contained in 6?

Suggestions.—How many times is $\frac{3}{7}$ contained in 1? If $\frac{3}{7}$ is contained $\frac{7}{3}$ times in 1, how many times is it contained in 6?

Ans., 14 times.

3. How many times is $\frac{5}{8}$ contained in 7?

Ans., $11\frac{1}{5}$ times.

4. How many times is $\frac{2}{3}$ contained in $\frac{5}{7}$?

Solution. $\frac{2}{3}$ is contained $\frac{3}{2}$ times in 1, and in $\frac{5}{7}$ it is contained $\frac{5}{7}$

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* See note on preceding page.

as many times as in 1. Hence $\frac{2}{3}$ is contained in $\frac{5}{7}$, $\frac{5}{7}$ of $\frac{3}{2}$, or $\frac{15}{14}$ times.

5. How many times is $\frac{4}{9}$ contained in $\frac{5}{11}$?

Suggestions. $1 \div \frac{4}{9} = \frac{9}{4}$. Hence $\frac{5}{11} \div \frac{4}{9} = \frac{5}{11}$ of $\frac{9}{4} = \frac{45}{44} = 1\frac{1}{44}$.

To Divide by a Fraction.

135. Rule.—*Invert the divisor and multiply the result by the dividend.*

Demonstration.—The divisor inverted shows how many times the divisor is contained in 1. Then in 2 it will be contained 2 times as many times as in 1; in 3, 3 times as many times; in 4, 4 times as many times; in $\frac{2}{3}$, $\frac{2}{3}$ as many times; in $\frac{5}{7}$, $\frac{5}{7}$ as many times, etc.

6. Divide 11 by $\frac{3}{7}$.

Solution. $\frac{3}{7}$ is contained $\frac{7}{3}$ times in 1, and in 11 it is contained 11 times as many times as in 1. Hence $11 \div \frac{3}{7} = 11$ times $\frac{7}{3} = \frac{77}{3} = 25\frac{2}{3}$.

7. Divide $\frac{5}{2}$ by $\frac{8}{11}$.

Solution. $\frac{8}{11}$ is contained $\frac{11}{8}$ times in 1, and in $\frac{5}{2}$ it is contained $\frac{5}{2}$ times as many times as in 1. Hence $\frac{5}{2} \div \frac{8}{11} = \frac{5}{2}$ times $\frac{11}{8} = \frac{55}{16} = 3\frac{7}{16}$.

Perform the following, giving the solution in full:

8. $8 \div \frac{2}{9} = 36$.	16. $12 \div \frac{5}{11}$.	24. $5 \div \frac{2}{3}$.
9. $\frac{4}{7} \div \frac{3}{11} = 2\frac{2}{21}$.	17. $\frac{15}{47} \div \frac{11}{13}$.	25. $\frac{1}{2} \div \frac{1}{3}$.
10. $\frac{5}{9} \div \frac{4}{7} = \frac{35}{36}$.	18. $\frac{2}{9} \div \frac{7}{5}$.	26. $\frac{1}{3} \div \frac{1}{5}$.
11. $11 \div \frac{2}{3} = 16\frac{1}{2}$.	19. $127 \div \frac{5}{19}$.	27. $\frac{1}{7} \div \frac{1}{8}$.
12. $17 \div \frac{8}{3} = 6\frac{3}{8}$.	20. $43 \div \frac{1}{2}$.	28. $\frac{6}{111} \div \frac{1}{41}$.
13. $\frac{11}{13} \div \frac{5}{9} = 1\frac{34}{65}$.	21. $71 \div \frac{4}{5}$.	29. $10 \div \frac{2}{5}$.
14. $\frac{7}{2} \div \frac{3}{13} = 15\frac{1}{6}$.	22. $\frac{8}{15} \div \frac{6}{21}$.	30. $100 \div \frac{1}{10}$.
15. $\frac{3}{2} \div \frac{5}{19} = 5\frac{7}{10}$.	23. $\frac{1}{10} \div \frac{3}{7}$.	31. $\frac{7}{103} \div \frac{302}{9}$.

32. Divide $4\frac{2}{3}$ by $\frac{7}{9}$.

Suggestion. $4\frac{2}{3} = \frac{14}{3}$. Now $\frac{14}{3} \div \frac{7}{9} = \frac{\cancel{14}^{2}}{\cancel{3}} \times \frac{\cancel{9}^{3}}{\cancel{7}} = 6$.

Perform the following, reducing the mixed numbers to

improper fractions, and then, putting the operation in the form of a problem in multiplication, cancel as much as possible:

33. $10\frac{2}{3} \div 3\frac{1}{4}$.	37. $42\frac{3}{5} \div 4\frac{11}{15}$.	41. $25\frac{5}{7} \div 2\frac{1}{7}$.
34. $15\frac{2}{3} \div 6\frac{1}{2}$.	38. $\frac{2}{111} \div 4\frac{1}{7}$.	42. $122 \div 7\frac{5}{8}$.
35. $1\frac{3}{21} \div \frac{8}{15}$.	39. $1\frac{1}{5} \div 1\frac{2}{3}$.	43. $\frac{16}{9} \div \frac{4}{9}$.
36. $11\frac{3}{7} \div 7\frac{13}{21}$.	40. $121\frac{1}{2} \div 23\frac{1}{3}$.	44. $\frac{25}{3} \div \frac{5}{3}$.

Complex Fractions.

136. An expression in the form of a fraction having a fraction in its numerator or denominator, or in both, is called a **Complex Fraction**; as $\frac{\frac{2}{3}}{4}$, $\frac{\frac{5}{6}}{\frac{3}{7}}$, $\frac{4\frac{1}{2}}{3}$, etc.

1. What is the numerator of $\frac{\frac{2}{3}}{4}$? If you multiply $\frac{2}{3}$ by 3, what does it become? (**127**). Then what is $\frac{\frac{2}{3}\times 3}{4\times 3} =$ to?

2. What is $\frac{\frac{5}{6}\times 6\times 7}{\frac{3}{7}\times 6\times 7} =$ to? What are the terms of $\frac{\frac{5}{6}}{\frac{3}{7}}$?

137. A fraction with a whole number for a numerator and a whole number for a denominator is called a **Simple Fraction.**

To Reduce a Complex Fraction to a Simple Fraction.

138. Rule.—*Multiply both terms of the complex fraction by the least common multiple of all the denominators of the partial fractions.*

Demonstration.—This does not alter the value of the complex fraction according to (**108**). It destroys the denominators of the partial fractions, because to multiply a fraction by a number equal to its denominator, we simply drop the denominator (**127**), and as

the multiplier contains each denominator of a partial fraction as a factor, all the denominators will disappear in the process.

3. Reduce $\dfrac{\frac{2}{9}}{\frac{5}{12}}$ to a simple fraction.

Solution.—The least common multiple of 9 and 12 is 36. Hence we have $\dfrac{\frac{2}{9}\times 36}{\frac{5}{12}\times 36}$, or $\dfrac{\frac{2}{\cancel{9}}\times 4\times \cancel{9}}{\frac{5}{\cancel{12}}\times \cancel{12}\times 3}=\frac{8}{15}$.

Reduce the following *Complex* fractions to simple fractions, whole or mixed numbers:

4. $\dfrac{\frac{5}{7}}{\frac{2}{3}}=\dfrac{15}{14}$.	9. $\dfrac{10\frac{1}{2}}{5\frac{1}{4}}$.	14. $\dfrac{121\frac{1}{2}}{327\frac{3}{4}}$.
5. $\dfrac{\frac{11}{45}}{\frac{22}{15}}=\dfrac{1}{6}$.	10. $\dfrac{\frac{3}{10}}{\frac{2}{5}}$.	15. $\dfrac{\frac{2}{3}}{111\frac{1}{2}}$.
6. $\dfrac{2\frac{5}{8}}{3\frac{1}{2}}$[*] $=\dfrac{3}{4}$.	11. $\dfrac{5\frac{1}{4}}{7\frac{1}{3}}$.	16. $\dfrac{10\frac{1}{2}}{\frac{5}{6}}$.
7. $\dfrac{2\frac{2}{3}}{5\frac{1}{7}}=\dfrac{14}{27}$.	12. $\dfrac{2}{3\frac{5}{7}}$.	17. $\dfrac{1114\frac{2}{3}}{56\frac{1}{3}}$.
8. $\dfrac{\frac{2}{11}}{5}=\dfrac{2}{55}$.	13. $\dfrac{\frac{2}{7}}{7\frac{5}{7}}$.	18. $\dfrac{49\frac{1}{2}}{12\frac{3}{8}}$.

Applications.

[Let it be borne in mind that *in all cases,* when the numbers are reasonably small and the combinations not too numerous, the solution is to be given without writing, and in ALL CASES the *Reasons for the Processes* are to be given in clear and logical statement.]

1. How many cents make \$1? How many cents are $\frac{1}{2}$ of a dollar? $\frac{1}{3}$ of a dollar? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{8}$? $\frac{1}{10}$?

[*] It is not necessary or best in such cases to reduce the mixed numbers to improper fractions. In this case multiply both terms by 8, thus $\frac{2\frac{5}{8}\times 8}{3\frac{1}{2}\times 8}=\frac{21}{28}=\frac{3}{4}$.

2. *One* is what part of 2? Of 3? 4? 5? 6? 7? 12? 57? 123? 347? 5280?

3. What part of 3 is 2?

Solution. 1 is $\frac{1}{3}$ of 3, and 2 being 2 times as great as 1, is 2 times as great a part of 3 as 1 is. Hence 2 is 2 times $\frac{1}{3}$, or $\frac{2}{3}$ of 3.

4. What part of 7 is 5? 3? 6? 2? 4? Give the solution as above.

5. What part of 13 is 3? 2? 10? 9? 12?

139. *We observe that all we have to do to ascertain what part of one number another is, is to write the former as the denominator and the latter as the numerator of a fraction.*

Of course we should always put results, when fractions, in simple fractions, and these in their lowest terms.

6. What part of 16 is 12? *Ans.*, $\frac{3}{4}$.

7. What part of 8 is $2\frac{2}{3}$? *Ans.*, $\frac{2\frac{2}{3}}{8} = \frac{1}{3}$.

8. What part of 100 is 25? 10? 5? 16? $12\frac{1}{2}$? $33\frac{1}{3}$? $66\frac{2}{3}$? $62\frac{1}{2}$? $37\frac{1}{2}$? $6\frac{1}{4}$? $16\frac{2}{3}$?

9. What part of 18 is 12? 6? 4? 2? 3? $5\frac{1}{3}$? $5\frac{1}{4}$?

10. What part of $\frac{4}{5}$ is $\frac{2}{3}$? *Ans.*, $\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{5}{6}$.

What part of $\frac{1}{2}$ is $\frac{1}{3}$?
What part of $\frac{1}{5}$ is $\frac{1}{3}$?
What part of $\frac{3}{7}$ is $\frac{1}{2}$?

11. How many times $\frac{2}{5}$ is $\frac{6}{7}$? *Ans.*, $\frac{\frac{6}{7}}{\frac{2}{5}} = 2\frac{1}{7}$.

How many times $\frac{2}{3}$ is $\frac{11}{12}$?
How many times $\frac{3}{11}$ is $\frac{5}{7}$?

12. What part of a year (52 weeks) is 13 weeks? 26 weeks? 24 weeks? 12 weeks? 14 weeks? 6 weeks?

13. 320 rods make a mile. What part of a mile is 40 rods? 80 rods? 120 rods? 60 rods?

14. There are 3 feet in a yard, and 12 inches in a foot. What part of a yard is $4\frac{1}{2}$ inches? 9 inches? 18 inches? 27 inches?

15. There are 320 rods in a mile, and $16\frac{1}{2}$ feet in a rod. How many feet are there in a mile? What part of a mile is 2640 feet? 1320? 660? 1980? 400? 1000?

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16. Which is the greater, $\frac{2}{7}$ or $\frac{11}{35}$?

Suggestion. $\frac{2}{7}=\frac{10}{35}$.

17. Which is the greater, $\frac{3}{4}$ or $\frac{13}{17}$? $\frac{4}{7}$ or $\frac{2}{3}$? $\frac{2}{9}$ or $\frac{7}{25}$? $\frac{17}{123}$ or $\frac{13}{100}$? $\frac{5}{11}$ or $\frac{6}{13}$? $\frac{4}{5}$ or $\frac{2}{3}$? $\frac{17}{50}$ or $\frac{11}{32}$?

18. Mr. Walter owned $\frac{2}{5}$ of a piece of land, Mr. Smith $\frac{3}{7}$, and Mr. Jones the remainder. Which owned the most, and which the least? How much did Mr. Jones own?

Answer. Mr. Smith owned the most, and Mr. Jones the least.

19. A sold to B $\frac{4}{7}$ of his farm, and then bought back $\frac{1}{3}$ of what he sold. Which then had the larger part of the farm? How much the larger part had he?

How do you compare two fractions in order to ascertain which is the greater? ***Write a rule.***

20. A bought $\frac{2}{3}$ of $\frac{5}{6}$ of a piece of land, and B bought $\frac{7}{8}$ of the remainder. Which bought the more of it? How much the more? How much remained?

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21. If 12 is $\frac{6}{7}$ of a number, what is $\frac{1}{7}$ of the number? What is $\frac{7}{7}$, or the whole of the number?

22. If 9 is $\frac{3}{4}$ of a number, what is the whole of the number?

Solution.—Since $\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$, if 9 is $\frac{3}{4}$ of a number, $\frac{1}{4}$ of that num-

ber is $\frac{1}{3}$ of 9, or 3; and if 3 is $\frac{1}{4}$ of a number, $\frac{4}{4}$, or the whole of it, is 4 times 3, or 12. Hence 12 is the number of which 9 is $\frac{3}{4}$.

23. If 15 is $\frac{3}{7}$ of a number, what is that number?

24. Mr. Harris bought 27 acres of land of Mr. Whipple, which was $\frac{3}{10}$ of Mr. Whipple's farm. How much had Mr. Whipple at first? How much had he left?

25. A sold 56 sheep to B, which was $\frac{7}{8}$ of his flock. How many sheep had A left? How many had he at first?

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26. $\frac{2}{3}$ of 12 is $\frac{4}{5}$ of what number?

Solution. $\frac{2}{3}$ of 12 is 8. Now if 8 is $\frac{4}{5}$ of a number, $\frac{1}{5}$ of the number is $\frac{1}{4}$ of 8, or 2. And if 2 is $\frac{1}{5}$, $\frac{5}{5}$, or the whole number, is 5 times 2, or 10. Hence $\frac{2}{3}$ of 12 is $\frac{4}{5}$ of 10.

27. $\frac{2}{7}$ of 21 is $\frac{3}{4}$ of what number?
$\frac{4}{5}$ of 15 is $\frac{3}{10}$ of what number?
$\frac{3}{10}$ of 40 is $\frac{4}{5}$ of what number?
$\frac{3}{9}$ of 27 is $\frac{3}{5}$ of what number?
$\frac{8}{9}$ of 27 is $\frac{8}{13}$ of what number?
$\frac{5}{9}$ of 81 is $\frac{9}{10}$ of what number?

28. A owned a farm of 260 acres, and sold $\frac{7}{13}$ of it, which was just equal to $\frac{5}{11}$ of B's farm. How many acres in B's farm?

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29. $\frac{3}{4}$ of 20 is $\frac{5}{9}$ of how many times 9?

Suggestion.—As above, we find that $\frac{3}{4}$ of 20 is $\frac{5}{9}$ of 27, which is 3 times 9.

30. $\frac{5}{8}$ of 24 is $\frac{3}{4}$ of how many times 10?
$\frac{4}{5}$ of 45 is $\frac{6}{11}$ of how many times 3?
$\frac{6}{7}$ of 35 is $\frac{5}{9}$ of how many times 2?
$\frac{8}{9}$ of 81 is $\frac{4}{5}$ of how many times 9?
$\frac{2}{3}$ of 15 is $\frac{5}{7}$ of how many times 7?

Suggestion.—Such examples afford an excellent drill when analyzed mentally as above, and should be so solved. They will also be found a neat written exercise in cancellation. Thus the last gives this form of operation: $\frac{2}{\cancel{3}} \times \overset{\cancel{5}}{\cancel{15}} \times \frac{\cancel{7}}{\cancel{5}} \times \frac{1}{\cancel{7}} = 2$.

140. In order to use cancellation to abridge computation, we first examine the problem and ascertain what numbers are to be multiplied together, and by what we are to divide. Then representing these operations by signs, proceed to cancel all factors which appear both as multipliers and divisors.

Cancellation is serviceable only in cases in which there are multiplications and divisions to be performed. When the processes are chiefly addition and subtraction, it is of no use.

31. If 5 yards of silk cost \$7, how much will 15 yards cost?

Suggestion.—In solving this we first *find what 1 yard costs, by dividing the cost of the given quantity by that quantity.* Then we multiply the cost of 1 (yard) by the number representing the required quantity (15 yards). The operations then are:

$$\frac{\textit{Cost of quantity, } 7}{\textit{Quantity, } 5} \times 15\text{, required quantity.}$$

Hence $\frac{7}{\cancel{5}} \times \overset{3}{\cancel{15}} = 21$, the required quantity.

32. If $\frac{3}{4}$ of a yard of silk cost \$6, what will $2\frac{1}{2}$ yards cost?

Suggestions.—We have

$$(6 \div \tfrac{3}{4}) \times 2\tfrac{1}{2}, \text{ or } \cancel{6} \times \frac{4}{\cancel{3}} \times \frac{5}{\cancel{2}} = 20.$$

33. If $\frac{7}{8}$ of an acre of ground cost \$245, what will $3\frac{3}{5}$ acres cost? *Ans.*, \$1008.

34. If $2\frac{3}{4}$ yards of cloth cost \$$3\frac{1}{2}$, what will $5\frac{1}{2}$ yards cost? *Ans.*, \$7.

2 times \$$3\frac{1}{2}$ gives the answer. Why is it?

35. If $\frac{4}{7}$ of an orange cost $2\frac{1}{2}$ cents, what will $\frac{3}{5}$ of an orange cost?

36. If 5 oranges cost 35 cents, what do 8 oranges cost?

Suggestion.—It should be seen clearly that the analysis is the same in both these last two examples. In each case we divide the cost of a number of articles by the number, to get the cost of 1; and then multiply the cost of 1 by the number of required articles. Thus we have

$$\text{For Ex. 35,}\quad 2\tfrac{1}{2} \div \tfrac{4}{7} \times \tfrac{3}{5};$$
$$\text{and for 36,}\quad 35 \div 5 \times 8.$$

This is the *practical* view of these examples; but let not the fuller analysis of such examples as 35 be neglected.

37. If $\frac{2}{7}$ of 6 yards of cloth cost $\$3\frac{3}{7}$, what will $\frac{3}{5}$ of 7 yards cost?

Practical Operation. $3\frac{3}{7} \div \frac{2}{7}$ of $6 \times \frac{3}{5}$ of 7, or

$$\frac{\overset{2}{\cancel{24}}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2} \times \cancel{6}} \times \frac{3 \times 7}{5} = \frac{42}{5} = 8\frac{2}{5}.$$

38. If $\frac{2}{3}$ of a yard of cloth cost $\$\frac{3}{4}$, what will $\frac{2}{5}$ of a yard cost?

Full Analysis.—If 2-thirds of a yard cost $\frac{3}{4}$ of a dollar, 1-third costs $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{3}{8}$ of a dollar; and if 1-third costs $\frac{3}{8}$ of a dollar, 3-thirds cost 3 times $\frac{3}{8}$, or $\frac{9}{8}$ of a dollar. Again, if 1 yard costs $\frac{9}{8}$ of a dollar, 1-fifth costs $\frac{1}{5}$ of $\frac{9}{8}$, or $\frac{9}{40}$ of a dollar, and 2-fifths cost 2 times $\frac{9}{40}$, or $\frac{9}{20}$ of a dollar.

39. If $\frac{5}{6}$ of twice my age is 30 years, what is my age? If $\frac{3}{7}$ of 4 times it is $22\frac{2}{7}$, what is my age? If $\frac{2}{5}$ of $\frac{1}{2}$ of it is 8, what is my age? If 3 times $\frac{2}{3}$ of it is 30, what is my age?

40. If $\frac{5}{12}$ of $\frac{3}{4}$ of the distance to a certain place is 48 miles, what is the distance? If $\frac{5}{8}$ of $\frac{3}{5}$ is 10 miles, what is the distance? If $\frac{1}{5}$ of $2\frac{2}{3}$ times the distance is $\frac{7}{8}$ of a mile, what is the distance?

41. If $\frac{2}{7}$ of a barrel of flour cost $\$1\frac{3}{7}$, what will $5\frac{2}{3}$ barrels cost? *Ans.*, $\$28.33\frac{1}{3}$.

42. If $\frac{2}{7}$ of 6 yards of cloth cost $\$2\frac{3}{7}$, how much will $\frac{3}{5}$ of 7 yards cost? *Ans.*, $\$5\frac{19}{20}$, or \$5.95.

43. If $\frac{3}{5}$ of $\$2\frac{1}{4}$ is the cost of a yard of cloth, what is the cost of $\frac{1}{2}$ of a yard of the same? Of 5 yards?

44. If $\frac{3}{5}$ of $\frac{5}{6}$ of a barrel of flour cost $\$3\frac{1}{2}$, what will $\frac{1}{3}$ of $\frac{4}{7}$ of 21 barrels cost?

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45. A vest costs $\$6\frac{2}{5}$, a hat $\$5\frac{1}{2}$, and a pair of boots $\$8\frac{2}{3}$; what is the cost of all?

46. Bought flour at $\$7\frac{3}{5}$ per barrel, and sold it at \$8; how much did I make on 50 barrels? Had I sold it at $\$7\frac{3}{4}$ per barrel, how much would I have made?

47. Bought 5 loads of potatoes, containing respectively $33\frac{1}{2}$, $27\frac{3}{4}$, $40\frac{1}{4}$, $35\frac{1}{2}$, and $29\frac{1}{4}$ bushels, and sold $12\frac{3}{4}$ bushels to each of 3 men, and $25\frac{1}{2}$ to each of 4 men. How many potatoes had I left?

48. Having on hand $19\frac{4}{5}$ tons of iron, if I sell $10\frac{1}{4}$ tons at \$45 per ton, and the remainder at \$43, what will I receive for it? *Ans.*, \$871.90.

49. Mr. Jones owns $\frac{3}{5}$ of a tract of land, and Mr. Smith buys $\frac{2}{3}$ of Mr. Jones's share, how much has Mr. Jones left? How much does Mr. Smith buy if the tract is 160 acres?
Answer to last, 64 acres.

50. What is a third and a half a third of 10? Of 12? Of 5? Of 7? Of 8? Of 9?

51. What part of a number is $\frac{1}{5}$ and $\frac{1}{2}$ of $\frac{1}{5}$ of it? $\frac{2}{3}$ and $\frac{1}{2}$ of $\frac{2}{3}$ of it? $\frac{1}{2}$ and $\frac{1}{3}$ of $\frac{1}{2}$ of it?

52. What is two-fifths and one-half of two-fifths of 5? Of 12? Of 7? Of 20? Of 6? Of 35?

53. What is $\frac{1}{7}$ and $\frac{3}{4}$ of $\frac{1}{7}$ of 8? Of 10? Of 12? Of 11?

54. Sold $5\frac{2}{7}$ yards of cloth at \$$3\frac{1}{2}$ per yard, and received in payment $32\frac{1}{2}$ pounds of butter at $15\frac{1}{2}$ cents per pound, $\frac{3}{4}$ of a ton of hay at \$15 per ton. What is the balance? and is it in my favor, or against me?

55. How much cheese, at $16\frac{2}{3}$ cents per pound, can be bought for $\frac{1}{2}$ a dollar?

56. How many raisins can be bought for \$1, at $17\frac{1}{2}$ cents per pound?

57. Bought a box of soap containing 70 pounds. Keeping it all summer it dried away $\frac{1}{3}$, when I sold it at $8\frac{2}{3}$ cents per pound. I gave 7 cents per pound. Did I make or lose? How much?

58. Bought 10 cords of wood for \$$5\frac{1}{2}$ per cord. Paid \$$\frac{3}{5}$ per cord for sawing, \$$\frac{2}{5}$ for splitting, and \$$\frac{1}{2}$ per cord for wheeling it into the shed and piling it. How much did the wood cost me in all? *Ans.*, \$70.

59. Owning 100 acres of land, I sold $27\frac{1}{2}$ acres to one man and $\frac{1}{2}$ the remainder to another. How much had I left?

60. A man cuts $35\frac{2}{3}$ cords of wood in $2\frac{1}{2}$ weeks, working time. What is the average of each day's work? How much does he earn per day, at 50 cents per cord for cutting?

61. If $3\frac{1}{2}$ yards of cloth cost \$$4\frac{3}{8}$, and 20 bushels of wheat bring \$$26\frac{2}{3}$, and a farmer brings in a load of $36\frac{1}{2}$ bushels of wheat and buys 40 yards of cloth, what is the balance, and to whom due?

Ans., There is a balance against the farmer of \$$1.33\frac{1}{3}$.

62. A room is 5 yards wide and $5\frac{1}{2}$ yards long. How much will it cost to carpet it with yard wide carpeting, at \$1.10 per yard?

63. From Chicago to Detroit, by way of the Michigan Central Railroad, is 284 miles. If an express train leaves Chicago at 9 o'clock in the morning, and arrives at Detroit at 6 o'clock and 45 minutes ($6\frac{3}{4}$ hours after noon), making 2 stops of 20 minutes each, and 12 stops of 5 minutes each on the way, what is the average rate per hour run by the train? *Ans.*, Nearly $35\frac{1}{7}$ miles.

Suggestion. 100 minutes are $1\frac{2}{3}$ hours, since 40 minutes are $\frac{40}{60}$, or $\frac{2}{3}$ of an hour.

64. A man engages to do a certain piece of work in 100 days. What part ought he to do in 10 days? In 25 days? In $12\frac{1}{2}$ days? In $16\frac{2}{3}$ days? In $33\frac{1}{3}$ days? In $62\frac{1}{2}$ days? In $6\frac{1}{4}$ days? In 11 days?

65. If 20 men require $7\frac{1}{2}$ barrels of flour for their subsistence 5 months, how much will 30 men require for a year? 17 men for 6 months? A barrel of flour is 196 pounds. At the above rate what is a man's daily allowance? *Ans.*, .49, or about $\frac{1}{2}$ a pound.

66. How long could 50 men subsist on a stock of provisions which would last 7 men 80 days? 10 men 100 days? 13 men 200 days? 5 men 8 days? 11 men 176 days?

67. A man bought 189 acres of land at $10 per acre, and $250\frac{1}{4}$ acres at $13 per acre. He sold $\frac{2}{3}$ of the former tract for $$18\frac{1}{2}$ per acre, and $\frac{2}{5}$ of the latter at $19 per acre. What would he make on the whole by selling the remainder at $20 per acre? *Ans.*, $3352.65.

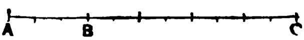
68. Bought at one time 320 acres of land at $$25\frac{1}{2}$ an acre, and at another 275 acres at $$31\frac{1}{4}$ an acre. Sold $\frac{3}{7}$ of the whole at $20, and the remainder at $30 per acre. Did I gain or lose? How much?

69. A boy, on being asked the time of day, said: "One-half the time past noon is $\frac{1}{4}$ of the time from now till midnight." What was the time?

Solution.—If $\frac{1}{2}$ the time past noon was $\frac{1}{4}$ the time to midnight, the *whole* time past noon was 2 times as great a part of the time to midnight, or $2 \times \frac{1}{4} = \frac{1}{2}$. Then if the time past noon was $\frac{1}{2}$ the time to midnight, the whole 12 hours would be divided into 3 equal parts, one part being the time *from* noon to the hour required, and 2 parts the time from this hour to midnight. Hence $\frac{1}{3}$ of 12, or 4 hours was the time past noon. It was, therefore, 4 o'clock in the afternoon.

70. A man on a journey of 120 miles had traveled a distance equal to $\frac{1}{3}$ of the remaining distance. How far had he traveled? When he had gone a distance equal to $\frac{2}{3}$ of the remaining, how far had he gone? How far when he had gone a distance equal to $\frac{5}{8}$ of the remaining distance?

71. If $\frac{2}{3}$ of the distance from A to B is $\frac{1}{4}$ of the distance from B to C, and the distance from A to C is 55, what is the distance from A to B? From B to C?



72. Two-thirds of the time from now to midnight is $\frac{2}{3}$ of the time from noon past to now. What o'clock is it?

73. Twice the time from now to midnight is the time past noon (*i. e.*, is the hour of the day). What time is it?

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74. A hound in chasing a fox runs 7 rods while the fox runs 5. How many rods does the hound gain on the fox in running 7 rods? In running 14 rods? In running 21 rods?

How many rods does the hound gain while the fox runs 5 rods? 10 rods? 15 rods?

If at the start the fox is 60 rods before the hound, how far will the hound run to catch the fox? How far will the fox run?

75. A fox is 40 rods before a hound, and runs 3 rods to the hound 5. How many rods must the hound run to overtake the fox? How far will the fox run?

76. A regular train starts from New York at 1 o'clock in the afternoon, and runs at the rate of 32 miles an hour. At 3 o'clock a special train is started in pursuit, and is run at 40 miles per hour. At what hour will the special overtake the regular train? How far from New York?

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77. If 4 men perform a certain piece of work in 3 days, how many men will it take to do 5 times as much work in 10 days?

Suggestion.—As the inquiry is about *men*, we must operate upon the given number of men, that is, upon the 4.* If 4 men can do the work in 3 days, it will take 3 times 4 men, or 12 *men* to do it in 1 day, and to do 5 times as much work will require 5 times 12 men, or 60 *men*. Now if 60 men can do the required amount of work in 1 day, it will require only $\frac{1}{10}$ as many men, or 6 *men* to do it in 10 days.

78. If 4 horses eat 16 bushels of grain in 8 days, how many bushels will 3 horses eat in 12 days?

Ans., 18 bushels.

79. If it require 7 men to cut a certain piece of grain in

* This point needs to be seen clearly. No division, multiplication, or other arithmetical operation can change the character of the number operated upon. Thus 2 times 4 men are 8 *men*, 3 times 6 days are 18 *days*, etc.

5 days, how many days will it take 13 men to cut 3 times as large a piece? *Ans.*, $8\frac{1}{13}$ days.

80. If 4 men can build 80 rods of fence in 6 days, how many rods can 10 men build in 5 days? In $2\frac{2}{3}$ days?

Suggestion.—*Indicate* the multiplications and divisions required, giving the reasons for them as above, and then cancel. Thus 1 man will build $\frac{80}{4}$ rods in 6 days, and $\frac{80}{4 \times 6}$ in one day. Then 10 men will build $\frac{10 \times 80}{4 \times 6}$ rods in one day, and $\frac{\overset{5}{\cancel{10}} \times \overset{20}{\cancel{80}} \times 5}{\cancel{4} \times \underset{3}{\cancel{6}}} = \frac{500}{3} = 166\frac{2}{3}$ in five days.

There are other methods of analyzing such examples. Thus 4 men in 6 days do 24 days work, and 10 men in 5 days do 50 days work. Then 10 men in 5 days will do $\frac{50}{24}$ times as much work as 4 men in 6 days. $\frac{50}{24}$ of $80 = 166\frac{2}{3}$.

81. If 5 pipes fill a cistern containing 120 barrels in 10 hours, how many pipes of the same size will it take to fill a cistern containing 200 barrels in 8 hours?

82. If 3 men can plow $15\frac{1}{2}$ acres in 4 days, how much can 5 men plow in $7\frac{2}{5}$ days? *Ans.*, $47\frac{19}{24}$ acres.

---

83. Two persons, 130 miles apart, start to travel toward each other, one at the rate of 25 miles a day, and the other at the rate of 32 miles. How long before they will meet? *Ans.*, $\frac{130}{57}$, or $2\frac{16}{57}$ days.

84. Two persons start from places $7\frac{1}{3}$ miles apart and travel the same way, one at the rate of $3\frac{2}{3}$ miles per hour, and the other at the rate of $4\frac{1}{2}$. How far apart will they be in 5 hours? *Ans.*, $11\frac{1}{2}$, or $3\frac{1}{6}$ miles.

85. Two persons on the same east and west line and 15 miles apart, start to travel towards each other. If they

continue these directions, one at the rate of $2\frac{3}{5}$ miles per hour, and the other at the rate of $3\frac{2}{3}$ miles per hour, how far will they be apart at the end of 6 hours?

Ans., $22\frac{3}{5}$ miles.

86. D bought of E $35\frac{1}{2}$ bushels of clover seed, for \$142, and afterwards sold to F $\frac{1}{4}$ of his purchase, at a profit of \$$1\frac{1}{4}$ per bushel. What did the part sold to F amount to?

Ans., \$$46\frac{19}{32}$.

87. At the rate of \$$61\frac{7}{8}$ for $11\frac{1}{4}$ cords of wood, what will be the cost of 6 loads of $\frac{7}{8}$ of a cord each, 4 loads of $\frac{3}{4}$ of a cord each, and 8 loads of $\frac{2}{3}$ of a cord each?

Ans., \$74.71 nearly.

88. A purchased of B 40 yards of cloth for \$260. He then sold to C $\frac{3}{5}$ of his purchase at a profit of \$$\frac{3}{8}$ per yard, and the remainder to D at a loss of \$$\frac{1}{8}$ per yard. What did A gain or lose by these several transactions?

Ans., Gained \$7.

---

89. What is the value of $\frac{1}{11}$ of $\frac{1}{12}$ of a vessel, if a person who owns $\frac{3}{11}$ of it sells $\frac{1}{9}$ of $\frac{7}{8}$ of his share for \$1750?

90. There are 60 seconds in a minute, and 60 minutes in an hour. At what rate per hour is a railroad train running which goes $\frac{1}{4}$ of a mile in 18 seconds?

91. Two land speculators went West to buy land. One bought $\frac{21}{23}$ of a section (640 acres), and the other $\frac{23}{25}$. Which bought the most land? How much?

92. If $\frac{3}{4}$ of a pound of silver dollars is worth \$$10\frac{2}{7}$, what is the weight of \$1 in ounces, 12 ounces making a pound?

93. $16\frac{1}{2}$ feet make a rod. What is the length and what the width in feet of a village lot 4 rods front and 8 rods deep?

94. If 36 pounds of sugar are worth 24 pounds of coffee, and 22 pounds of coffee are worth 55 pounds of rice; how many pounds of rice can be bought for 16 pounds of sugar?

Solution. 1 pound of coffee is worth $\frac{55}{22}$ of a pound of rice, and 24 pounds are worth 24 times $\frac{55}{22}$, or $\frac{24 \times 55}{22}$ pounds of rice. But this is the worth of 36 pounds of sugar. Hence 1 pound of sugar is worth $\frac{24 \times 55}{22 \times 36}$ pounds of rice, and 16 pounds are worth $\frac{24 \times 55 \times 16}{22 \times 36}$ pounds of rice. Cancelling and reducing, we have $26\frac{2}{3}$ pounds of rice.

95. If 40 bushels of potatoes are worth 45 bushels of corn, and 18 bushels of corn are worth 14 hundred weight of hay, and 35 hundred weight of hay are worth 4 barrels of flour; how many barrels of flour are 75 bushels of potatoes worth?

Suggestions.—As the answer is to be in barrels of flour, we operate on the 4 barrels.

1 hundred weight of hay is worth $\frac{4}{35}$ barrels of flour.

14 hundred weight of hay are worth $\frac{4 \times 14}{35}$ barrels of flour.

1 bushel of corn is worth $\frac{4 \times 14}{35 \times 18}$ barrels of flour.

45 bushels of corn are worth $\frac{4 \times 14 \times 45}{35 \times 18}$ barrels of flour.

1 bushel of potatoes is worth $\frac{4 \times 14 \times 45}{35 \times 18 \times 40}$ barrels of flour.

75 bushels of potatoes are worth $\frac{4 \times 14 \times 45 \times 75}{35 \times 18 \times 40}$ barrels of flour.

Cancelling and reducing, we have the answer.

96. If 8 bushels of wheat are worth as much as 3 cords of wood, 9 cords of wood as much as 3 tons of hay; how many bushels of wheat should be given for 5 tons of hay?

---

97. How is the value of a proper fraction affected by *adding* the same number to both its terms? Try it by

adding 3 to both terms of $\frac{5}{7}$. By adding 1 to both terms of $\frac{2}{3}$. Try other cases.

98. How is an improper fraction whose value is greater than 1 affected by adding the same number to both its terms? Try it by adding 2 to both terms of $\frac{5}{4}$; of $\frac{7}{3}$; of $\frac{11}{5}$. By adding 1 to both terms of $\frac{8}{3}$; of $\frac{13}{7}$. By adding 20 to both terms of $\frac{3}{2}$; of $\frac{5}{3}$.

---

99. Of two unequal numbers, show that the greater is equal to the less + the difference between the numbers.

Also that the sum of the two is equal to twice the less + the difference.

Also that if you *add* the difference to the *sum*, you have twice the greater.

Also that if you *subtract* the difference from the *sum*, you have twice the less.

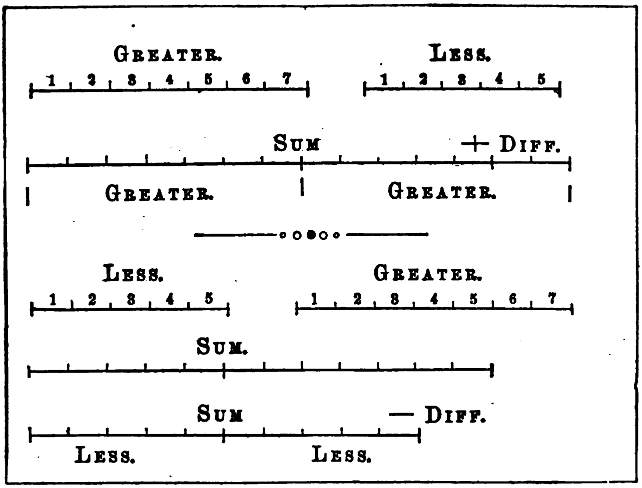
Suggestion.—The greater is made up of the less and the difference. Hence the sum of the greater and the less is twice the less + the difference, and if we take away the difference we shall have twice the less.

100. The sum of two numbers is 198, and their difference 52. What are the numbers? Sum 295 and difference 117. What are the numbers?

Principle.

141. *If to the sum of two numbers the difference be added, it gives twice the greater; but if from their sum their difference be subtracted, it gives twice the less number.*

Illustration.



101. If the sum of two numbers is 70 and the difference 16, what are the numbers? Sum 222, difference 114? Sum 386, difference 214? Illustrate as above.

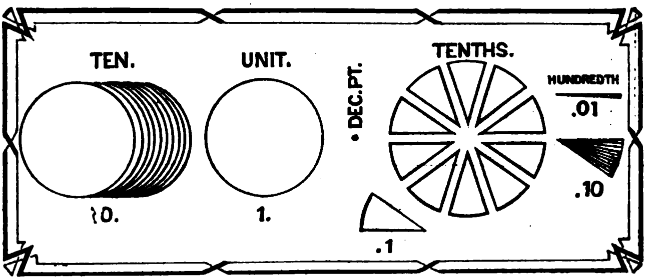
102. James and John together have 86 cents, and John has 15 more than James. How many have each?

103. Two men, A and B, agree to work for C for $\$1\frac{1}{2}$ per day's work. Together they work 38 days, and A works 5 days more than B. How much must each receive?

[The TEACHER'S HAND-BOOK furnishes over 600 exercises for class drill in Common Fractions, a large number of which are adapted to oral, or mental work.

CHAPTER III.

DECIMAL FRACTIONS.



SECTION I.

DEFINITIONS, AND READING AND WRITING DECIMAL FRACTIONS.

1. If we divide any thing into 10 equal parts, what is *one* of the parts called? What are *two* of the parts called? 3 of them? 4 of them? 5 of them?

2. If we divide any thing first into 10 equal parts (as the counter in the picture), and then divide each one of these parts into 10 equal parts, into how many equal parts will the whole be divided? What then is *one* such part called? What 2? 3? 7? 8? 12? 35? 82?

3. If we divide each of the 100 hundredths into which any thing may be divided into 10 equal parts, into how many equal parts will the whole be divided? What is one of these parts called?

4. How many tenths in a unit? How many hundredths in a tenth? How many thousandths in a hundredth?

142. Dividing any whole, or a unit, first into 10 equal parts (tenths), and these again into 10 equal parts (hundredths), and these again into 10 equal parts (thousandths), etc., is called the *Decimal Division.*

143. We have learned in Common Fractions that one-tenth may be represented thus $\frac{1}{10}$; 2-tenths, thus $\frac{2}{10}$; 3-tenths, thus $\frac{3}{10}$, etc. So also that 1-hundredth may be written $\frac{1}{100}$; 2-hundredths, $\frac{2}{100}$; 7-hundredths, $\frac{7}{100}$; 4-thousandths, $\frac{4}{1000}$, etc. But there is another way to represent such fractions as these, that is, those which arise from the *Decimal Division.* It is this: tenths are represented by putting a dot, called a **Decimal Point,** at the left of the figure telling how many tenths; thus .1 is one-tenth; .2 is 2-tenths; .3 is 3-tenths, etc. So also .01 is one-hundredth; .02 is 2-hundredths; .03 is 3-hundredths, etc.

144. Fractions which arise from the *Decimal Division* (**142**), and are represented by means of the *Decimal Point,* and without the denominator expressed, are called **Decimal Fractions.**

For brevity Decimal Fractions are often called simply *Decimals,* although strictly speaking, all numbers, whole or fractional, expressed in the *Decimal,* or common notation, are Decimals.

145. Counting to the right from the *Decimal Point,* the *first* place is *Tenths* place, the second *Hundredths,* the third *Thousandths,* the fourth *Ten-thousandths,* etc.

Thus the *orders* of decimal fractions succeed each other as we go from units order towards the right, just as the orders of whole numbers do as we go to the left, according to the following

DECIMAL NUMERATION TABLE.

Hundred Billions.	Ten Billions.	Billions.	Hundred Millions.	Ten Millions.	Millions.	Hundred Thousands.	Ten Thousands.	Thousands.	Hundreds.	Tens.	Units.	DECIMAL POINT.	Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.	Ten-Millionths.	Hundred-Millionths.	Billionths.	Ten-Billionths.	Hundred-Billionths.
3	3	3	3	3	3	3	3	3	3	3	3	.	3	3	3	3	3	3	3	3	3	3	3

Orders of Whole Numbers. Orders of Decimal Fractions.

5. Read the following, and tell what they mean; .2; .5; .04; .07; .006; .001; .0008; 00007; .000003.

Reading Decimals.

1. In .23 how many *tenths,* and how many *hundredths?* 2 *tenths* are how many *hundredths?* Then 2 tenths and 3 hundredths, are how many hundredths?

How then is .23 read? *Ans.*, 23 hundredths.

2. In .536 how many tenths; how many hundredths, and how many thousandths? 5 tenths and 3 hundredths make how many hundredths? 53 hundredths with 6 thousandths make how many thousandths?

How then is .536 read?

To Read any Decimal Fraction.

146. Rule.—*I. Numerate the fraction; that is, begin at the decimal point and name the orders to the right,*

and bear in mind the name of the lowest, or right hand order.

II. Read the expression just as a whole number, and then pronounce the name of the lowest or right hand order.

We numerate to find what the denominator of the fraction is, since, as in whole numbers, the whole may be read as so many of the lowest denomination (**21**).

We then read the number as written, since it tells how many parts are represented, that is, it is the numerator; and having read it pronounce the name of the lowest order, since this tells the denomination or kind of parts, that is, it is the denominator.

This method of reading is just the same in principle as the method of reading a *common* fraction.

3. Read .23478. Of what order is the 8? What then is the denominator of the fraction? What is the numerator? How would you read $\frac{23478}{100000}$?

4. Read .00234. What is the denominator? What the numerator? What is the difference between .00234 and $\frac{234}{100000}$?

Read each of the following, also writing each in the form of a common fraction:

5. .304;	9. .56;	13. .01;	17. .58437;
6. .0027;	10. .506;	14. .001;	18. .00236;
7. .00801;	11. .3004;	15. .782564;	19. .00023;
8. .0101;	12. .0501;	16. .586423;	20. .90801.

We read .00801, 801 hundred-thousandths. As a common fraction it is written $\frac{801}{100000}$.

N. B.—Pupils *write* a rule for representing a fraction which is expressed decimally, as a common fraction.

21. Read 23.5.

This is the same as $23\frac{5}{10}$ and is read in the same way, that is "23, and 5-tenths."

22. Read 7248.347.

In reading whole numbers and decimals as mixed numbers, it may promote clearness to omit *and* in each case except before the fraction. True it is a little harsh; but perhaps the perspicuity compensates for this. In this way 7248.347 will be read "7 thousand 2 hundred forty-eight, *and* 3 hundred forty-seven thousandths."

Read the following:

23.	48.21;	30.	437.543621;	37.	152.342;
24.	107.056;	31.	1002.000508;	38.	793.546;
25.	5006.00205;	32.	7080.070936;	39.	100.001;
26.	5400.0064;	33.	5006.000807;	40.	888.999;
27.	1000.0001;	34.	52111.111111;	41.	.4343;
28.	7281.5436;	35.	5333.333333;	42.	3.1416;
29.	54037.02501	36.	6420.022022;	43.	2.71828.

Principle.

147. *Annexing 0's to a decimal does not alter its value, since according to our method of reading, we annex an equal number to the denominator, and hence multiply both terms of the fraction by the same number.*

1. Explain the following:

$.5 = .50 = .500 = .5000 = .50000.$

$.25 = .250 = .2500 = .2500000.$

$.012 = .0120 = .012000 = .01200000.$

To Write Decimals.

148. Rule.—*Write the numerator as a whole number. Then beginning at the* RIGHT, *apply the decimal numeration, calling the right-hand figure tenths, the next at the left hundredths, etc., filling all vacant orders with 0's, till the name of the order designated by the denominator is reached; at the left of this write the decimal point.*

1. Write six hundred twenty-five hundred-thousandths.

Process.—The numerator is 625. Now beginning at the 5 *call* it tenths, the 2, hundredths, the 6, thousandths, and prefixing a 0 call it ten-thousandths, and then another 0 and call it hundred-thousandths. At the left of this last 0 place the decimal point, thus, .00625.

Reasons for the Rule.—The numerator being a whole number is so written. The reason for beginning at the right to apply the decimal numeration is simply to get the right number of places after the decimal point. True the orders are thus *falsely* named, but the expedient is convenient, and if this *trial* numeration from right to left gives the right number of orders, the *true* numeration from *left to right* cannot fail to be right.

2. Write fifteen hundredths. Nineteen thousandths. Six ten-thousandths. Twenty-four thousandths. Five hundred thousandths. Thirty-nine millionths. One hundred thousandths. Forty-nine hundredths. Ten ten-millionths. Fifty-two thousandths. Eight hundred-thousandths. Eight hundred thousandths. Seventy-one hundredths. Ninety-one millionths. Seventeen ten-thousandths. Two thousand eight hundred forty-five ten-thousandths. Three hundred sixteen thousandths.

3. Write sixty-nine, and nine hundred three thousandths.

Suggestion.—It will be well for beginners to write the whole number first, and then, if the decimal is not readily written, *write it in another place*, according to the rule given above, and then annex it to the whole number.

4. Write seven hundred three thousand, and two hundred seven ten-thousandths.

Suggestion.—The whole number is 703000. The decimal, written according to the rule, is .0207. Written together they are 703000.0207

Write the following:

5. Five, and two hundred sixty-three millionths.

6. Nine hundred eighty, and four thousandths.

7. Two, and eighty-five billionths.

8. Two hundred, and seventy-four ten-thousandths.

9. Eight thousand two hundred, and thirty-two thousandths.

10. Four hundred fifty-two hundred-thousandths.

11. Sixty-five, and five hundred twenty-one thousandths.

12. Eighty-two, and sixty-five billionths.

13. 7 hundred 63 thousand twenty, and one hundred eight millionths.

14. Seven thousand five hundred 23 millionths.

15. Four hundred seventy-five hundred-thousandths.

16. Forty-five, and three hundred and seventy-five ten-thousandths.

17. Three billion seven hundred fifty-five million 2 hundred 26 thousand, and 5 hundred forty-three millionths.

18. Three, and one thousand four hundred sixteen ten-thousandths.

19. Nine hundred 27 million 3 hundred 64 thousand 5 hundred, and 2 thousand 5 hundred 68 ten-millionths.

20. Write the following as decimals: $405\frac{17}{100}$; $300\frac{5}{10}$; $1\frac{27}{1000}$; $57\frac{802}{1000000}$; $1002\frac{1804}{100000000}$; $7\frac{5}{10000}$; $6\frac{7}{10000}$.

SECTION II.

REDUCTIONS.

To Reduce Common Fractions to Decimals.

1. How many times as many *tenths* are there in any number as there are units? How many times as many hundredths as units? How many times as many thousandths as units.

2. How many tenths in 2? In 7? In 23? In 407? How many hundredths in each of the above numbers? How many thousandths? How many millionths?

3. How do you find how many tenths there are in any number of units? How many thousandths? How many ten-thousandths? How many millionths?

4. How many tenths in $\frac{3}{5}$?

Solution.—Since $\frac{3}{5}$ is $\frac{3}{5}$ of a unit, and there are 10 times as many tenths in any number as there are units, there are 10 times $\frac{3}{5}$, or $\frac{30}{5}$ *tenths* in $\frac{3}{5}$. $\frac{30}{5} = 6$; hence there are 6 tenths in $\frac{3}{5}$, or $\frac{3}{5} = .6$.

5. As above reduce $\frac{1}{2}$; $\frac{4}{5}$; $\frac{1}{5}$; $\frac{2}{5}$; and $\frac{3}{5}$ to tenths, explaining the operation.

6. How many hundredths are $\frac{3}{4}$?

Solution.—Since $\frac{3}{4}$ is $\frac{3}{4}$ of a unit, and there are 100 times as many 100ths as units in any number, there are 100 times $\frac{3}{4}$, or $\frac{300}{4}$ hundredths in $\frac{3}{4}$. $\frac{300}{4} = 75$; hence there are 75 100ths in $\frac{3}{4}$, or $\frac{3}{4} = .75$.

7. Reduce $\frac{1}{4}$; $\frac{3}{25}$; $\frac{3}{20}$; $\frac{7}{25}$; and $\frac{9}{20}$ to hundredths.

8. Reduce the following to thousandths: $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{3}{40}$; $\frac{29}{500}$; and explain as above.

To Reduce a Common Fraction to a Decimal.

149. Rule.—*Annex* 0's *to the numerator, and divide the result by the denominator, continuing to annex* 0's *till the division is exact, or until as many decimal places are obtained as desired. Point off as many places of decimals in the quotient as there have been* 0's *annexed, prefixing* 0's *to the significant figures, if necessary.*

Demonstration.—One 0 annexed multiplies a number by 10; two by 100; three by 1000, etc. (**60**). Hence the number of 0's annexed indicates the denomination of the decimal, or the number of places there are to be in the decimal, according to the decimal notation.* Dividing by the denominator is reducing the fraction to a whole number (of tenths, hundredths, thousandths, etc.).

* Notation means "method of writing."

9. Reduce $\frac{1}{16}$ to a decimal.

Operation.
16) 100 (.0625
96
40
32
80
80

Explanation.—In this case we have virtually annexed 4 0's, or multiplied $\frac{1}{16}$ by 10000, which reduces it to 10000ths. Thus $\frac{1}{16} = \frac{10000}{16}$ ten-thousandths, or 625 ten-thousandths. This in the decimal notation is written .0625.

Another Explanation.—We can explain the process as we go along with our work of dividing, thus: 16 is not contained in 1, but one is 10 tenths. Again, 16 is not contained in 10 so that there are no *tenths* in the quotient. Hence we write 0 in tenths place. Now 10 tenths make 100 hundredths, and as 16 is contained in 100 6 times, we have 6 hundredths in the quotient, with 4 hundredths remaining, etc. This is the explanation given in Division (**80**).

This process may also be explained as multiplying and dividing the given fraction by the same number. Thus $\frac{3}{5}$ multiplied by 10 is $\frac{30}{5} = 6$. This divided by 10 is .6 (**91**).

Reduce the following common fractions to decimals, carrying the decimal to *millionths* if the division is not exact:

10. $\frac{15}{16} = .9375$.	19. $13\frac{7}{16}$.	28. $128\frac{3}{11}$.
11. $\frac{6}{25} = .24$.	20. $40\frac{15}{16}$.	29. $13\frac{5}{12}$.
12. $\frac{9}{20} = .45$.	21. $\frac{2}{7}$.	30. $\frac{4}{9}$.
13. $\frac{21}{8} = 2.625$.	22. $13\frac{5}{9}$.	31. $\frac{5}{9}$.
14. $5\frac{1}{4} = 5.25$.	23. $\frac{21}{37}$.	32. $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}$.
15. $16\frac{1}{2} = 16.5$.	24. $12\frac{2}{3}$.	33. $\frac{139}{225}$.
16. $103\frac{3}{8} = 103.375$.	25. $1\frac{5}{6}$.	34. $\frac{1}{99}, \frac{2}{99}, \frac{3}{99}$.
17. $\frac{1}{3} = .33333+$.*	26. $3\frac{1}{5}$.	35. $\frac{102}{13}$.
18. $\frac{12}{13} = .923076+$.	27. $10\frac{1}{25}$.	36. $\frac{1256}{17}$.

When we are annexing 0's in order to extend a decimal, if at any time we have a remainder which is the same as we have had before, the same figures will recur in the quotient. Such a decimal is called a **Repeating or Circulating Decimal**, or simply a *Repetend*.

* The + sign when thus used signifies that the division does not terminate.

A repetend is indicated by writing a dot over the first and last repeating figures; thus $.3\dot{4}57\dot{6}$ means .34576576576, etc.; $\dot{5}\dot{7}$ means .575757, etc.

Principle.

150. *When reducing a common fraction to a decimal, if the decimal does not terminate, the farther we extend it the nearer we approach to the value of the common fraction.*

Thus it is easy to see that $\frac{1}{3} = .3$ nearly, but .33 more nearly, and .333 still more nearly, etc. When we wish to use only a few places of the decimal, if the first figure of the part we would drop is 5 or more, it is more accurate to increase the last figure we use by 1; thus instead of using .27 for .276, it is more accurate to use .28 so instead of 3.1415 for 3.14159 +, 3.1416 is more accurate.

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151. In United States Currency, since a dime is a tenth of a dollar, a cent a tenth of a dime, and a mill a tenth of a cent, these are respectively tenths, hundredths, and thousandths of a dollar, and are so written as we have already seen (page 79). So also we have learned that it is customary to write and read dimes, or tenths of dollars, with cents; thus we read \$4.75 not "4 dollars, 7 dimes, and 5 cents, but "4 dollars, and 75 cents."

Thus we are enabled to treat fractions of a dollar, or cents and mills, as other decimals.

Examples.—Represent the following fractions in decimals, that is, in cents and mills

\$$5\frac{1}{2}$; \$$127\frac{1}{4}$; \$$1\frac{1}{8}$; \$$200\frac{3}{4}$; \$$1\frac{1}{3}$; \$$12\frac{1}{6}$; \$$12\frac{2}{7}$.

To Reduce Decimals to Common Fractions.

152. Rule.—*Suppress the decimal point, and underneath the numerator of the fraction as thus obtained write*

the denominator, which is 1, *with as many* 0's *annexed as there were places in the decimal. Reduce the fraction thus obtained to its lowest terms.*

This is simply writing what the fraction means, and is a direct consequence of the method of writing decimals. In fact we have already performed the operation (**146**), and only mention it here to refresh the memory and give a little more practical exercise.

Express the following decimals as common fractions:

1. .5.	7. 23.625	13. 1.06.	19. 3.75.
2. .35.	8. 10.3	14. 1.10.	20. 3.075.
3. .125.	9. 2.4	15. 1.07.	21. 15.6.
4. .375.	10. .0025	16. 1.08.	22. 15.60.
5. .25.	11. .0005	17. 1.125.	23. 15.600.
6. .75.	12. 1.04	18. .225	24. 1.12½.

SECTION III.

ADDITION AND SUBTRACTION.

153. The rules for Addition and Subtraction of Decimal Fractions are exactly the same as for whole numbers, and need not be repeated.

1. Add 715.25, 3051.039, 4.8, and 25.3156.

Solution.—Writing the numbers so that like orders shall fall in the same column, we add exactly as in whole numbers. Thus in the ten-thousandths column there are only 6, so we write it in the sum. In the thousandths column there are 5 and 9 which make 14 thousandths, which are 1 hundredth and 4 thousandths, since 10 thousandths make 1 hundredth. In like manner we proceed through the other orders.

$$\begin{array}{r} 715.25 \\ 3051.039 \\ 4.8 \\ 25.3156 \\ \hline 3796.4046 \end{array}$$

Perform the following additions:

2.	3.	4.	5.
715.206	605.271	742	$ 86.45
34.73	342.156	85.6	$243.045
1280.008	83.805	207.006	$580.50*
7.5643	1.27	84	$ 73.00*
187.4	483.5	556.38	$100.40

Accountants are required to add long columns of figures with rapidity and accuracy. The following are specimens. Let them be added with care.

6.	7.	8.
$ cts.	$ cts.	$ cts.
8.37	.78	673.28
4.33	.47	597.84
7.62	.53	3426.87
.48	2.75	219.48
.97	1.20	8.37
2.50	4.37	167.84
6.19	8.29	5986.32
10.00	13.85	6749.31
4.28	2.00	4863.27
8.07	.62	7542.35
4.37	.25	2986.28
9.48	1.37	379.87
4.21	9.83	2.59
13.26	6.75	69.80
1.20	8.43	4060.75
.57	20.48	309.71
3.08	6.00	124.87
4.96	1.00	8520.06
.85	1.50	2493.28
4.00	7.69	48.75

* In writing United States Currency, especially for addition and subtraction, it is customary to fill vacant places of cents orders with 0's.

9. From 782.19 take 325.536.

782.19
325.536
456.654

Solution.--Writing so that the various orders of the subtrahend shall fall under like orders in the minuend, we subtract as in whole numbers. Thus, as there are no thousandths in the minuend from which to take the 6 thousandths of the subtrahend, we take one of the 9 hundredths, which makes 10 thousandths, and subtract the 6 thousandths, obtaining 4 thousandths for the remainder. 3 hundredths from 8 hundredths leaves 5 hundredths. As 5 tenths cannot be taken from 1 tenth, we take a unit from the 2, which makes 10 tenths. This with the 1 tenth of the minuend makes 11 tenths, from which taking the 5 tenths there remain 6 tenths. Thus we proceed through all the orders as in whole numbers.

Examples for Practice.

1.	2.	3.	4.
408.207	\$23.56	\$100.00	1000
57.0583	\$ 7.50	\$ 47.23	325.008
351.1487	\$16.06	\$ 52.77	674.992

5.	6.	7.	8.
1765.004	700.584	48	1
843.02	23.	.546	.003
921.984			

9. From 8.1 take 3.547.
10. From 123 take 78.256.
11. From 3.46 + 10 take 5.7.
12. From .46 take .0037.
13. From 71.86 + 4.313 take 18.5 + 20.007.
14. From 10 take one tenth.
15. From 1 take one thousandth.
16. From \$1 subtract 15 cents.
17. From \$5.68 take \$1.17.
18. Add 7.52, 10.478, 600.5, .87 and 23. *Sum,* 642.368.

19. From the sum of \$347.63 and \$20.59 take the sum of \$100 and \$57.13.

20. From the sum of 8.7, .32, 6.08, 25, and 2.45 take the sum of .75, 1.25, 13, and 3.5.

Applications.

1. A man earned \$5.27 one week, \$6.52 another, and \$7.15 another. Out of this he bought a pair of boots for \$4.75, and a hat for \$3.38. How much remained?

Ans., \$10.81.

2. A fence builder laid up 20.5 rods one day, 17.02 rods the next, 31.25 rods the next, and 27 rods the next. A wind blew down 52.37 rods, and so displaced 10.5 rods more as to make it necessary to lay it over. How much remained of his work?

3. My grocery bill for the several days of a week was \$3.73, \$1.10, \$4, \$2.05, \$1.88, and \$3. On this I paid during the week, at one time \$2, and at another \$3.50. How much did I still owe?

4. Write the following in the decimal form and then add them: $6\frac{1}{4}$, $12\frac{1}{2}$, $5\frac{3}{8}$, $6\frac{5}{8}$, $\frac{3}{5}$, $\frac{3}{4}$. *Sum*, 32.1.

5. From the sum of $\frac{5}{6}$, .25, 10, $\frac{2}{5}$, and $13\frac{1}{2}$, subtract the sum of $1\frac{1}{2}$, $\frac{4}{5}$, 8, and 2.56.

6. A steamer sailed in 6 successive days 305.24, 206.5, 182.402, 343.25, 287.13, and 313.008; and a sail vessel starting from the same port sailed after her, making 105 miles the 1st day, $204\frac{3}{4}$ the 2d, $98\frac{1}{2}$ the 3d, $110\frac{5}{8}$ the 4th, $212\frac{5}{6}$ the 5th, and $113\frac{2}{5}$ the 6th. How much in advance was the steamer at the end of the 6 days?

Ans., $792.421\frac{2}{3}$.

7. My meat bill for the 7 days of a week was, 1st day

\$1.37, 2d day \$0.88, 3d day \$1, 4th day 75 cents, 5th day \$1.25, 6th day 97 cents, 7th day \$1.10. At the beginning of the week the market man owed me \$5.00. How did the account stand at the close of the week ?

8. During the week a grocer sold from a bin of potatoes which contained $87\frac{1}{2}$ bushels at the first, $6\frac{3}{4}$, $\frac{1}{2}$, 17, $25\frac{1}{2}$, 42.25, 18.5, and 31 bushels; having in the mean time bought and put in 2 loads of $33\frac{1}{2}$, and 41.75 bushels, respectively. How many potatoes remained in the bin ?

9. How much less than a thousand is 156.75, $428\frac{5}{8}$, 243.125, and $89\frac{1}{2}$?

10. From a piece of cloth containing $41\frac{3}{4}$ yards, I sold $\frac{1}{2}$ a yard, $1\frac{3}{4}$, 8, 5.25, and 21.125 yards. What was the remnant ?

11. A man's property is worth \$7528 ; but he owes \$347.50 to one man, \$1000 to another, $\$75\frac{3}{4}$ to another, and $\$15\frac{1}{2}$ to another. How much would he be worth if his debts were paid ?

SECTION IV.

MULTIPLICATION AND DIVISION.

Multiplication of Decimals.

154. The *process* of Multiplication of Decimals is precisely the same as that of whole numbers, the only thing needing farther attention being the position of the *Decimal Point* in the product.

1. Multiply 732.53 by 37.846.

Solution.—Multiplying the multiplicand by the parts of the multiplier in succession, we then add the partial products as in whole

numbers. First, to multiply 732.53 by 6 thousandths, if we first multiply by 6 we have 4395.18. But as our multiplier was to be .006 or $\frac{6}{1000}$ we have to divide this product by 1000 (**130**), which is done by removing the point 3 places to the left, that is, cutting off 3 more figures (**86**). Hence we have for this partial product 4.39518. Again, multiplying by 4 we get 2930.12. But as this 4 is .04, we have to divide by 100, or remove the point two places to the left. Hence this partial product is 29.3012. *Now we notice that with each succeeding figure to the left there will be* 1 *less figure to cut off, in order to divide by the denominator of that order. So if we carry each partial product* ONE *place further to the left than the preceding it will bring the decimal point in it exactly under the one above, and hence like orders will stand in the same columns in the partial products.*

```
       732.53
        87.846
    ----------
       4.39518
      29.3012
     586.024
    5127.71
   21975.9
   -----------
   27723.33038
```

Perform the following, giving the reasoning in each case as above:

```
      2.                 3.               4.
    356.28             5.2704            .256
    175.6               .0341            .053
   --------          ----------        --------
    213.768           .00052704        .000768
   1781.40            .0210816         .01280
  24939.6             .158112          --------
  35628              ----------        .013568
  ---------           .17972064
  62562.768
```

5. 46.702 × 81.54.	8. 8246 × 3.14.
6. 1.764 × .0547.	9. .347 × 28.
7. .0058 × .073.	10. 80.008 × .0007.

We observe from the above that the number of decimal places in the product is fixed by the multiplication *by the lowest order in the multiplier*. Now this requires us to move the decimal point as many places to the left (of where it is in the multiplicand) as there are decimal places in the multiplier.

Principle.

155. *In Multiplication of Decimals the number of decimals in the product must equal the number in both factors.*

Demonstration.—The preceding paragraph gives the reason for this principle. It may also be given thus:—Any number wholly or partly decimal may be read as so many of the lowest order mentioned as are expressed by the figures regardless of the point. (For example 4.5 is 45 *tenths;* 43.75 is 4375 hundredths, etc.) In reading in this way the denominator of each factor becomes 1 with as many 0's annexed as there are decimal places in that factor. Hence the product of these denominators, which will be the denominator of the product (page 143), will contain as many 0's as there are decimal places in both factors. Therefore the number of decimal places in the product equals the number in both the factors.

11. Multiply 34.8 by 3.76 explaining the pointing off according to the last paragraph.

Solution. $34.8 = \frac{348}{10}$, and $3.76 = \frac{376}{100}$. Hence $34.8 \times 3.76 = \frac{348}{10} \times \frac{376}{100}$. We therefore multiply the numbers together as the numerators, disregarding the decimal points. This product is 130848. Now as the denominator of the multiplicand is 10 and that of the multiplier 100, the product is thousandths. Therefore we point off three decimals, which is equivalent to writing 1000 as a denominator.

```
   34.8
   3.76
   2088
  2436
 1044
130.848
```

Examples for Practice.

1. $327 \times .6 = 196.2$.
2. $327 \times .06 = 19.62$.
3. $327 \times .006 = 1.962$.
4. \$375.6 $\times$ .125 = \$46.95.
5. 78.05×3.47.
6. $.543 \times .0027$.
7. $.00279 \times .008 = .00002232$.
8. \$48.275 $\times$ 6.421.
9. $5832 \times .05$.
10. \$70.01 $\times$ .0001.
11. $34 \times .000025$.
12. 47×3.5.
13. \$73.6 $\times$ .0002.
14. $56400 \times .01$.
15. $150 \times .1$.
16. \$79 $\times$ .1.
17. 34.76×3.1416.
18. $875 \times .4343$.
19. \$17.58 $\times$ 2.002.
20. $.8 \times 1.1$.
21. 4.5×4.5.
22. 3.712×3.712.

Division of Decimals.

156. The *process* of Division of Decimals is precisely the same as that of Division of Whole Numbers, the only thing needing further attention being the position of the Decimal Point in the Quotient.

1. Of what operation is Division the converse? * To what does the Dividend correspond? To what the Divisor and Quotient?

2. In Multiplication how must the number of Decimals in the product compare with those in the multiplier and multiplicand together? Then in Division of Decimals how must the number of Decimals in the dividend compare with those in the divisor and quotient together?

3. Divide 988.234 by 25.87.

Operation.

```
25.87) 988.234 (382 Quotient unpointed.
       776 1
       212 13
       206 96
         5 174
         5 174
```

Explanation.—Dividing as in whole numbers, the unpointed quotient is 382. Now there are *two* decimals in the divisor, and as there are *three* in the dividend, there must be *one* in the quotient (155). Hence the quotient is 38.2.

4. Divide 25 by .005.

```
.005) 25.000
       5 000
```

Solution.—As there must be as many decimals in the dividend as in the divisor (at least), we annex *three* 0's as decimals, in this case.

156 (a). Practical Suggestion.—*By writing the first figure in the quotient directly over the figure in the first subtrahend which arises from multiplying the units of the dividend by this figure, the decimal point in the quotient will fall over the point in the dividend.*

---

* The young learner would say "opposite." In mathematics one problem is the converse of another when what is given in one is required in the other.

5. Divide 6052.74 by 4.379.

```
         1382.219+
4.379 ) 6052.74
        4379
        16737
        13137
         36004
         35032
           9720
           8758
            9620
            8758
             8620
             4379
             42410
             39411
              2999
```

Here we observe that the product of the 4 units of the divisor falls under the 6 of the dividend; hence we place the 1 of the quotient over the 6 of the dividend. Dividing, the decimal point in the divisor falls over the point in the dividend.

In this case the dividend used is 6052.740000. Hence, as there are 3 decimals in the divisor and 6 in the dividend, there must be 3 in the quotient.

Principle.

157. *In Division of Decimals the number of decimals in the divisor and quotient together must equal the number in the dividend used.*

Examples for Practice.

1. 688.1875 ÷ 5.5 = 125.125.
2. .440946 ÷ .561 = .786.
3. 242.451 ÷ 1.2245 = 198.
4. \$183.375 ÷ \$.489 = 375.
5. .1728 ÷ 12 = .0144.
6. 343 ÷ .007 = 49000.
7. \$67.8632 ÷ 32.8 = \$2.069.
8. \$983 ÷ 6.6 = \$148.939 +.
9. 12 ÷ .002 = 6000.
10. 147.828 ÷ 9.7 = 15.24.
11. \$37.4 ÷ \$4.5 = 8.311 +
12. 7.85 ÷ 3.43 = 2.2886 +.
13. .478 ÷ .58 = .824 +.
14. .9009 ÷ .4051 = 2.223 +.
15. 2.25 ÷ 18 = .125.
16. 22.36 ÷ 4.3.
17. \$3.25 ÷ \$.3.
18. 87.9 ÷ .3.
19. \$22.5 ÷ .005.
20. .001638 ÷ .07.
21. .08 ÷ 32.
22. \$.643 ÷ \$1.05.
23. .278 ÷ .07.
24. 14 ÷ .7854.
25. 3.1416 ÷ 4.
26. \$8.371 ÷ 500.
27. \$583.71 ÷ \$120.
28. 4.737 ÷ 3000.
29. 42 ÷ 600.
30. 125 ÷ 25000.

Suggestion.—In case the divisor has 0's at its right, *neglect them* in the process of dividing, and obtain the quotient of the dividend divided by the significant figures. Then remove the decimal point to the left as many places as there have been 0's neglected. *This is the same as the corresponding case in division of whole numbers.* Thus to divide 4.737 by 3000, we neglect the 3 0's and divide by 3, obtaining the quotient 1.579. Now this quotient is to be divided by 1000, the other factor of 3000 (**89**). This is done by removing the decimal point 3 places to the left (**86**).

$$\begin{array}{l} 3|000\,)\,\underline{4.737} \\ \qquad\;\; 1.579 \text{ Quot. of } 4.737 \div 3 \\ \text{True quot., } .001579 \end{array}$$

Perform the following:

31. $7.82 \div 10$.
32. $\$540 \div 100$.
33. $.56 \div 10$, by 100.
34. $.072 \div 900$.
35. $1800 \div .0006$.
36. $\$640 \div \$8000$.
37. $.05 \div 100$.
38. $235 \div 50$.
39. $473 \div 2300$.
40. $5.276 \div 11200$.
41. $30.03 \div 710$.
42. $64 \div .64$.

Applications.

[**Note.**—In these problems when *common* fractions occur, put them into the forms of *decimals* before performing the operation required.]

1. What is the cost of $3\frac{3}{4}$ yards of cassimere at $\$2\frac{1}{4}$ per yard?

Operation.

$$\begin{array}{r} \$2.25 \\ \underline{3.75} \\ 1225 \\ 1575 \\ \underline{675} \\ \$8.4475 \end{array}$$

$8.4475, or $8 and 45 cents.

In all business operations it is customary to drop from results all fractions of a cent less than ½, or .5 of a cent, and to call all fractions of a cent equal to or greater than .5 an additional cent. This will always be done in these examples.

2. What cost $14\frac{1}{2}$ cords of wood at \$$6\frac{3}{4}$ per cord?

Ans., \$97.88.

3. Bought of the groceryman $17\frac{3}{8}$ pounds of sugar at $11\frac{5}{8}$ cents per pound, and handed him in payment a \$5 bill; how much change should I receive? *Ans.*, \$2.98.

4. How many dozen hats, at \$$5\frac{3}{4}$ each, can be bought for \$$103\frac{1}{2}$? *Ans.*, $1\frac{1}{2}$ dozen.

5. Bought a case of 1 dozen pairs of boots for \$112.50. What was that per pair? *Ans.*, \$$9\frac{3}{8}$.

6. If a railroad train averages $23\frac{3}{5}$ miles an hour, including stops, how long will it be in running 288 miles?

7. If a railroad train runs 350 miles in $19\frac{1}{2}$ hours, but stops 3 times 20 minutes each, and 10 other times 6 minutes each, what is its average rate per hour while running?

8. How much will it cost to fence my house lot which is 4 rods by 8, on two adjacent sides, if I pay \$$1\frac{3}{4}$ per foot for the fence, a rod being $16\frac{1}{2}$ feet? *Ans.*, \$346.50.

9. If a ship sails at the rate of 130.75 miles per day, in what time will she make a trip of $69\frac{1}{3}$ miles?

Ans., .53 + of a day.

Bills of Goods.

158. A Bill of Goods is a written statement in proper form, which the seller gives to the buyer, specifying the amount and price of each article, and the aggregate value of the whole.

The person who buys is called *Debtor*, and the person who sells is called *Creditor*.

The letter @ made in this way is used in such bills for "at," and is followed by the price of a unit, as 1 yard, 1 pound, 1 gallon, etc.

Find the amount of each of the following Bills of Goods.

10. ANN ARBOR, *Dec.* 1st, 1874.

MR. JAMES SMITH

Bought of PHILIP BACH

$16\frac{1}{4}$ yds. Sheeting	@	22 cts.	$3.58
$7\frac{3}{4}$ yds. Flannel	@	$62\frac{1}{2}$ cts.	4.84
$\frac{1}{2}$ doz. Hdkfs *	@	$37\frac{1}{2}$ cts.	2.25
$2\frac{3}{4}$ yds. Drilling	@	$15\frac{3}{4}$ cts.	.43
		Rec'd Pay't,	$11.10

PHILIP BACH.

11. DETROIT, *Nov.* 25th, 1874.

MR. J. B. ANGELL

Bought of S. C. JOHNSON

5 Cans Oysters	@	$37\frac{1}{2}$ cts.	
$19\frac{1}{4}$ lbs. Turkey	@	$12\frac{1}{2}$ cts.	
20 lbs. Raisins	@	$17\frac{3}{4}$ cts.	
		Rec'd Pay't,	$7.84

S. C. JOHNSON.

12. ANN ARBOR, *Oct.* 7th, 1874.

MRS. A. E. PARKS

To COLE & TREMAIN, *Dr.*†

For	10 lbs. Granulated Sugar	@	$11\frac{1}{2}$ cts.	
"	5 Heads Celery . . .	@	$7\frac{1}{2}$ cts.	
"	$6\frac{3}{4}$ lbs. Butter	@	$31\frac{1}{4}$ cts.	
"	1 lb. Tea			$1.40
"	$2\frac{1}{2}$ lbs. Mocha Coffee . .	@	$42\frac{3}{4}$ cts.	
"	$10\frac{1}{4}$ lbs. Codfish	@	$10\frac{1}{2}$ cts.	
	Chg'd in Acct.			

* Hdkfs means handkerchiefs.

† This form of Bill-head means just the same as the preceding. Mrs. Parks is the *Debtor*, and Cole and Tremain are the *Creditor*.

When an account at a store has been running some time, the merchant often makes a copy of it for the creditor upon settlement. The following is such a transcript. What was due on the account?

13. MR. AMOS WHITE

In Acct. with C. H. MILLEN & Co.

Dr.

May 1st, 1874,	To	10 yds. calico . .	@	$8\frac{3}{4}$ cts.	
	"	6 spools thread .	@	$6\frac{1}{4}$ cts.	
	"	$2\frac{1}{4}$ yds. sheeting .	@	$17\frac{3}{4}$ cts.	
June 15th, 1874,	To	3 yds. Cassimere	@	\$$2\frac{1}{4}$ . .	
	"	Trimmings, \$1.25			
Aug. 21st, 1874,	To	4 Table Cloths .	@	\$1.75 .	
	"	3 Prs. Cotton Hose	@	28 cts.	

Cr.

Sept. 1st, 1874,	By Cash		\$2.00
		Due	\$

ANN ARBOR, *Dec.* 31*st*, 1874.

Goods Sold by the 100 or 1000.

14. What is the amount of a bill of 5728 ft. of lumber at \$$37\frac{1}{2}$ per thousand feet?

Explanation.—Were we to analyze this process we should first find how many 1000 feet there were by dividing 5728 by 1000, obtaining 5.728. Then, if 1 thousand cost \$$37\frac{1}{2}$, 5.728 will cost 5.728 times \$$37\frac{1}{2}$. Hence we see that we have to multiply together 5728 and $37\frac{1}{2}$ and divide by 1000; but, as the *order* of the operation is immaterial, we take the most convenient one. Hence

Operation.

$$\begin{array}{r} 5728 \\ 37\frac{1}{2} \\ \hline 2864 \\ 40096 \\ 17184 \\ \hline \$214.700 \end{array}$$

159. *To find the cost of articles sold by the* 1000, *multiply together the price per thousand and the number*—

using the number or the price for multiplier, as is most convenient—and then cut off 3 decimals from the product, or remove the decimal point 3 places to the left.

If the price is given per ton and the quantity in pounds, proceed in the same way, and divide the result by 2, *since a ton is* 2000 *pounds.*

If the articles are sold by the 100, *proceed in the same way, except that* 2 *instead of* 3 *figures are to be cut off as decimals, or the point removed* 2 *places to the left.*

15. What is the cost of 72568 brick, at $6.75 per M.?*

16. What is the cost of 85670 lath, at 23 cts. per C.?*

17. It is estimated that a certain public building will require 2 million bricks in its construction. What will these cost at $6¼ per M.?

18. I wish to put a board fence around a lot which is 10 rods by 20 (16½ feet make a rod). I find that for 1 panel (length) of fence (16 feet) it will take 1 board 1 foot wide, which measures 16 square feet; one 8 inches wide, which measures 10⅔ square feet; 4 boards each 5 inches wide, which together measure 26⅔ square feet. How much will the boards for the whole cost at $17½ per M.? Observe that I shall have to pay for boards for *whole* panels. They will not cut the boards for me at the lumber yard.

Ans., $57.04.

19. If it takes 8750 shingles for the roof of my house, how much will they cost at $4.75 per M.?

20. What is the cost of 117800 lath at $2.50 per M.?

21. What is the cost of a load of hay weighing 1875 *lbs.* at $12.50 per ton (2000 *lbs.*).

---

* M. stands for 1000 (27), and C. for 100.

Operation.

```
    1875
   12.50
   -----
    9375
   3750
  1875
  --------
2)23.43750
  --------
 $11.72
```

22. A farmer brought me 5 loads of hay which he had weighed at the public scales; they weighed 2785½ *lb.*, 3056¾ *lb.*, 2907¼ *lb.*, 3000 *lb.*, and 3172½ *lb.* each, including the wagon which weighed 950 *lb.* What is the value of the hay at $15½ per ton?

23. What will a load of coal weighing 1725 *lb.* cost at $11½ per ton?

24. I paid a coal dealer $9.92 for a load of coal weighing 1725 pounds. How much was that per ton?

25. I paid a coal dealer $9.92 for a load of coal, at $11½ per ton. How many pounds of coal should there have been?

26. What is the cost of 15500 *lb.* of plaster at $7.50 per ton?

27. A man paid $58.12½ for a car load of plaster at $7½ per ton. How many pounds should there have been?

28. If 31000 *lb.* of plaster cost $116.25, what is that per ton?

29. If freight from New York to Detroit is 35½ c. per hundred, how much will be the transportation on 6 boxes of goods weighing severally 347⅜, 158¼, 527⅝, 250, 348⅛, and 629⅞ *lb.*?

30. If railroad freight on wheat from Detroit to New York is 37½ cents per 100 pounds, what is that per bushel of 60 pounds?

31. November 25, 1874, freights by water from Milwau-

kee to Buffalo are quoted at $5\frac{1}{2}$ c. (*i. e.*, $5\frac{1}{2}$ c. per bushel). What is this per 100 *lb.?*

32. Which would be better, to charter a vessel which would carry 1200 tons from Milwaukee to Buffalo for $2500, or to pay freight at the rate of $5\frac{1}{2}$ c. per bushel of wheat?

33. What is the freight from Detroit to New York on a car load of wheat—375 bushels—at $37\frac{1}{2}$ c. per 100 *lb.?*

34. Expressage from Ann Arbor to East Saginaw is now $$1\frac{1}{4}$ per hundred. What will this add to the cost of butter per pound if I ship 6 crocks weighing in gross (*i. e.*, crocks and all) $35\frac{1}{2}$ *lb.* each, the crocks weighing 4 *lb.* each, and the return expressage to be paid on the crocks?

Ans., A little more than $1\frac{1}{2}$ cents.

35. This fall (1874) freight on wheat by vessel from Toledo, O., to Oswego, N. Y., is quoted at 14 *c.* (14 *c.* per *bush.*). What is the freight on a schooner's cargo of 1200 tons?

36. Which will cost the more, to ship 1000 bushels of wheat by water at $15\frac{1}{2}$ cents per bushel, or by railroad at $31\frac{1}{4}$ *c.* per 100 *lb.?* How much?

---

37. What ought eggs to be per pound when they are selling at $18\frac{3}{4}$ *c.* per dozen, if they average $9\frac{1}{2}$ eggs to a pound?

38. If a compositor (one who sets type) receives 40 *c.* per 1000 m's for setting, how much will he receive for setting a book of 296 pages, which measure 1576 m's to a page?

39. What is the amount of a bill of lumber for 25371 *ft.* boards @ $$37\frac{1}{2}$ per M., 1483 *ft.* scantling @ $$18\frac{3}{4}$ per M., 21000 lath at 21 *c.* per C., and 7342 shingles at $$6\frac{1}{2}$ per M.?

CHAPTER IV.

DENOMINATE NUMBERS.

SECTION I.

Tables.

160. An Abstract Number is a *mere* number; that is, a number not applied to any specified things. Thus *ten, seven,* 146, $\frac{2}{3}$, $4\frac{3}{4}$, are abstract numbers.

161. A Concrete Number is a number applied to some specified thing, as *ten* men, *seven* trees, 146 feet, $\frac{2}{3}$ of an acre, $4\frac{3}{4}$ pounds, etc.

162. Denominate Numbers means, literally, *Named Numbers;* but the term is applied only to concrete numbers which represent *money, weight,* or *measure.* Thus $5, 10 *lb.*,* 3 *gal.*, are denominate numbers; but 5 *men,* 10 *trees,* 3 *stones,* are not.

163. In denominate numbers the different *Orders,* as of money, weight, or measure, are called **Denominations.** Thus *dollars, dimes,* and *cents* are denominations of money; and *rods, feet,* and *inches* are denominations of measure.

164. A Compound Number consists of several related denominations written together, and to be read as one number. Thus 4 *gal.* 2 *qt.* 1 *pt.* is a compound number; so also is 6 *mi.* 25 *rd.* 10 *ft.**

---

* It is presumed that the pupil will have become familiar with these common abbreviations before he reaches this point in his course.

UNITED STATES MONEY.

Present United States Coinage.

Gold. **Silver.**

Table.

10 *m.* = 1 *c.*
10 *c.* = 1 *d.*
10 *d.* = $1.
$10 = 1 *Eagle.*

E.

The *Trade dollar* is coined only for foreign use. Silver 5-cent and 3-cent pieces, and 2-cent pieces, though in common use, are not now coined. The mill was never coined.

Gold coin is .9 pure gold and .1 alloy of silver and copper. Silver coin is .9 pure silver and .1 copper. *Nickel* coin (so-called) is ¾ copper and ¼ nickel. *Bronze* is copper, tin, and zinc.

d.

$

A gold dollar weighs 25.8 *gr.*; a silver trade dollar, 420 *gr.*; the dollar of 1878, 412½ *gr.*; and the smaller silver pieces at the rate of 385.8 *gr.*

Nickel. **Bronze.**

Canada Currency

is the same as United States, although much use is still made of English currency.

ENGLISH, OR STERLING MONEY.

165. The Denominations of English money are Pounds, Shillings, Pence, and Farthings, represented respectively by *£.*, *s.*, *d.*, and *far.*

FARTHING.

The *Gold Coin* whose value is £1, is called a *Sovereign* (*sov.*). Its value in American gold is $4.8665.* From this value and the following table, the pupil will be able to calculate the value of any English coin.

Copper.

Table.

4 *far.*	=	1 *d.*
12 *d.*	=	1 *s.*
20 *s.*	=	£1.

A *Guinea* is a gold coin equal to 21*s.* A *Crown* is a silver coin equal to 5*s.* A *Florin* is a silver 2-shilling piece.

Silver.

1. What is the value in our coin of an English shilling? Of a penny? Of a farthing? Of a crown? A guinea?

2. What is £5 12s. in dollars and cents?

3. What English coin is very nearly a half-dollar? What is very nearly a half-eagle?

Gold.

SOVEREIGN.

4. What is 10s. 6d. in our coin? What 5s. 10d.?

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* This "par of exchange," as it is called, is often much less than the commercial value in bank notes. At this time £1 = $5.47 in our national bank notes.

FRENCH COINS.

166. The French coins most frequently mentioned in this country are the *Franc* (*frank*), the *Napoleon*, and the *Centime* (*centeem'*).

The *Centime* is $\frac{1}{100}$ of a Franc, just as our cent is $\frac{1}{100}$ of a dollar. A *Napoleon* is a 20-franc gold piece. The franc is silver, and the centime bronze. The abbreviation for francs is fr., and centimes are written as decimals, thus 5 fr.17 is 5 francs and 17 centimes.

BRONZE.

FRANC.—Silver.

NAPOLEON.—Gold.

1. A Franc is $.193. How much is a Centime? A Napoleon?

2. How near is a 5-franc piece to $1?

3. The French *Gold* coins are 100, 40, 20, 10, and 5 franc pieces. How much is each in our coin?

4. The French *Silver* coins are 5, 2, and 1 franc pieces, and 50 and 25 centime (*i. e.*, half and quarter francs) pieces. How much is each in our coin?

5. The *Bronze* French coins are 10, 5, 2, and 1 centime pieces. How much is each in our coin?

6. $1{,}000{,}000^{fr}$ are how many dollars and cents? $25^{fr}.37$? $15^{fr}.75$? $258^{fr}.60$? $100^{Nap.}$? $550 = how many francs?

GERMAN COINS.

167. The standard denominations of German coins are **The Mark** (23.8 *cents*), and **The Pfennig** = $\frac{1}{100}$ of a mark.

20-MARK.—Gold.

Silver.

The Prussian *Silver Thaler* (74.6 c.), and the *Silver Groschen* ($2\frac{1}{2}$ c.), are the coins as yet most frequently referred to in this country, the mark and pfennig having been made the standards in 1872.

1. The German *Gold* coins are 5-mark, 10-mark, and 20-mark pieces. What is the value of each in our coin?

2. The German *Silver* coins are 20-pfennig, 1-mark, 2-mark, and 5-mark pieces. What is the value of each in our coin?

3. The German *Copper* coins are 5-pfennig, and 10-pfennig pieces. What is the value of each in our coin? How many pfennigs make a cent?

4. If gold is worth $1\frac{1}{8}$ in our paper currency, what is the value in currency of a book quoted at 12.24 marks? At 56 pfennigs? At 3 *Th.* 8 *s. gr.*? At 1 *Th.* 12 *s. gr.*? At 21.54 marks? What are hundredths of marks?

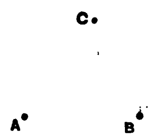
5. $100 gold = how many marks?

6. How many pfennigs make $\frac{1}{4}$ of a dollar?

7. How many marks make $1 gold?

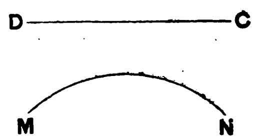
MEASURES OF EXTENSION.

168. A Point is a place without size.



We represent points by dots on paper, or on our slates, but the point is to be thought of as just the centre of the dot. We put letters by the points to designate them, and so speak of the point **A**, the point **B**, the point **C**, etc., meaning the centre of the dot by which the letter stands.

169. A Line is the path of a point in motion.



If we conceive a point at **D** and then as moving to some other position, as to **C**, its *imaginary* path is a line. So also if the point does not move in a *straight* line but in a curved line, as from **M** to **N**, its imaginary path is a line, all the same. When we draw a mark with a pencil to represent a line, the pencil point represents the point, and the mark made represents the line. A *true* line has no width, although the mark by which we represent it has. We may think of the line as in the centre of the mark just as we think of the point as in the centre of the dot. The ends of a line are points. A line is designated by letters placed at its ends, as the line **DC**, the line **MN**, etc.

170. A Straight Line is the path of a point moving all the time in the same direction. Generally, when we say "Line" we mean a "straight line."

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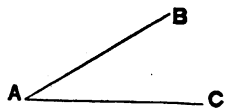
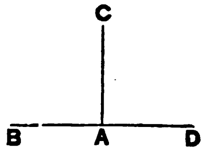
171. An Angle is the opening between two lines which meet. The point where the lines meet is called the **Vertex** of the angle.



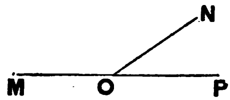
In common language we call an angle a *Corner*. Thus the corner or opening between the two lines **BA** and **CA**, is the angle **BAC**.

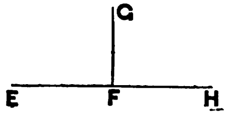
In designating an angle we use a letter placed at the vertex, or three letters, one on each of the lines, with the letter which stands at the vertex named between them. Thus the angle above may be spoken of as the "angle **A**," or the "angle **BAC**."

172. When one straight line, as **CA**, meets another, as **BD**, so as to make the angles **CAB** and **CAD** equal (*i. e.* just alike), the angles are called **Right Angles**. A right angle is a *square corner*.

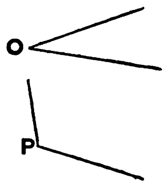


Are the angles **MON** and **NOP** equal? Are they right angles? Which is the larger angle (corner)? Are **EFG** and **GFH** equal? Are they right angles?

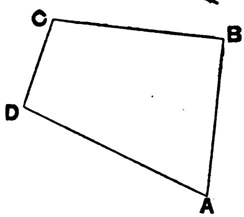




173. An Acute Angle is an angle which is less than a right angle, and an **Obtuse Angle** is one which is greater than a right angle.



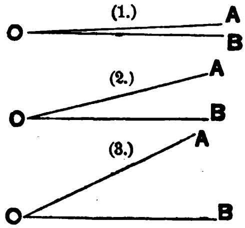
An acute angle is a sharp corner, and an obtuse angle is a blunt corner. Which is the acute angle, **O** or **P**? In this figure there is one acute angle and one right angle, and there are two obtuse angles. Point them out, designating them by the proper letter, or letters.

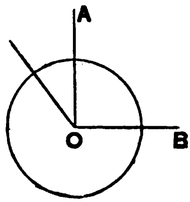


174. An Angle may be conceived as produced by a line revolving about one of its extremities (ends).

Thus suppose the upper line to revolve around its left-hand extremity, as if the two lines were two sticks fastened together at

one end by a hinge. When the lines are shut together (they are nearly so in 1) there is no angle, or we call the angle 0. When the line **AO** has revolved a little so as to make a small opening, as in 2, the angle is quite acute. As **AO** revolves the angle enlarges until it becomes a right angle as **AOB** in the figure at the left. Then as **AO** moves on the angle becomes obtuse.





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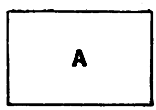
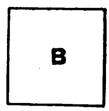
175. A space which has length and breadth but not thickness, is called a **Surface.**

176. A Rectangle is a plane (flat) surface, or figure, bounded by four straight lines and having all its angles right angles. **A** is a rectangle.



177. A Square is a rectangle having all its sides equal each to each. **B** is a square.



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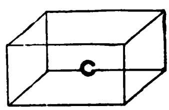
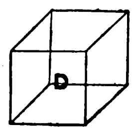
178. A Solid, or **Body,** is a space having length, breadth and thickness.

179. A Parallelopiped is a solid, or space, bounded by six rectangles, and having all its angles right angles. **C** represents a parallelopiped.



180. A Cube is a parallelopiped having all its faces (sides) squares. **D** represents a cube. A cube each of whose edges is 1 inch, is a *Cubic Inch;* one whose edges



are each 1 foot, is a *Cubic Foot;* one whose edges are each 1 yard, a *Cubic Yard,* etc.

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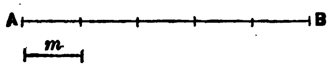
181. A *Line,* or *Distance,* is *measured* by another line or distance.

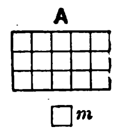


Thus the line **AB** may be measured by the line *m*. If *m* is 1 the line **AB** is 5, since it contains *m* 5 times, *i. e.* **AB** is 5 times as long as *m*.

182. The lines or distances commonly used in measuring length, or distance, are an *Inch,* a *Foot,* a *Yard,* a *Rod,* and a *Mile.*

The line **A** is an inch. A foot is 12 times as long. A yard is 3 times as long as a foot. A rod is $5\frac{1}{2}$ times as long as a yard. A mile is 320 rods.

183. A *Surface is measured* by another surface,—usually by a square. **The Area** of a surface is the number of times it contains the measure.



Thus the surface **A** may be measured by the surface *m*. If *m* is 1, we readily see that **A** is 15, for **A** is 15 times as large as *m; i. e.,* if we were to cut out 15 pieces of paper like *m*, it would take 15 of them to cover the figure **A**. Therefore the *area* of **A** is 15, if *m* is the measure.

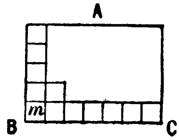
184. The surfaces commonly used for measuring surface are the *Square Inch, Square Foot, Square Yard, Square Rod, Square Mile,* and the *Acre.*

The whole square on page 200 is a square inch. A square foot is a square each of whose sides is 1 foot long. A square yard is a square 1 yard on a side. A square rod is a square 1 rod on a side. A square mile is a square 1 mile on a side. An acre is 160 square rods; it has no corresponding measure of length.

Principle I.

185. *The* **Area** *of a* **Rectangle** *is the product of its length by its width.**

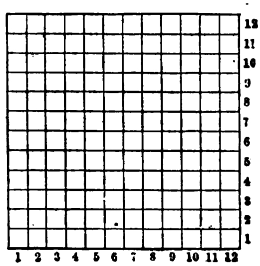
Illustration.—If surface **A** is 7 long and 5 wide; and *m*, the measure, is 1 on a side, we can apply 7 of the measures along one side of the surface, as from **B** to **C**, making a row of 7 measures. Now we can apply the measure so as to make as many such rows as there are units in the width—in this case 5; and 5 times 7 measures (*m*) make 35 measures. Hence the area of **A** is 35, *m* being the measure.



1. A square foot is 12 inches on a side. How many square inches in a square foot?

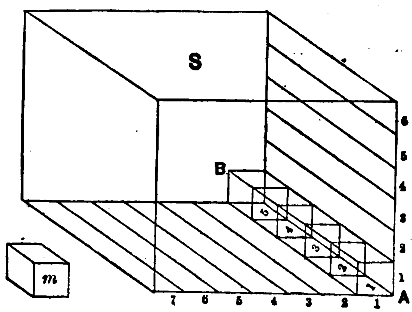
2. A square yard is 3 feet on a side. How many square feet in a square yard?

3. Show that there are $30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet in a square rod.



4. Show that there are 102400 square rods in a square mile. 160 square rods make an acre. How many acres in a square mile?

186. A Solid is measured by another solid—usually by a cube. The **Volume,** or **Contents,** of a solid is the number of times it contains the measure.



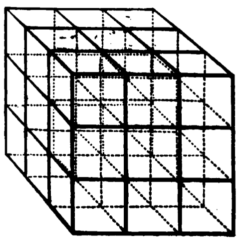
* With some grades of pupils it will be well for the teacher to introduce the terms *Base* and *Altitude*.

Thus the solid **S** may be measured by means of the cube *m*. If *m* is 1 we readily see that **S** is 210, for **S** is 210 times as large as *m*; *i. e.* if we were to make a pile as large as **S** of such blocks as *m*, it would take 210 such as *m* to make it.

Principle II.

187. *The* **Volume** *of a* **Parallelopiped** *is the product of its three dimensions, that is, of its length, breadth, and thickness.*

Illustration.—If solid **S** (**186**) is 7 long, 5 wide, and 6 high; and *m*, the measure, is a cube 1 on each edge, we can place 5 of the measures along one side of the bottom (base) of the parallelopiped **S**, as from **A** to **B**, and 7 such rows will cover the base. Thus it takes 7×5 of the measures to cover the base 1 deep. Then, as the parallelopiped is 6 high, it will take 6 such layers, or $7 \times 5 \times 6$, to fill the whole space. Hence the volume of the parallelopiped **S** is $7 \times 5 \times 6$, or 210, *m* being the measure.



1. How many cubic feet in a cubic yard?

2. How many cubic feet in a parallelopiped 8 feet long, 4 feet wide, and 4 feet high? This is called a *Cord* of wood. *Ans.*, 128.

3. How many cubic inches in a cubic foot?

188. Abbreviations.—*in.* stands for inch or inches, *ft.* for foot or feet, *yd.* for yard or yards, *rd.* for rod or rods, *mi.* for mile or miles, *sq.* for square, *cu.* for cubic, *A.* for acre, *cd.* for cord.

189. **Tables.**

LONG MEASURE.		SQUARE MEASURE.		CUBIC MEASURE.
12 *in.* = 1 *ft.*	∴*	144 *sq. in.* = 1 *sq. ft.*,	and	1728 *cu. in.* = 1 *cu ft.*
3 *ft.* = 1 *yd.*	∴	9 *sq. ft.* = 1 *sq. yd.*,	and	27 *cu. ft.* = 1 *cu. yd.*
$5\frac{1}{2}$ *yd.* = 1 *rd.*	∴	$30\frac{1}{4}$ *sq. yd.* = 1 *sq. rd.*		128 *cu. ft.* = 1 *cd.*
320 *rd.* = 1 *mi.*		160 *sq. rd.* = 1 *A.*		
		640 *A.* = 1 *sq. mi.*		

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190. Cloth, ribbons, laces, etc., are sold by the yard in length, irrespective of the width, the differing widths being considered in fixing the price. For such measurements the yard is divided into halves, quarters, and eighths.

Ex. How many inches is a half a yard? How many in a quarter? An eighth? A sixteenth?

191. For measuring land, Surveyors use a *Chain* 4 rods long, and made of 100 links of equal length.

1. How many feet in 4 rods? How many inches in 66 feet? Then how long is 1 link of the Surveyor's chain?

2. How many rods in a mile? Then how many chains in a mile?

192. The *Public Lands* of the United States which have been surveyed during the present century have been laid out in **Townships,** which are squares 6 miles on a side. These are divided in what are called **Sections.** A Section is a Square Mile.

1. How many sections in a Township?

2. How many acres in a section? In a half-section? A quarter-section? An eighth of a section?

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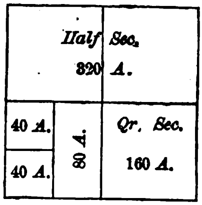
* This sign ∴ means "therefore." The teacher should be sure that the pupil sees the connection, and dependence of these tables.

3. What part of a section is 320 acres? 240? 160? 120? 80? 40?

4. How many 80 acre lots in a section? A half-section? A quarter-section?

5. This figure represents the way in which a section is usually divided. How long and how wide is a quarter-section, or 160 A.? 80 A.? 40 A.?



[*See Appendix* for the method of our Public Land Surveys.]

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193. In measuring boards the *surface measure* alone is considered unless the board is more than an inch thick.

1. How many inches in length is a board 10 *ft.* long? If such a board is 8 inches wide, how many square inches in it? Then how many square feet?

Suggestion.—Observe that we have $\frac{10 \times 12 \times 8}{144}$, which by cancellation becomes $\frac{20}{3}$, or $6\frac{2}{3}$.

Why do we multiply the 10 by 12? Why this product by 8? Why divide by 144?

2. How many square feet in 6 boards each 14 *ft.* long and 15 *in.* wide?

Suggestion.—We have $\frac{14 \times 12 \times 15 \times 6}{144} = \frac{14 \times 15 \times 6}{12}$

Why is it that in measuring boards we may multiply the length IN FEET *by the wihth in* INCHES, *and divide the product by* 12 *to get the square feet?*

3. How many square feet in a load of boards in which there are 10 boards 12 *ft.* long and 10 *in.* wide, 8 boards 16 *ft.* long and 8 *in.* wide, 20 boards 14 *ft.* long and 10 *in.* wide? How much would this load amount to at $32 *per M.?* *Ans.*, $13.40.

When the lumber is $1\frac{1}{4}$ *in.* thick, $\frac{1}{4}$ is to be added to the superficial measure; when $1\frac{1}{2}$ *in.* thick, $\frac{1}{2}$ is to be added; when 2 *in.* thick, the superficial measure is to be doubled. Why?

4. How many feet of boards in a load consisting of 30 pieces of $1\frac{1}{4}$ *in.* stuff, 12 *ft.* long 6 *in.* wide; 40 pieces of $1\frac{1}{2}$ *in.* stuff, 10 *ft.* long and 9 *in.* wide; and 20 bolts of "siding" ($\frac{1}{2}$ *in.* stuff) 12 *ft.* long and 5 *in.* wide, with 6 pieces in each bolt? *Ans.*, 1,275 feet.

Lumber is sometimes sawed so that the boards are an inch or two wider at one end than at the other. In such cases the width is measured in the middle.

5. How much flooring $1\frac{1}{4}$ *in.* thick will it take for the floors of 10 rooms, 2 being 18×15 *ft.* (18 *ft.* by 15 *ft.*) each, 2 15×12 *ft.* each, 1 12×16 *ft.*, 1 12×12 *ft.*, 2 10×12 *ft.* each, and 2 10×10 feet? How much will it cost at $32 per M.? *Answer to last*, $67.04.

6. Wall paper is usually 18 inches wide, and 8 yards make a roll. How many rolls must we buy to paper a room 16×18 *ft.* whose walls are 10 *ft.* high, no allowance being made for doors and windows? What will it cost at 40 *c.* per roll if an allowance of $\frac{1}{4}$ be made for doors and windows?

Suggestion.—How far is it around the room? How many widths of paper will it take to go around? It is 68 *ft.* around the room, and as the paper is $1\frac{1}{2}$ *ft.* wide, it will take $68 \div 1\frac{1}{2}$ strips, that is, 45 *whole* strips and $\frac{1}{3}$ of the width of a strip. Hence we shall have to buy 46 strips of 10 feet each, or 460 feet. Now there are 24 *ft.* in a roll; hence we must buy $460 \div 24$, or $19\frac{1}{6}$ rolls. As they do not usually cut rolls, we shall have to buy 20 rolls.

7. How many rolls of wall paper must we buy to paper 2 rooms, one 12×16 *ft.* and the other 15×18, each 10 *ft.* high, there being 7 doors and windows in one room, and 6

in the other, each of which is $3\frac{1}{4}$ *ft.* wide: it being understood that the strips are not to be pieced, but that the 4 *ft.* which each roll will overrun will piece out above the doors and windows? What will the papering cost at 60 *c.* per roll for the paper and 20 cents per roll for putting it on, to which must be added bordering at 15 *c.* per yard, and 20 *c.* for putting on every 8 *yds.* of bordering?

We shall have to buy *whole* rolls, and shall be charged for putting on a fraction of a roll, as for a whole roll.

Ans., $27.90.

8. My house lot is a corner lot 4 rods by 8. What will the lumber cost for a 2 *in.* plank walk, 6 *ft.* wide, at $18 *per M.*, with 3 by 4 *in.* scantling for stringers, the stringers to be laid crosswise and so that each 12 *ft.* plank shall rest on 4 stringers, the scantling costing $12 *per M.*

9. Carpenters often charge by the *square* 10 by 10 *ft.* for laying floors, ceiling, roofing, etc. What will the flooring of 4 rooms with $1\frac{1}{4}$ *in.* plank cost, at $20 *per M.* for the plank, 75 *c.* per square for laying, and 20 lbs. of nails at 5 *c.* *per lb.*, the rooms measuring respectively 18 *ft.* square, 15 by 18 *ft.*, 12 by 15 *ft.*, and 16 *ft.* square?

10. Mr. A. owned 100 acres of land lying in the form of a rectangle. It extended $\frac{1}{4}$ of a mile on the road; what part of a mile did it extend back from the road?

11. What is the difference between 10 rods square and 10 square rods?

12. How many bricks (a brick is $2 \times 4 \times 8$ *in.*) laid

on the side will it take to pave a cellar, 15 × 18 *ft.* on the bottom? How many if laid on the edge?

13. How many tiles 6 *in.* square will lay a floor 20 × 36 *ft.*?

14. What is the cost of plastering a room 20 × 24 *ft.* and 10½ *ft.* high, including the ceiling, at 25 *c.* per square yard, there being 3 doors 7½ *ft.* high, and 3 *ft.* 9 *in.* wide, and 4 windows 6 *ft.* high and 3½ *ft.* wide?

15. How many yards of cloth 27 *in.* wide will it take to line 12 *yds.* 1¼ *yd.* wide?

16. How much does it cost to carpet a room 16 *ft.* by 20 *ft.* with carpet ¾ *yd.* wide, costing $1.60 *per yd.*, allowing a half-length strip to piece the width and 4 *in.* waste on each strip in cutting?

17. How many acres of land in each of the following pieces:

200 rods by ½ a mile?
1200 *ft.* by 60 rods?
208.7 *ft.* square?

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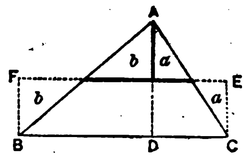
18. In surveyor's measure 80 chains make a mile. How many square chains in a square mile?

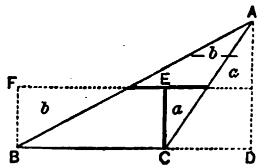
19. A piece of land 20 chains square contains how many acres? How many square rods in a square chain?

20. How many square chains in an acre? How many acres in a piece of land 7 chains wide and 12 chains long?

21. How does this figure show that the area of the triangle ABC is the same as the area of a rectangle, FBCE, having the same base BC and ½ the altitude AD?



22. How does this figure show that the area of the triangle ABC is the same as the area of a rectangle, FBCE, having the same base BC and ½ the altitude AD?



Cut two triangles of the above forms from card-board. Then cut the triangles in the heavy lines and place the pieces together so as to make the rectangles.

23. What is the area in acres of a triangular piece of ground whose base is 7.52 *ch.* and altitude 5.32 *ch.*?

Ans., 2 *A.*+.

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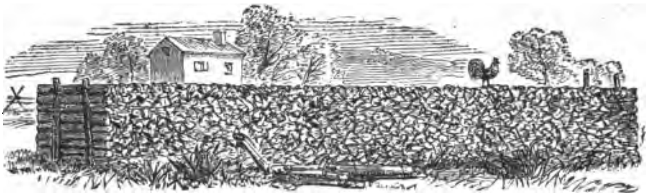
194. Sea Depths are measured in Fathoms. A **Fathom** is 6 feet.

1. How many feet in a mile? How many fathoms in a mile? In 3 miles?

2. At a certain place the sea was reported as 900 fathoms deep. How much more than a mile deep was it?

3. How many miles in depth is that place in the Atlantic Ocean which is reported as 2640 fathoms deep?

4. In our Western forest many of the trees are 100 feet high. How many fathoms deep would a lake be which would submerge these standing forests?



1. How many cubic feet in a pile of 4 foot wood (that is, sticks of wood 4 feet long) piled as in the picture, the pile being 32 feet long and 5 feet high? How many cords?

2. How many cords of wood in a pile of 4 foot wood 130 *ft.* long and 6 *ft.* high?

Suggestion.—A convenient way to solve such an example is this

$$\frac{\overset{65}{\cancel{130}} \times \cancel{4} \times \overset{3}{\cancel{6}}}{\cancel{4} \times \cancel{4} \times 8} = \frac{195}{8} = 24\tfrac{3}{8}.$$

The pupil should give the reasons for this process.

3. How many cords in a pile of 6 *ft.* wood 148 feet long and 8 feet high? *Ans.*, $55\frac{1}{2}$.

4. How many cords in a pile of wood 36 *ft.* long, 4 *ft.* wide, and 4 *ft.* high? When wood is piled in this way how many feet in length of the pile does it take to make a cord?

5. Why does $\frac{250}{8}$ give the number of cords in a pile of 4 *ft.* wood, 4 *ft.* high, and 250 *ft.* long?

6. How many cords in three piles of 4 *ft.* wood, the first 36 *ft.* long and 4 *ft.* high, the second 42 *ft.* long and 5 *ft.* high, and the third 20 *ft.* long and 6 *ft.* high?

Query.—Why, in estimating the number of cords in a pile of 4 *ft.* wood, is it only necessary to multiply the length by the height, and divide by 4 × 8, or 32?

7. How many cords in a pile of 4 *ft.* wood 200 *ft.* long and 7 *ft.* high? How many if the wood be 3 feet long? If 2 *ft.* long? How many in each case if the pile be 6 *ft.* high? 5 *ft.* high? 8 *ft.* high?

8. How much wood in a load consisting of 3 lengths of 4 foot wood, the load being 3 *ft.* 2 *in.* wide, and 2 *ft.* 6 *in.* high?

Suggestion.—Since 2 in. $= \frac{2}{12} = \frac{1}{6}$ of a foot, and 6 in. $= \frac{1}{2}$ a foot, this load is equivalent to a pile $9\frac{1}{2}$ ft. long and $2\frac{1}{2}$ high. Hence we have $\frac{9\frac{1}{2} \times 2\frac{1}{2}}{4 \times 8} = \frac{19 \times 5}{4 \times 4 \times 8} = \frac{95}{128}$, or a cubic foot less than $\frac{3}{4}$ of a cord. ($\frac{96}{128}$ cu. ft. are $\frac{3}{4}$ of a cord.)

9. How much wood in a load consisting of 3 lengths of 4 *ft.* wood, the load being 3 *ft.* wide and 2 *ft.* 8 *in.* high?

Suggestion. $\frac{9 \times 2\frac{2}{3}}{4 \times 8} = \frac{9 \times 8}{3 \times 4 \times 8} = \frac{3}{4}$.

Note.—Wood-racks are sometimes narrower at the bottom than at the top, and hence the width of the load should be measured half way up, as this gives the *average* width.

10. How much wood in a load consisting of 2 lengths of 4 *ft.* wood, the average width of the load being 2 *ft.* 9 *in.* and the height 3 *ft.*? *Ans.*, $\frac{1}{2}$ *cd.* and 2 *cu. ft.*

11. How many cubic yards in a ditch $\frac{1}{2}$ *mi.* long, and *which averages* 3 *ft.* deep and 4 *ft.* wide?

12. At 20 *c.* per cubic yard what does it cost to excavate a cellar 20 *ft.* by 30 *ft.*, and $6\frac{1}{2}$ *ft.* deep?

13. At 30 *c.* per cubic foot what cost a stick of timber $15\frac{1}{2}$ *in.* square and 20 *ft.* long?

14. What is the cost of digging a trench 650 *ft.* long, $2\frac{1}{2}$ *ft.* wide at top and $1\frac{1}{2}$ *ft.* wide at bottom, and averaging $3\frac{1}{4}$ *ft.* deep, at 25 *c. per cu. yd.*?

The average width is 2 *ft.*

15. The cross section of a certain R. R. tunnel is 525 *sq. ft.*, and the length of the tunnel $\frac{3}{8}$ of a mile. How many cubic yards were removed in the excavation?

16. How many cubic feet in 3 sticks of timber which measure respectively 4×6 *in.* and 14 *ft.* long, 8×10 *in.* and 16 *ft.* long, 1 *ft.* square and 20 *ft.* long?

17. How much will the brick cost, at $5.25 *per M.*, for a building with main part 40 *ft.* front and 32 *ft.* deep, and walls 26 *ft.* high, with a rear part 20 *ft.* wide and 30 *ft.* deep, and walls 20 *ft.* high; all the walls 17 inches thick (including mortar), $\frac{1}{7}$ being allowed for mortar, also there being 1 doorway 6 *ft.* wide and 8 *ft.* high, 5 doorways $3\frac{1}{2}$ ft. wide and 7 *ft.* high, 22 windows 3 *ft.* wide and $6\frac{1}{2}$ *ft.* high, and 13 windows 3 *ft.* wide and $5\frac{3}{4}$ *ft.* high?

18. What is the difference between a cube 3 *ft.* on an edge, and 3 cubic feet?

19. How many tons (2000 *lb.* each) of ice can be packed in an ice-house 50 *ft.* long, 20 *ft.* wide, and 12 *ft.* high, a cubic foot of ice weighing 58.125 *lb.*?

20. In a room 125 *ft.* by 90 *ft.*, with ceiling 30 *ft.* high, and seating 2500 persons, how soon will the air have been all breathed, allowing 10 *cu. ft.* per minute for each person?

21. Milwaukee bricks are $8\frac{1}{2} \times 4\frac{1}{4} \times 2\frac{3}{8}$ *in.* If it takes 40,000 common bricks for a particular structure, how many Milwaukee bricks will it take?

22. Philadelphia and Baltimore bricks are $8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}$ *in.* How many such bricks would be required for the house in Ex. 21?

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195. Doyle's Rule for calculating the amount of square-edged inch boards which can be sawed from a round log is this: *From the diameter in inches subtract 4; the square of the remainder will be the number of square feet of inch boards yielded by a log 16 ft. in length.**

The yield of logs of the same diameter is in the ratio of their lengths. The log is scaled, *i. e.*, its diameter is measured at the top end.

1. What is the board measure of a log 16 *ft.* long and 18 *in.* in diameter? One 20 *in.* in diameter? 30 *in.*?

2. What is the board measure of a log 12 *ft.* long and 2 *ft.* in diameter? 31 *in.*? 40 *in.*? 37 *in.*?

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* This rule, so admirable in its simplicity, is the foundation of the table in *Scribner's* popular *Lumber and Log Book*, which is said to have a larger sale than all other books of the kind together, and is a generally recognized standard among lumbermen. Nevertheless, in a scientific point of view, the rule is but a rude approximation. See the Author's SCIENCE OF ARITHMETIC, p. 277.

3. What is the board measure of a log 18 *ft.* long and 36 *in.* in diameter? Of one 15 *ft.* long? 10 *ft.*? 20 *ft.*?

MEASURES OF CAPACITY.

196. The Capacity of a vessel is the amount which it contains—its *contents* (**186**).

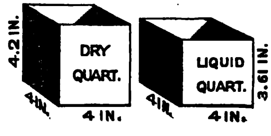
There are two varieties of measures of capacity in common use, viz., *Liquid Measure* and *Dry Measure.*

197. Liquid Measure is used in measuring liquids or in estimating the capacity of vessels designed to contain liquids, as water, milk, oil, molasses, alcohol, etc. The *Denominations,* or measures, are gills (*gi.*), pints (*pt*), quarts (*qt.*), gallons (*gal.*), and barrels (*bbl.*).

198. Dry Measure is used in measuring grain, seeds, fruit, etc., or in estimating the capacity of vessels designed to contain such articles. The Denominations are pints (*pt.*), quarts (*qt.*), pecks (*pk.*), and bushels (*bu.*).

199. The denominations of like name in these two measures do not represent the same amounts. The *Dry pint* and *quart* are about $\frac{1}{6}$ larger than the *Liquid pint* and *quart;* or more exactly, the *Dry quart* contains 67.2 cubic inches, and the *Liquid quart* 57.75 cubic inches.

Illustration.—These two figures represent boxes each of which is 4 inches square on the bottom; but the *Dry quart* is 4.2 inches deep while the liquid quart is but 3.61 inches deep.



1. 4 quarts make a gallon. How many cubic inches make a gallon?

2. 8 quarts make a peck and 4 pecks make a bushel. How many cubic inches in a bushel?

3. Which is the more, $1\frac{1}{4}$ *cu. ft.* or 1 *bu.*?

Ans., The difference is less than $\frac{1}{3}$ *pt.* ($\frac{2}{7}$).

200. **Tables.**

LIQUID MEASURE.	DRY MEASURE.
4 *gi.* = 1 *pt.*	
2 *pt.* = 1 *qt.*	2 *pt.* = 1 *qt.*
4 *qt.* = 1 *gal.*	8 *qts.* = 1 *pk.*, or 2 *gal.*
$31\frac{1}{2}$ *gal.* = 1 *bbl.**	4 *pk.* = 1 *bu.*

Physicians and Apothecaries use a kind of liquid measure of which the denominations are *Minims* (♏), *Fluid Drachms* (f ʒ), *Fluid Ounces* (f ℥), *Pints*, and *Gallons*. The pint and gallon are the same as the common *Liquid Pint* and *Gallon*, but are designated by the abbreviations (O.) (Latin *octarius*, pint), and *Cong.* (Latin *congius*, gallon). 60 ♏ = 1f ʒ, 8f ʒ = 1f ℥, and 16 f ℥ = 1 (O.).

Physicians in making prescriptions frequently call a minim a *drop*, a fluid drachm a *teaspoonful*, 4 fluid drachms a *tablespoonful*, a fluid ounce 2 *tablespoonfuls*, 4 fluid ounces a *teacupful*, and a pint 4 *teacupfuls*.†

1. How many pints in a gallon? In 5 gallons? 7 gallons? A barrel?

2. How many quarts in a barrel? In 5 *bbl.*? In 4 *bbl.*?

* Barrels are made of various sizes from 30 to 40 or even 56 gallons; but in estimating the capacity of cisterns, vats, etc., $31\frac{1}{2}$ *gal.* is usually considered a barrel. There is no definite measure in use called a hogshead. Any large cask is frequently so called.

† These measures are very indefinite, and in fact are much in excess of what they are called. Thus a drop of most liquids is much more than a *minim*. A common teaspoon holds nearer 90 than 60 drops of water, and we more frequently find teacups that hold $\frac{1}{2}$ a pint than a gill.

3. How many pints in 20 *gi.*? In 16 *gi.*?

4. How many quarts in 128 *pt.*? Then how many gallons in 128 *pt.*?

5. How many barrels in 78,506 *gal.*? In 250,082 *gal.*? In 856.47 *gal.*? *Answer to one,* 7939.11 +.

6. How many gallons in 412,578 *qt.*? Then how many barrels?

7. How many barrels in 642,800 *qt.*? In 500,000 *qt.*? In 1,000,000 *pt.*? *Answer to last,* 3968.25 +.

8. How many cubic feet in rectangular (square-cornered) box 3 *ft.* long 3 *ft.* wide and 2 *ft.* high? How many cubic inches? Then how many gallons? How many barrels? *Answer to last,* A little more than $4\frac{1}{4}$.

9. How many barrels in a rectangular box 5 *ft.* by 3 *ft.** and 4 *ft.* high? How many bushels?

Suggestion.—In finding how many bushels a vessel contains it is convenient to observe that $\frac{1728}{2150.4} = \frac{9}{11.2}$. Hence in this case $\frac{5 \times 3 \times 4 \times 1728}{2150.4} = \frac{5 \times 3 \times 4 \times 9}{11.2} = \frac{5 \times 3 \times 9}{2.8} = 48.2 +$.

10. A box which would hold a cord of wood would hold how many bushels of wheat? How many barrels of water? *Ans.,* 102.85 + *bu.,* or 30.4 *bbl.* nearly.

11. How many bushels in a barrel of $31\frac{1}{2}$ gallons. *Ans.,* 3.4 nearly.

12. It is customary to heap the measure in measuring apples, potatoes, corn in the ear, ashes, and some other substances, so that about 5 pecks are sold as a bushel. According to this method of measuring, how many bushels does a barrel of $31\frac{1}{2}$ *gal.* contain. *Ans.,* $2\frac{3}{4}$ nearly.

* The size of a rectangle is thus expressed.

13. How many bushels of wheat does a rectangular bin 5 *ft.* by 5 *ft.* and 5 *ft.* deep contain? How many of potatoes?

Answer, 100.44 + of wheat, and $\frac{4}{5}$ of 100.44 + or 80.35 + of potatoes.

14. According to the note under the table, how many drops make a teaspoonful? How many teaspoonfuls make a teacupful? (See note at the bottom of page 210.)

15. A man sold me 32 quarts which he called a bushel of strawberries; but he measured them in a liquid quart measure instead of a dry quart measure. What part of a bushel did I get? How many liquid quarts should he have given me?

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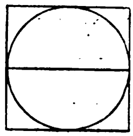
16. I have a cistern the bottom of which is a circle 6 *ft.* across (*i. e.* in diameter), and 6 *ft.* deep, the sides being perpendicular. How many gallons of water does it contain? How many barrels?

Note.—It is found out in Geometry that if a circle is enclosed by a square, as in the figure, the area of the circle is .7854 of the area of the square. Hence



201. *To obtain the area of a circle multiply the diameter (distance across it) by itself, and this product by .7854.*

And to obtain the volume of a cylinder (such a body as the cistern) multiply the area of the base (bottom) by the height.

Solution of Ex. 16. $\frac{6 \times 6 \times 6 \times .7854 \times 1728}{231}$ = 1269.043 +. Hence the cistern contains a little more than 1269 *gal.*, or $40\frac{1}{3}$ *bbl.* nearly.

17. How many barrels does a cylindrical cistern contain

the diameter being 8 *ft.*, and the depth 8 *ft.*? One whose diameter is 12 *ft.* and depth 10 *ft.*?

For the first we have $\frac{8 \times 8 \times 8 \times .7854 \times 1728}{231 \times 31\frac{1}{2}}$.

MEASURES OF WEIGHT.

202. There are *Three Varieties* of measures of weight; viz., *Avoirdupois* * *Weight*, *Troy* † *Weight*, and *Apothecaries Weight*.

203. Avoirdupois Weight is the common weight used by grocers, and for most ordinary purposes. The denominations are *Ounces* (*oz.*), *Pounds* (*lb.*), *Hundred Weight* (*cwt.*), and *Tons* (*T.*).

204. Troy Weight is the weight used for weighing gold, silver, and precious stones, and in philosophical experiments. The denominations are *Grains* (*gr.*), *Pennyweights* (*pwt.*), *Ounces* (*oz.*), and *Pounds* (*lb.*).

205. Apothecaries Weight is used in medical prescriptions. The denominations are *Grains* (*gr.*), *Scruples* (℈), *Drams* (ʒ), *Ounces* (℥), and *Pounds* (*lb.*). Drugs and medicines are bought and sold in the quantity by Avoirdupois Weight.

Physicians in writing prescriptions use the Roman Notation (**26**) to designate the number of grains, scruples, drams, or ounces, and write them *after* the symbol. In writing thus, a final i is written j; thus *gr.* vj is 6 grains, ℈ij is 2 scruples, ʒiij is 3 drams, ℥iv is 4 ounces, etc.

In wholesale transactions in coal and iron, and in the U. S. Custom Houses, 112 *lb.* are called a *cwt.*; hence 28 *lb.* is a *Quarter* (of a *cwt.*), and 2240 *lb.* make a *Ton*. This is sometimes called the *Long Ton*.

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* From three French words *avoir du poids*, meaning "to have weight."

† From *Troyes* in France, whence this weight came into use.

Tables.

206. AVOIRDUPOIS WEIGHT.

$16\ oz. = 1\ lb.$

$100\ lb. = 1\ cwt.$

$20\ cwt. = 1\ T.$

TROY WEIGHT.	APOTHECARIES WEIGHT.
$24\ gr. = 1\ pwt.$	$20\ gr. = 1\ sc.$, or ℈.
	3 ℈ $= 1\ dr.$, or ʒ.
$20\ pwt. = 1\ oz.$	8 ʒ $= 1\ oz.$, or ℥.

$12\ oz. = 1\ lb.$, or ℔.

Table showing the Weight of a Bushel of the principal grains and seeds, as established by Law in the several States.

Barley.	Ill., Ind., Ia., Ky., Mich., Minn., Mo., N. C., N. J., Ohio, Wis. **48** *lb.* Mass., Or., Vt. 46 *lb.*; W. T. 45 *lb.*; La. 32 *lb.*; Pa. 47 *lb.*; Cal. 50 *lb.*
Buckwheat.	Mich., Minn., Or., Wis. 42 *lb.*; Ia., Ill., Ky., Mo. 52 *lb.*; Ind., N. C., N. J. 50 *lb.* Cal. 40 *lb.*; Mass., Vt. 46 *lb.*; N. Y., Pa. 48 *lb.*; Conn. 45 *lb.*
Clover Seed.	Ill., Ind., Ia., Ky., Mich., Minn., Mo., N. Y., Ohio, Or., W. T., Wis. **60** *lb.* N. J. 64 *lb.*
Indian Corn.	Conn., Del., Ind., Ia., Ill., Ky., La., Mass., Mich., Minn., N. J., Ohio, Or., Pa., Vt., W. T., Wis. **56** *lb.* Cal., Mo. 52 *lb.*; N. C. 54 *lb.*; N. Y. 58 *lb.*
Oats.	Cal., Ill., Ind., La., Mich., Minn., N. Y., Ohio, Pa., Vt., Wis., **32** *lb.* Me., Mass., N. C., N. H., N. J. 30 *lb.*; Ia., Mo. 35 *lb.*; W. T. 36 *lb.*; Conn. 28 *lb.*; Ky. 100 *lb.* to 3 *bu.*
Rye.	Conn., Ind., Ia., Ill., Ky., Mass., Mich., Minn., Mo., N. J., N. Y., Ohio, Or., Pa., Vt., W. T., Wis. **56** *lb.* Cal. 54 *lb.*; La. 52 *lb.*
Timothy Seed.	Ill., Ind., Ia., Ky., Mo. **45** *lb.* N. Y. 44 *lb.*; Wis. 46 *lb.*
Wheat.	**60** *lb.* in all except Conn. In Conn. 56 *lb.*

Peas, *Beans*, and *Potatoes* are usually weighed at 60 *lb.* to the bushel.

196 *lb.* *Flour* make a barrel. 200 *lb.* *Pork* or *Beef* make a barrel.

1. How many ounces in $\frac{1}{2}$ *lb.* of butter? In $\frac{1}{4}$ *lb.*? In $\frac{1}{2}$ *lb.* of gold? In $\frac{1}{4}$ *lb.* of silver?

2. On the common grocer's scales what part of a pound is 8 *oz.*? 4 *oz.*? 12 *oz.*?

3. How many *oz.* in 5 *lb.* of sugar? In 5 *lb.* of gold?

4. How much does a barrel of water weigh, if 1 *pt.* weighs 1 *lb.*? How much a gallon?

5. In a certain hay field there were 1250 heaps which would average 85 *lb.* each. How many tons in the field?

6. How many barrels of pork will be cut from 120 hogs which average 175 *lb.* each? *Ans.*, 105 *bbls.*

7. How many barrels of flour will be made from 2160 *bu.* of wheat, if it yield 40 *lb.* to the bushel?

8. How many scruples in 10 drams? How many grains in ℈vij? In ʒv? In ℥iij?

9. In *gr.* xxxij how many scruples?

10. A man ordered a 3 *oz.* gold watch-case; but when it came it weighed only 57 *pwt.* How much did it fall short of the required weight?

11. How much will a gold watch-case weighing $2\frac{1}{2}$ *oz.* cost, at $0.90 per *pwt.* and $20 for making?

12. In Michigan, how many bushels of wheat in a ton? How many in Minnesota? In Iowa? In Connecticut?

13. How many bushels of barley in a ton in Illinois? In Kentucky? In California?

14. What cost 15 *lb.* 12 *oz.* of butter at 32 *c.* per *lb.*? 7 *lb.* 8 *oz.*? 10 *lb.* 7 *oz.*? How much is the butter per *oz.*?

207. The denominations of like name in Troy and Apothecaries weights are the same, but the Avoirdupois pound is heavier, whereas the ounce is lighter, than the pound and ounce of the other weights. The Troy pound is 22.79 *cu. in.* of water, and the Avoirdupois pound is $\frac{175}{144}$ times as much, or 27.69 *cu. in.* of water. A pint of water ($28\frac{7}{8}$ *cu. in.*) is a little more than 1 *lb.* Avoirdupois.

The only difference between the Troy and the Apothecaries tables is in the subdivision of the ounce. In Troy weight there are two subdivisions—pennyweights and grains—whereas in Apothecaries there are 3—drams, scruples, and grains; but in each, 480 grains make an ounce. 7000 Troy grains are equal to a pound Avoirdupois.

15. How many grains are equal to an ounce Avoirdupois? How many to an ounce Troy or Apothecaries?

A Troy ounce is about $1\frac{1}{11}$ as heavy as an Avoirdupois ounce.

MEASURES OF TIME.

208. The denominations of time are *Seconds* (*sec.*), *Minutes* (*min.*), *Hours* (*hr.*), *Days* (*da.*), *Weeks* (*wk.*), *Months* (*mo.*), *Years* (*yr.*), and *Centuries* (*Cen.*).

209. **Table.**

60 *sec.*	=	1 *min.*
60 *min.*	=	1 *hr.*
24 *hr.*	=	1 *da.*
7 *da.*	=	1 *wk.*
365 *da.*	=	1 common *yr.*
366 *da.*	=	1 Leap *yr.*
100 *yr.*	=	1 *Cen.*

In *Computing Interest,* 30 *da.* = 1 *mo.* For many purposes 4 *wk.* are called a month

Of the 12 Calendar Months which make up the year, September, April, June, and November have 30 *da.* each. All the others except February have 31 *da.* each. In common years February has 28 *da.*, in Leap Year 29 *da.* What two exceptions are there to the law that each alternate month has 31 days?

210. Every year whose number is divisible by 4, except the centennial years which are not divisible by 400, is a **Leap Year.**

Thus 1840, 1844, 1880, 1876, 1600, 1200, 2000, are leap years. 1551, 1842, 1883, 1500, 1100, 1900 are not leap years.

The reason for *Leap Year* is this: A year is the time it takes the earth to go around the Sun. But this is a little more than 365 days.

Instead of reckoning this part of a day, it is neglected, and a *whole* day is added to the year every 4th year (in general). But as this is a little too much, the centennial years (in general), although they are the 4th years, are reckoned as common years (365 *da.*). But this again is rejecting too many leap years: so that every centennial year which is divisible by 400 is made a leap year. With this correction the error does not amount to a day in 100,000 years.

1. How many seconds in $\frac{1}{2}$ a minute? In $\frac{1}{4}$ of a minute? In 2 *min.*? In 10 *min.*? In $2\frac{1}{2}$ *min.*?

2. How many minutes in $\frac{1}{2}$ an hour? In $\frac{1}{4}$ an hour? In $\frac{3}{4}$ an hour? In $1\frac{1}{2}$ *hr.*? In 3 *hr.*? In $5\frac{1}{4}$ *hr.*?

3. What part of a day is 12 *hr.*? 6 *hr.*? 1 *hr.*? 5 *hr.*?

4. How many days in 48 *hr.*? In 36 *hr.*? In 80 *hr.*?

5. How many whole weeks in a common year? How many days over? From this how does it appear that any particular date in such years, as July 4th, falls one day later in the week in each succeeding year?

6. Show that from Mar. 1st of Leap Year, to Mar. 1st of the following year, dates fall *two* days later in the week than the corresponding dates of the preceding year.

7. What months of common years begin on the same day of the week? Why? What of Leap Years?

8. If Jan. 11th is Tuesday, what is Feb. 11th? Why?

9. If Apr. 6th is Saturday, what is May 6th? Why?

10. If May, June, or Aug. 1st, of a common year, falls on Sunday, show that no other month of that year begins on Sunday. How is this in Leap Year?

11. To-day, Apr. 17th, is Saturday, on what day of the week does the 4th of July fall this year?

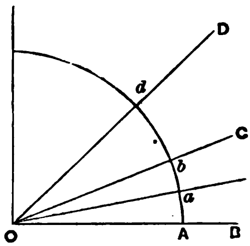
12. Point out the leap years in the following: 1540, 1320, 1000, 800, 560, 2100, 2290, 2400, 1875, 1876, 1877, 1878, 1879, 1880, 1881, 1882, 1883, 1900.

14. How many days in May, August, and October together? In June, December, and September? In February and March in 1876? In 1878?

[See TEACHER'S HAND-BOOK for other exercises and interesting matter.]

CIRCULAR OR ANGULAR MEASURE.

211. A Degree is $\frac{1}{360}$ part of the circumference of a circle, or an angle measured by this part of a circumference, and is designated thus 1°.

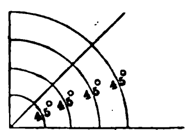


In the figure from **A** to *a* is 10 degrees, written 10°. From **A** to *b* is 20°, and the angle **BOC** is an angle of 20°. From **A** to *d* is 45°, and the angle **BOD** is an angle of 45°. A quarter of a circumference, or a right angle, is 90°.

212. A Minute (as circular or angular measure) is $\frac{1}{60}$ part of a degree, and is designated thus 1′. 5 minutes of this measure is written 5′.

213. A Second (as circular or angular measure) is $\frac{1}{60}$ part of a minute, and is written 1″. 12 seconds is written 12″.

N. B.—It will be observed that a degree *circular* measure is not a definite length; thus a degree on the circumference of the earth is about $69\frac{1}{7}$ miles, while a degree on such a circle as the one in the last figure is so small that we could not see it. But a degree as the measure of an angle is a definite thing. Thus if we draw an angle equal *to ½ a right angle,* and then strike a circum-



Angle of 45°

ference with the vertex of the angle as a centre, there will be 45° of the circumference included between its sides whether the circle be large or small.

The teacher should give the pupil exercises in guessing at the size of angles as expressed in degrees, until a definite conception is secured.

214. **Table of Circular or Angular Measure.**

$60'' = 1'$, $60' = 1°$, $90° = 1$ Quadrant or Right Angle.

215. A *Geographic* or *Nautical* (marine) mile is 1′ of the equator, or about $1\frac{1}{8}$ common miles.

216. Degrees of Latitude being distances north or south of the equator, and measured on equal circles (great circles which pass through the poles), are nearly equal; * but degrees of Longitude vary from $69\frac{1}{6}$ miles at the equator to 0 at the poles. In the latitude of Ann Arbor, Mich., a degree of longitude is 51.1 miles. At 60° latitude a degree of longitude is 34.53 miles.

1. How many minutes in 2°? In $1\frac{1}{8}$°? In In 10°? In .5 of a degree? In .3?

2. How many degrees in 240′? In 580′? In 200′? In 1260′ how many degrees and decimals of a degree?

3. How many seconds in $\frac{1}{2}$ a minute? In .75 of a minute? In .25 of a minute?

4. On a circumference which is 1800 *ft.* in length (around), how long is a degree? A minute? A second?

5. The circumference of a circle is 3.1416 times its diameter (nearly). Show that a degree on the circumference of the earth is about $69\frac{1}{7}$ miles, the radius of the earth being 3962 miles.

* In consequence of the flattening of the earth at the poles, degrees of latitude are not absolutely equal. A degree of latitude at any place is $\frac{1}{360}$ part of the circumference of a circle which has the same curvature as the meridian at that place.

LONGITUDE AND TIME.

217. As the sun appears to pass around the earth from east to west, and as the hours of the day are determined by its position, any given hour comes to a place at the east *before* it does to a place at the west; thus it is noon at New York *before* it is noon at Detroit, Mich. Hence when it is noon at Detroit it is past noon at New York. In like manner, when it is noon at Detroit it is *before* noon at Omaha, Neb.

Principle.

218. *When it is any given hour of the day at any given place, it is Later at places Eastward and Earlier at places Westward.*

219. To ascertain just how great this *Difference in Time* is, we have only to consider that as the sun appears to go around the earth, that is, passes over 360° of longitude, in 24 hours, in 1 *hr.* it passes over $360 \div 24$, or 15° of longitude. Then 1° of longitude makes a difference of $\frac{1}{15}$ of 1 *hr.* (60 *min.*), or 4 *min.* in time. And 1′ of longitude makes a difference of $\frac{1}{60}$ of 4 *min.* (240 *sec.*), or 4 *sec.* in time.

220. **Table of Longitude and Time.**

15° Longitude make 1 *hr.* diff. in Time.
1° Longitude makes 4 *min.* diff. in Time.
1′ Longitude makes 4 *sec.* diff. in Time.

1. Adrian, Mich., is in 84° west longitude, and Fort Kearney, Neb., is in 99° west longitude. When it is 9 o'clock A. M. at Adrian, what time is it at Fort Kearney? When it is noon at Fort Kearney, what time is it at Adrian?

How many degrees *west* of Adrian is Fort Kearney?

2. New York City is in longitude 74° west, and San Francisco in $122\frac{1}{2}$° west. What is the difference in time? When it is 6 A. M. at New York, what time is it at San Francisco? When it is 4 P. M. at San Francisco, what time is it at New York? *Answer to last,* 7:14 P. M.

3. In the latitude of Ann Arbor, Mich., 51.1 miles make a degree of longitude. Detroit is 38 *mi.* east of Ann Arbor. What is the difference in time?

Detroit is $\frac{380}{511}$ of a degree east of Ann Arbor; hence the difference in time is 4 times $\frac{380}{511}$, or about 3 *min.* More accurately, 2 *min.* 58.4 *sec.*

4. What is the difference in longitude between two places whose difference in time is $\frac{1}{2}$ an hour?

Every 4 *min.* in time makes how much difference in longitude?
Every 4 *hr.* in time makes how much difference in longitude?

5. What is the difference in longitude between two places whose difference in time is $3\frac{1}{2}$ *hr.?* $2\frac{1}{2}$ *min.?* 2 *hr.* 15 *min.* 20 *sec.?*

2 *hr.* make 30° diff. in long.; 15 *min.* make 3° 45′ diff. in long.; and 20 *sec.* make 5′ diff. in long. *Ans.* 33° 50′.

221. *Knowing the difference in longitude between two places, how do you find the difference in time?*

Require the pupils to *write* out a rule. Several different ones can be made. Encourage pupils to make them.

222. *Knowing the difference in time between two places, how do you find the difference in longitude?* Write rules.

223. *Knowing the distance in miles between two places in the same latitude, and the length of a degree of longitude at that latitude, how do you find the difference in time?* Write rules. See Ex. 3.

224. *What is the converse of the last question?* Write it and the rule for solving it. (See bottom of page 179.)

6. New York being in Longitude 74° west and New Orleans in 90° west, when it is 2 P. M. at New York, what time is it at New Orleans? When it is 11 A. M. at New Orleans, what time is it at New York?

Answers, 56 *min.* past noon, and 4 *min.* past noon.

7. What is the difference in time between Chicago, 87° 37′ 37″ W. Long., and Cincinnati, 84° 27′ W. Long.? When it is 5 P. M. at Chicago, what is the time at Cincinnati? When it is 8 A. M. at Cincinnati, what time is it at Chicago?

8. The difference in time between two places being 1 *hr.* 22 *min.* and 32 *sec.*, what is the difference in longitude?

1 hour in time makes 15° in longitude. 20 *min.* make 5°. 152 *sec.* make 38′. Hence 1 *hr.* 22 *min.* 32 *sec.* in time make 20° 38′ in longitude.

9. A telegraphic signal given from Washington, D. C., at 3:20 P. M. was received at St. Louis, Mo., at 2:27 P. M. The longitude of Washington being 77° 0′ 15″, what is that of St. Louis?

10. The difference in time between Buffalo and Cleveland being 11 *min.*, what is the difference in longitude?

11. Boston, Mass., and Ann Arbor, Mich., are in about the same latitude; and the difference in time is 51 *min.* At this latitude 51.1 *mi.* make a degree of longitude. What is the distance in a direct line from Ann Arbor to Boston? *Ans.*, 651.525, or about 651½ *mi.*

12. Berlin, Prussia, is in longitude 13° 23′ 53″ E., and Boston is in longitude 71° 4′ 9″ W. When it is 4 A. M. at Berlin, what time is it at Boston?

13. Annapolis, Md., and Cincinnati, O., are both in latitude 39° N. (nearly). On this parallel a degree of longitude is 53.47 *mi.*; and the distance between the places is 422.413 *mi.* What is the difference in time?

14. The difference in time between Logansport, Ind., and Omaha, Neb., is 39 *min.*; and the distance between them is 509 *mi.* How many miles make a degree of longitude at this latitude?

15. In the latitude of Milwaukee, Wis. (43° 2′), 50½ *mi.* make a degree of longitude. How far east or west of Milwaukee do you have to go to make a difference in time of 15 *min.*? of 20 *min.*? Of ½ an hour? Prairie du Chien has the same latitude. A man's watch being set to Milwaukee time he found it 13½ *min.* fast at Prairie du Chien. How far is it in a direct line from one place to the other?

16. The difference in time between Detroit and Chicago is 19 *min.* If my watch is set to Chicago time, how will it compare with Detroit time? The longitude of Detroit is 82° 58′ west. What is the longitude of Chicago?

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225. Of Paper 24 **Sheets** are called a **Quire,**
20 **Quires** " " a **Ream,**
and 2 **Reams** " " a **Bundle.**

226. In Counting, 12 **Things** are called a **Dozen,**
12 **Dozens** " " a **Gross,**
12 **Gross** " a **Great Gross,**
and 20 **Things** are called a **Score.**

1. How much does a quire of paper cost when we pay $2.50 per ream?

2. At 20 c. per quire, what cost ½ a ream of paper?
3. At 50 c. a ream, what cost 2 bundles of paper?
4. At $1.50 for a ½ ream, what cost ¼ of a quire of paper?
5. At 5 c. per dozen, what cost 84 screws?
6. How many eggs in 30 dozen?
7. How many score in 100?

SECTION II.

REDUCTION.

227. Reduction of Denominate Numbers is the process of changing the denomination in which a number is expressed without altering the value represented by it. If the change is from higher denominations to lower, the reduction is said to be *Descending;* if from lower to higher, *Ascending.*

1. In 4 yd. 2 ft. 9 in. how many inches?

Operation.	**Explanation.**
4 *yd.* 2 *ft.* 9 *in.*	Since 3 *ft.* make a yard, in any number of yards there are 3 times as many feet as yards. Hence in 4 *yd.* there are 3 times 4, or 12 *ft.*, which with the 2 *ft.* make 14 *ft.*
3	
14 *ft.*	
12	
177 *in.*	

Since 12 *in.* make a foot, in any number of feet there are 12 times as many inches as feet. Hence in 14 *ft.* there are 12 times 14, or 168 *in.*, which with the 9 *in.* make 177 *in.* Hence 4 *yd.* 2 *ft.* 9 *in.* are 177 *in.*

Is this reduction descending, or ascending? Why?

2. In 5127 *pt.* of water, how many barrels, gallons, quarts, and pints: or reduce 5127 *pt.* liquid measure to the higher denominations of that measure?

Operation.	Explanation.
$2\overline{)5127}$ *pt.*	Since every 2 *pt.* make a quart, in 5127 *pt.*
$4\overline{)2563}$ *qt.* 1 *pt.*	there are as many quarts as 2 is contained
$\overline{640}$ *gal.* 3 *qt.*	times in 5127, *i. e.*, 2563 *qt.* and 1 *pt.*
$31.5\overline{)640.0}($ 20 *bbl.*	[Let the pupil fill out the explanation.]
630	
$\overline{10.0}$ *gal.*	∴ 5127 *pt.* = 20 *bbl.* 10 *gal.* 3 *qt.* 1 *pt.*

To Reduce Denominate Numbers.

228. Rule.—Reduction Descending *is performed by beginning with the highest denomination given, multiplying this by the number which it takes of the next lower to make one of this higher, and adding to this product the given number of this next lower denomination. Reduce this sum to the next lower denomination in a like manner, and add to the result the given number of this lower. Proceed in this manner till the denomination required is reached.*

II. **Reduction Ascending** *is performed by dividing the given number by the number of this denomination which it takes to make one of the next higher denomination, and treating the quotient thus arising in like manner, proceeding thus till the desired denomination is reached.**

Examples for Practice.

Each of the following will afford an exercise for both forms of reduction :

1. 3 *da.* 5 *hr.* 17 *min.* = 278220 *sec.*†

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* It is scarcely expedient to give an abstract demonstration of a rule so mechanical as this. In fact, the author has doubts about the expediency of giving the *rule* at all. Be careful, however, that the *rationale* of the process is understood, and can be given with ease and elegance.

† These examples should be assigned in both ways, each making two exam-

2. 442 *pt.* = 6 *bu.* 3 *pk.* 5 *qt.*
3. 6 *gal.* 3 *qt.* 1 *pt.* = 55 *pt.*
4. £32 12*s.* 10*d.* = 7834*d.*
5. 8 *lb.* 6 *oz.* 12 *pwt.* 16 *gr.* = 49264 *gr.*
6. ℥ vj. ʒ iij. ℈j. = 154 *sc.*
7. 78692 *gr.* = 13 *lb.* 7 *oz.* 18 *pwt.* 20 *gr.*
8. 5382 *oz.* (avoirdupois) = 3 *cwt.* 36 *lb.* 6 *oz.*
9. 78562 *cu. ft.* = 613 *cd.* 98 *cu. ft.*
10. 4268 *sq rd.* = 26.675 *A.*
11. $\frac{3}{4}$ of a *bbl.* = 189 *pt.*

Suggestion.—This should be solved thus:

$$\frac{3}{\cancel{4}} \times \frac{63}{\cancel{2}} \times \cancel{4} \times \cancel{2} = 189.$$

Why is $\frac{3}{4}$ multiplied by $\frac{63}{2}$? Is this quite in accordance with the rule (**227**)?

The converse process is thus performed

$$\frac{189}{2 \times 4} \div 31\tfrac{1}{2}, \text{ is } \frac{189}{2 \times 4} \times \frac{2}{63} = \frac{3}{4}.$$

Does this process differ from that given in the rule (**227**)?*

12. $4\frac{3}{4}$ cords = 608 *cu. ft.*
13. $\frac{2}{3}$ *gal.* = $5\frac{1}{3}$ *pt.* $\frac{1}{2}$ *pt.* = $\frac{1}{16}$ *gal.*
14. 2 *A.* 110 *sq. rd.* = 430 *sq. rd.*
15. 6875988 *cu. in.* = 31 *cd.* 11 *cu. ft.* 276 *cu. in.*
16. 12 *bbl.* 19 *gal.* 2 *qt.* = 3180 *pt.*
17. 368 *pt.* = 5 *bu.* 3 *pk.*
18. 30630 *min.* = 21 *da.* 6 *hr.* 30 *min.*
19. 172800″ = 48°.

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ples, one in reduction descending and one in reduction ascending. Thus this one will be given: "Express 3 *da.* 5 *hr.* 17 *min.* in seconds;" and again: "Express 278220 *sec.* in days, hours, and minutes." In assigning them for work on the black-board the two examples may be assigned to two students standing together, and thus one will "prove" the other's work.

* It should be the aim of the teacher to show that we need no *special* rules for reduction of denominate fractions.

20. 17 *sq. ft.* 27 *sq. in.* = 2475 *sq. in.*
21. 75288 *gr.* = 13 *lb.* 17 *pwt.*
22. 3.25 *gal.* = 26 *pt.*

Does the process by which we perform this constitute an exception to the general rule (227)?

23. 12 *ft.* 9 *in.* = 12.75 ft.

How are inches reduced to feet? 9 ÷ 12 = what decimal? Does this process constitute an exception to the rule (227)?

24. 3 *in.* = $.083\frac{1}{3}$ *yd.*

How are inches reduced to feet? How are feet reduced to yards? 3 ÷ 12 = .25. .25 ÷ 3 = $.083\frac{1}{3}$.

25. £4.67 = £4 13*s.* 4*d.* 3.2 *far.*

How is the £ .67 reduced to shillings? How is .4*s.* reduced to pence?

26. $\frac{7}{30}$ *wk.* = 1 *da.* 15 *hr.* 12 *min.*

$\frac{7}{30} \times 7 = 1\frac{19}{30}$. $\frac{19}{\cancel{30}_5} \times \cancel{24}^4 = 15\frac{1}{5}$. $\frac{1}{5} \times 60 = 12$.

$\therefore \frac{7}{30}$ *wk.* = 1 *da.* 15 *hr.* 12 *min.*

$12 \div 60 = \frac{1}{5}$. $15\frac{1}{5} \div 24 = \frac{\cancel{76}^{19}}{5} \div \cancel{24}^6 = \frac{19}{30}$. $1\frac{19}{30} \div 7 = \frac{49}{30} \div 7 = \frac{7}{30}$.

N.B.—Let the pupil trace this work till he sees clearly that the processes form no exceptions to the general rules for reduction.

27. .475° = 28′ 30″.
28. ℥ iv ℈ ij = $\frac{7}{12}$ *oz.*
29. .345 of a *bbl.* of flour = 67 *lb.* $9\frac{2}{2}\frac{2}{5}$ *oz.*
30. 16 *lb.* 8 *oz.* = .275 *bu.* of wheat, or $\frac{11}{40}$ *bu.*
31. 43 *rd.* 11.7 *in.* = .13456 *mi.* +.
32. $\frac{5}{6}$ *oz.* = 16 *pwt.* 16 *gr.*
33. $\frac{4}{5}$ *bu.* = 3 *pk.* 1 *qt.* $1\frac{1}{5}$ *pt.*
34. 213 *rd.* 1 *yd.* $2\frac{1}{2}$ *ft.* = $\frac{2}{3}$ *mi.*

35. A pile of wood 4 *ft.* long, 3 *ft.* high, 4 *ft.* wide = $\frac{3}{8}$ *cd.*

36. ℥ ix ʒ j ℈ij gr. viij = .76875 *lb.*

37. 3 *lb.* 10 *oz.* Troy = 3 *lb.* 2$\frac{1}{2}$ *oz.* Avoirdupois, nearly.

38. 528 *chains* = 6 *mi.* 192 *rd.*

39. 2$\frac{3}{4}$ *mi.* = 220 *ch.*

40. 3 *ch.* 4 *lk.* = 200.64 *ft.*

41. 2$\frac{1}{2}$ *A.* = 25 *sq. ch.*

42. A lot 12 *ch.* 24 *lk.* by 15 *ch.* 40 *lk.* = 18.8496 *A.*

$$12.24 \times 15.4 \div 10 = 18.8496.$$

43. 3 *A.* = 300000 *sq. lk.*

44. A lot 81 *ch.* 52 *lk.* by 34 *ch.* 2 *lk.* = 277.3 *A.* +.

45. 756 *cu. ft.* = 1306368 *cu. in.*, or 5.9$\frac{1}{16}$ *cd.*

46. .24 *bu.* wheat = 14 *lb.* 6.4 *oz.*

47. 75 *sq. rd.* 12 *sq. yd.* 4 *sq. ft.* = 20530$\frac{3}{4}$ *sq. ft.*

48. 165888 *cu. in.* = $\frac{3}{4}$ *cd.*

49. 8410 *lk.* = 1 *mi.* 4.1 *ch.*, or 1.05125 *mi.*

50. 24$\frac{1}{2}$ *lb.* = $\frac{1}{8}$ *bbl.* flour.

51. 5 *f.* ʒ 36 ♏ = .7 *f.* ℥.

52. .008 *mi.* = 2 *rd.* 9.24 *ft.*

53. \$68 = 680 *d.* = 6800 *ct.* = 68000 *m.*

54. 3256 *m.* = 325.6 *ct.* = 32 *d.* 5 *c.* 6 *m.* = \$3.256.

55. 1250 *fr.* = \$241.25. \$10 = 51 fr. 81 c. +

56. 5 *Nap.* = \$19.30. \$100 = 25 Nap. 18 fr. 13 c. +.

57. 5 fathoms = —— *ft.?* 256 *ft.* = —— fathoms?

58. 2 *mi.* 34 *rd.* 3 *yd.* = —— *yd.?*

59. 2000 *yd.* = —— *mi.?*

60. 1 *mi.* = —— *yd.?* 40 *rd.* = —— *yd.?*

61. 100 *yd.* = —— *rd.?* $\frac{2}{3}$ *yd.* = what part of a *rd.?*

62. A lot 5 *rd.* by 8 = what part of an acre?

63. 3$\frac{3}{4}$ *A.* = —— *sq. yd.?* 5 *sq. rd.* = —— *A.?*

64. A lot 7.21 *ch.* by 3.40 *ch.* = 2.4514 *A.*
65. 5 *A.* 110 *sq. rd.* 1 *sq. yd.* = 5.6877 *A.*, very nearly.
66. $\frac{2}{3}$ *cd.* = —— *cu. ft.?* 12 *cu. ft.* = what part of a *cd.?*
67. 5 *cd.* 120 *cu. ft.* = —— *cu. yd.?*
68. 1376 *cu. yd.* = 290$\frac{1}{4}$ *cd.* 1 *cu. yd.* = —— *cd.?*
69. 3 *bbl.* 20 *gal.* = —— *qt.?*
70. 4000 *qt.* = 31 *bbl.* 23$\frac{1}{2}$ *gal.* = 31$\frac{47}{63}$ *bbl.*
71. 10 *gal.* 3 *qt.* 1 *pt.* = 43.5 *qt.*
72. 126$\frac{3}{4}$ *qt.* = 31 *gal.* 2 *qt.* 1$\frac{1}{2}$ *pt.* = 1 *bbl.* 1$\frac{1}{2}$ *pt.*
73. 43 *qt.* = 5 *pk.* 3 *qt.*, or 5$\frac{3}{8}$ *pk.*
74. $\frac{1}{12}$ *bu.* = 5$\frac{1}{3}$ *pt.*
75. 3.416 *bu.* = 3 *bu.* 1 *pk.* 5$\frac{1}{3}$ *qt.* nearly.
76. 2 *bu.* 3 *pk.* 5 *qt.* 1$\frac{1}{2}$ *pt.* = 187.5 *pt.*
77. 1 *bu.* 1 *pk.* 1 *qt.* 1 *pt.* = How many pecks?
78. 2 *bbl.* 15 *gal.* 110$\frac{1}{4}$ *pt.* = 91.78125 *gal.*
79. 5287 *qt.* = —— *bbl.* —— *gal.* —— *qt.* —— *pt.?*
80. 786 *gal.* = —— *pt.?*
81. 4 *lb.* 8 *oz.* 12 *pwt.* 16 *gr.* = 27184 *gr.*
82. 3 *lb.* 10 *oz.* 20 *gr.* = 46$\frac{1}{24}$ *oz.*
83. 342$\frac{2}{5}$ *oz.* = 28 *lb.* 6 *oz.* 8 *pwt.*
84. 1 *T.* 13 *cwt.* 58 *lb.* = 3358 *lb.*
85. 7129 *lb.* = 3 *T.* 11 *cwt.* 29 *lb.*
86. 7129 *lb.* = 3 *T.* 3 *cwt.* 73 *lb.* U. S. Customs weight.
87. 52 *rd.* 4 *yd.* 2 *ft.* = $\frac{109}{660}$ *mi.*
88. 1$\frac{1}{12}$ *bbl.* = 34 *gal.* 1 *pt.*
89. 2 *hr.* 52 *min.* 48 *sec.* = $\frac{3}{25}$ *da.*
90. $\frac{5}{896}$ *bu.* = $\frac{5}{14}$ *pt.*
91. 11 *da.* = $\frac{11}{30}$ *mo.* = $\frac{11}{360}$ *yr.* in computing interest.
92. 2 *mo.* 13 *da.* = .2028 − *yr.* in computing interest.
93. 2 *yr.* 5 *mo.* 7 *da.* = 2.436 *yr.*+, as above.
94. 11 *yr.* 10 *mo.* 21 *da.* = 11.89 *yr.*+, as above.
95. 4 *yr.* 10 *mo.* 24 *da.* = 58.8 *mo.*, as above.

96. 3.725 *yr.* = 3 *yr.* 8 *mo.* 21 *da.*, as above.
97. 7 *mo.* 6 *da.* = .6 *yr.*, as above.
98. 1 *yr.* 8 *mo.* 12 *da.* = 1.7 *yr.*, as above.
99. 1 *yr.* 4 *mo.* 24 *da.* = 1.4 *yr.*, as above.

SECTION III.

ADDITION.

1. There are three casks which contain 2 *gal.* 3 *qt.* 1 *pt.*, 5 *gal.* 2 *qt.* 1½ *pt.*, and 4 *gal.* 1 *qt.* 1½ *pt.* How much do they all contain?

How many pints are 1½, 1½, and 1? How many *quarts* does this make? 2 *qt.*, 1 *qt.*, 2 *qt.* and 3 *qt.* make how many *quarts?* How many gallons does this make, and how many quarts over? 2 *gal.*, 4 *gal.*, 5 *gal.*, and 2 *gal.*, make how many gallons?

2. There are 5 pieces of rope whose respective lengths are 2 *yd.* 2 *ft.* 3 *in.*, 4 *yd.* 1 *ft.* 7 *in.*, 3 *yd.* 2 *ft.* 5 *in.*, 5 *yd.* 2 *ft.* 10 *in.*, and 3 *yd.* 2 *ft.* What is the entire length?

Operation.

2 *yd.*	2 *ft.*	3 *in.*
4 *yd.*	1 *ft.*	7 *in.*
3 *yd.*	2 *ft.*	5 *in.*
5 *yd.*	2 *ft.*	10 *in.*
3 *yd.*	2 *ft.*	
20 *yd.*	2 *ft.*	1 *in.*

Explanation.—We write numbers of the same denomination in the same column, because such are more conveniently added together. We then begin the addition with the *lowest* denomination, because we can thus tell whether there will arise any of the higher denominations from adding the lower, and if there does can add it in with the higher denominations as we go along. [Were we to *commence* with the highest denomination we should have to revise our results after having added all the columns. Let the pupil try it.]

In this example the sum of the inches column is 25 *in.* = 2 *ft.*

1 *in.* The sum of the feet column together with the 2 *ft.* from the inches column, is 11 *ft.* = 3 *yd.* 2 *ft.* The sum of the yards column together with the 3 *yd.* from the feet column, is 20 *yd.*

To add Compound Numbers.

229. Rule.—*Write the numbers so that like denominations shall stand in the same column. Beginning with the column of the lowest denomination, add it and find how many of the next higher the sum makes, reserving this number in mind, and writing the remainder of this sum under its own column. Add the next higher column together with what arose of this denomination from the preceding column, and treat the sum as before. Proceed in this manner till all the columns are added.*

Pupil give the reasons (Demonstration) in order. He is to tell, 1. Why we write the numbers as the rule directs. 2. Why we begin with the lowest denomination. 3. Why we reduce the several sums to the higher denominations. (See "Explanation" above.)

3. What is the amount of £105 1*s.* 2*d.* 3*fr.*, £218 11*s.* 5*d.* 2*fr.*, £199 17*s.* 9*d.* 2*fr.*, and £77 18*s.* 3*d.* 3*fr.*?

Ans., £601 8*s.* 9*d.* 2*fr.*

4. What is the sum of 8 *lb.* ℥ xj ʒ vj ℈ ij, 9 *lb.* ℥ x ʒ vij ℈ j, 4 *lb.* ℥ vij ʒ iij ℈ j, 17 *lb.* ℥ viij ʒ iij ℈ j, and 45 *lb.* ℥ xj ʒ iij ℈ j? *Ans.*, 87 *lb.* ℥ ij.

5. What is the sum of 10 *rd.* 3 *yd.* 1 *ft.* 7 *in.*, 7 *rd.* 2 *yd.* 2 *ft.* 5 *in.*, 3 *rd.* 4 *yd.* 1 *ft.* 9 *in.*, 5 *rd.* 2 *yd.* 1 *ft.* 10 *in.*, and 13 *rd.* 4 *yd.* 11 *in.*? *Ans.*, 41 *rd.* 1 *yd.* 1 *ft.*

6. What is the sum of 145 *bu.* 3 *pk.* 1 *qt.*, 163 *bu.* 1 *pk.* 3 *qt.*, 275 *bu.* 2 *pk.* 7 *qt.*, 45 *bu.* 3 *pk.* 6 *qt.*, and 73 *bu.* 1 *pk.* 5 *qt.*? *Ans.*, 704 *bu.* 6 *qt.*

7. What is the amount of £13 17*s.* 11*d.* 1*fr.*, £22 14*s.* 9*d.* 1*fr.*, £37 18*s.* 6*d.* 3*fr.*, and £46 13*s.* 7*d.* 2*fr.*?

Ans., £121 4*s.* 10*d.* 3*fr.*

8. A silversmith bought of A 3 *lb.* 9 *oz.* 14 *pwt.* 16 *gr.* of silver, of B 9 *lb.* 11 *oz.* 17 *pwt.* 18 *gr.*, of C 1 *lb.* 8 *oz.* 19 *pwt.* 21 *gr.*, and of D 3 *lb.* 7 *oz.* 12 *pwt.* 16 *gr.* How much silver did he buy?

Ans., 19 *lb.* 2 *oz.* 4 *pwt.* 23 *gr.*

9. What is the amount of 45 *cd.* 23 *cu. ft.* 25 *cu. in.*, 273 *cd.* 75 *cu. ft.* 684 *cu. in.*, 97 *cd.* 18 *cu. ft.* 384 *cu. in.*, 250 *cd.* 64 *cu. ft.* 197 *cu. in.*, and 264 *cd.* 84 *cu. ft.* 848 *cu. in.*? *Ans.*, 931 *cd.* 9 *cu. ft.* 410 *cu. in.*

10. What is the sum of 86 *sq. yd.* 7 *sq. ft.* 46 *sq. in.*, 245 *sq. yd.* 8 *sq. ft.* 89 *sq. in.*, and 265 *sq. yd.* 7 *sq. ft.* 128 *sq. in.*? *Ans.*, 598 *sq. yd.* 5 *sq. ft.* 119 *sq. in.*

11. What is the amount of 5 *bbl.* 20 *gal.* 3 *qt.* 1½ *pt.*, 7 *bbl.* 25 *gal.* 2 *qt.*, 28 *gal.* 1 *qt.*, 3 *bbl.* 30 *gal.* 1¾ *pt.*, 2 *pt.* 1½ *pt.*, 13 *gal.*, 2 *qt.* 1¾ *qt.*?

Ans., 18 *bbl.* 24 *gal.* 3 *qt.* ¼ *pt.*

12. What is the sum of ¾ of a bushel, 2.64 pecks, .5 *bu.*, 12⅞ *qt.*, 10½ *pk.*, and 320 *pt.*, in bushels?

Ans., 9.92875 *bu.*

13. What is the sum of 8 *yd.* 2 *ft.*, 75½ *ft.*, 2005 *in.*, 4.35 *yd.*, 28⅝ *ft.*, 3⅜ *yd.*, .25 *ft.*, and 226 *ft.* in *rods*?

Ans., 33.168 + *rd.*

14. Mr. E. Jones owned 1¼ sections of land in one township, 80 acres in another, a quarter section in another, a 40 acre lot in another, and a piece of land 40 rods by 40 chains in another. How much land had he in all?

Ans., 2 sections.

SECTION IV.

SUBTRACTION.

1. From 3 *lb.* 8 *oz.* 16 *pwt.*, subtract 1 *lb.* 3 *oz.* 12 *pwt.* 17 *gr.*

Operation.

3 *lb.* 8 *oz.* 16 *pwt.*
1 *lb.* 3 *oz.* 12 *pwt.* 17 *gr.*
2 *lb.* 5 *oz.* 3 *pwt.* 7 *gr.*

Explanation.—We write the denominations of the subtrahend under like denominations in the minuend, because it is more convenient to subtract a number of any denomination from another of like denomination. We begin to subtract at the lowest denomination, so that if there should chance not to be as many of any particular denomination in the minuend as in the subtrahend, we can take one from the next higher denomination in the minuend, and put it with the number in this deficient denomination. Thus in this example there are *no* grains represented in the minuend; but we can take one of the 16 *pwt.* which makes 24 *gr.*, and subtracting 17 *gr.* from it, have 7 *gr.* left. Then 12 *pwt.* from 15 *pwt.* leaves 3 *pwt.*, etc.

2. From 14 *bu.* 3 *pk.* 4 *qt.* 1 *pt.* subtract 8 *bu.* 3 *pk.* 7 *qt.* *Ans.*, 5 *bu.* 3 *pk.* 5 *qt.* 1 *pt.*

To Subtract Compound Numbers.

230. Rule.—*Write the subtrahend under the minuend so that its denominations shall fall under the corresponding denominations of the minuend. Begin with the lowest denomination, and take the number represented in each denomination of the subtrahend from the number in the corresponding denomination in the minuend, and write the remainder underneath. If the number in any denomination in the minuend is less than the corresponding number in the subtrahend, take 1 of the next higher*

denomination of the minuend in which there are any, and reducing it to this lower denomination and uniting it with what there may be in this denomination, perform the subtraction. Observe when passing to the higher denominations how much remains in them.

3. From £10 12*s.* 7*d.* take £6 8*s.* 5*d.*

4. From £17 0*s.* 3*d.* take £9 10*s.* 5*d.*

Operation.

£17	0*s.*	3*d.*
£ 9	10*s.*	5*d.*
£ 7	9*s.*	10*d.*

Explanation.—As 5*d.* can not be taken from 3*d.*, we take £1 = 20*s.*, and taking 1 of the 20*s.*, which makes 12*d.*, subtract 5*d.* from 12 + 3*d.*, or 15*d.* Then we have 10*s.*, which we subtract from the 19*s.* remaining of the £1. Finally we take £9 from £16.

5. From 10 *bu.* 3 *pk.* 4 *qt.* take 4 *bu.* 1 *pk.* 2 *qt.*
6. From 1 *bu.* 1 *pk.* 1 *qt.* take 2 *pk.* 1 *qt.* 1 *pt.*
7. From $\frac{1}{2}$ *bu.* take 3 *qt.* and 2 *pt.* *Rem.*, 1 *pk.* 4 *qt.*
8. From 5 *bbl.* 24 *gal.* 2 *qt.* take 1 *bbl.* 27 *gal.* 3 *qt.*
Rem., 3 *bbl.* 28 *gal.* 1 *qt.*
9. From 4 *lb.* 8 *oz.* 12 *pwt.* take 2 *lb.* 5 *oz.* 17 *gr.*
Rem., 2 *lb.* 3 *oz.* 11 *pwt.* 7 *gr.*
10. From 1 *lb.* take 15 *pwt.* 20 *gr.*
Rem., 11 *oz.* 4 *pwt.* 4 *gr.*
11. From 5 *mi.* 100 *rd.* 12 *ft.* take 2 *mi.* 30 *rd.* 15 *ft.*
Rem., 3 *mi.* 69 *rd.* 13 *ft.* 6 *in.*
12. From 27 *A.* take 110$\frac{1}{4}$ *sq. rd.*
Rem., 26.3109375 *A.*
13. From 2$\frac{1}{2}$ cords take 120.32 *cu. ft.*
Rem. 1.56 cords.
14. From 5 *yd.* take 5 *ft.* *Rem.*, 3$\frac{1}{3}$ *yd.*
15. From 2 *T.* 12 *cwt.* take 3420 *lb.*
Rem., 17 *cwt.* 80 *lb.*
16. From $\frac{1}{4}$ a section take 51 *A.* 45 *sq. rd.*

17. From ℥ vij ʒ iij ℈ j take ℥ iij ʒ v ℈ ij.
Rem., ℥ iij ʒ v ℈ ij.

18. Sold from a barrel of molasses 10 *gal.* 3 *qt.* 1 *pt.* How much remained?

19. From the sum of 3 *lb.* 13 *oz.*, 2 *lb.* 5 *oz.*, and 6 *lb.* 11 *oz.*, take 12 *lb.* 10 *oz.*

20. From ¼ *yd.* take 7 *in.*

To Find the Time between Two Dates.

231. There are several methods in use for estimating the time between two dates; of these we shall consider two.

1. To find the exact time;

2. To find the time as commonly reckoned in computing interest, that is, reckoning 30 days as a month, and 12 months as a year.

1. What is the *exact* time in days between Jan. 12th, 1856, and May 15th, 1874?

Solution.—Between Jan. 12th, 1856, and Jan. 12th, 1874, are 1874 − 1856 = 18 *yr.*, in which there are 5 Februaries containing 29 days each.* Hence in these 18 *yr.* there are 365 × 18 + 5 = 6575 days. Then from January 12th to May 15th, 1874, there are 19 + 28 + 31 + 30 + 15 = 123 days. Therefore there are in all 6575 + 123 = 6698 days between these dates.

2. What is the exact time in days between Apr. 5th, 1860, and Nov. 24th, 1875? *Ans.*, 5711 days.

3. Reckoning 30 *da.* a month, and 12 *mo.* a year, what is the time between Aug. 17th, 1871, and Feb. 3d, 1875?

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* That is, those in the Leap Years 1856, 1860, 1864, 1868, and 1872.

Operation.	
1875 *yr.* 2 *mo.* 3 *da.*	
1871 *yr.* 8 *mo.* 17 *da.*	
3 *yr.* 5 *mo.* 16 *da.*	

Explanation.—The later date (larger number) is the 1875th year, 2d month, 3d day. The earlier is the 1871st year, 8th month, and 17th day.*

Find the time between the following dates, reckoning 12 *mo.* a year, and 30 *da.* a month:

4. March 16th, 1850, and Dec. 5th, 1871.
5. Aug. 23d, 1846, and Apr. 2d, 1827.
6. Dec. 22d, 1620, and Jan. 19th, 1875.
7. May 5th, 1872, and July 15th, 1873.
8. Mrs. J. was born Dec. 5th, 1825. What is her exact age in calendar years, months, and days on the 20th day of January, 1875? *Ans.*, 49 *yr.* 1 *mo.* 15 *da.*

231. *a.* **In reckoning calendar months and exact days, when subtracting the days, "borrow," and when adding "carry,"** ***according to the number of days which make the month next preceding that in which the period terminates;*** **and when obtaining** ***a date*** **by subtraction, should a 0 fall in months order, transfer 1** ***yr.*** **= 12** ***mo.*** **to that order.**

9. What is Mr. O.'s age March 12th, 1875, he having been born July 24th, 1827? *Ans.*, 47 *yr.* 7 *mo.* 16 *da.*

The month "borrowed" is February, 28 ***da.***

10. What is Mary's age Aug. 15th, 1875, she having been born July 20th, 1868? *Ans.*, 7 *yr.* 26 *da.*

The month "borrowed" is July, 31 ***da.***

11. A note dated July 25th, 1857, matures in 5 *yr.* 3 *mo.* 24 *da.* When is it due? *Ans.*, Nov. 18th, 1862.

The month "carried" (filled out) is the 10th, 31 ***da.***

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* A full explanation of this process requires that we understand that the earlier date is really 1874 *yr.* 1 *mo.* 3 *da.* after the Christian era, and the later 1870 *yr.* 7 *mo.* 17 *da.* after the same time.

1874 *yr.* 1 *mo.* 3 *da.*	
1870 *yr.* 7 *mo.* 17 *da.*	
3 *yr.* 5 *mo.* 16 *da.*	

12. June 8th, 1877, I have a note which has run 2 *yr.* 5 *mo.* 10 *da.* What is its date?

Subtracting, we have 1875 0 29, which is Dec. 29, 1874.

13. Feb. 12, 1876, Henry is 10 *yr.* 1 *mo.* 15 *da.* old. When was he born? *Ans.*, Dec. 28th, 1865.

14. What is the exact time in days from Sept. 7th, 1873, to Dec. 10th, 1875? *Ans.*, 824 *da.*

Subtracting, we have 2 *yr.* 3 *mo.* 3 *da.* The 2 *yr.* = 730 *da.*, and the 3 *mo.* = 91 *da.*

SECTION V.

MULTIPLICATION.

1. Multiply 5 *yd.* 2 *ft.* 8 *in.* by 7.

Operation.

5 *yd.*	2 *ft.*	8 *in.*
		7
41 *yd.*	0 *ft.*	8 *in.*

Explanation.—The multiplier is written under the lowest denomination of the multiplicand as matter of custom.* We commence the multiplication with the lowest denomination, since by so doing we can find how many of the next higher denomination any particular product makes, and thus add it in with the next product as we pass along. Thus 7 times 8 *in.* are 56 *in.* = 4 *ft.* 8 *in.* Writing the 8 *in.* under inches, we reserve the 4 *ft.* to be added to the next product. 7 times 2 *ft.* are 14 *ft.*, which with the 4 *ft.* from the preceding product, make 18 *ft.* = 6 *yd.* 0 *ft.* Finally, 7 times 5 *yd.* = 35 *yd.*, which with the 6 *yd.* from the preceding product makes 41 *yd.* Hence 7 times 5 *yd.* 2 *ft.* 8 *in.* are 41 *yd.* 8 *in.*

2. Multiply 5 *lb.* 13 *oz.* by 8. *Prod.*, 46 *lb.* 12 *oz.*

1. Where do you write the multiplier? Why?

2. Where do you begin to multiply? Why?

3. What do you do with the product arising from multiplying any particular denomination by the multiplier? Why?

Pupil *write* the rule and the reasons for it; that is, the *Rule* and the *Demonstration.*

3. Multiply £7 9*s.* 5*d.* by 6. *Prod.*, £44 16*s.* 6*d.*

4. Multiply 1 *cd.* 112 *cu. ft.* by 10.
Prod., 18 *cd.* 96 *cu. ft.*

5. Multiply 12 *gal.* 3 *qt.* 1 *pt.* by 7. By 100.

6. Multiply 4 *oz.* 12 *pwt.* 21 *gr.* by 16. By 132.

7. Multiply 17 *A.* 35 *sq. rd.* 4 *sq. yd.* 5 *sq. ft.* by 23.
Prod., 396 *A.* 8 *sq. rd.* 14 *sq. yd.* 36 *sq. in.*

8. Multiply 4 *mi.* 110 *rd.* 17 *ft.* by 127. By 250.

9. Multiply 5 *da.* 15 *hr.* 13 *min.* 20 *sec.* by 341.

10. Multiply $10° 12' 14''$ by 45. By 6. By 13. By 10.

11. Multiply 3 *lb.* ℥ viij ʒ ij ℈j by 4. By 12. By 24.

SECTION VI.

DIVISION.

1. Divide 23 *da.* 15 *hr.* 51 *min.* by 7.

Operation.

7)23 *da.* 15 *hr.* 51 *min.*
3 *da.* 9 *hr.* 7 *min.* $17\frac{1}{7}$ *sec.*

Explanation.—We begin the division with the *highest* denomination, since if there is any remainder it can be reduced to the next lower denomination, combined with what is expressed in this denomination, and the whole sum divided at once. Thus 23 *da.* ÷ 7 = 3 *da.* and 2 *da.* (= 48 *hr.*) remainder. Then 48 *hr.* + 15 *hr.* = 63 *hr.*, which divided by 7 gives 9 *hr.* 51 *min.* ÷ 7 = 7 *min.* and 2 *min.* (or 120 *sec.*) remainder. 120 *sec.* ÷ 7 = $17\frac{1}{7}$ *sec.*

2. Divide 12 *lb.* 15 *oz.* by 8. *Quot.*, 1 *lb.* $9\frac{7}{8}$ *oz.*

3. Divide 125 *bbl.* 17 *gal.* 3 *qt.* by 36.

Operation.

36) 125 *bbl.* 17 *gal.* 3 *qt.* (3 *bbl.* 15 *gal.* $1\frac{17}{36}$ *qt*
108
17 *bbl. rem.*
$31\frac{1}{2}$
$8\frac{1}{2}$
17
51
17
$552\frac{1}{2}$ *gal.*
36
192
180
$12\frac{1}{2}$ *gal. rem.*
4
50 *qt.*
3 *qt.*
53 *qt.*
36
17 *qt. rem.*

Explanation.—Dividing 125 *bbl.* by 36 we find the quotient 3 *bbl.* and a remainder 17 *bbl.* Reducing this to gallons and adding the 17 *gal.* in the given number, we have $552\frac{1}{2}$ *gal.* This divided by 36 gives a quotient 15 *gal.* and a remainder $12\frac{1}{2}$ *gal.* This reduced to quarts makes, with the 3 *qt.* of the given number, 53 *qt.* Dividing this by 36, we have $1\frac{17}{36}$ *qt.*

To Divide Compound Numbers.

232. Rule.—*I. Write the divisor on the left hand of the dividend and the quotient underneath the dividend, or at its right, according as you divide by short or long division.*

II. Beginning with the highest denomination, divide it and write the quotient of this denomination in its place. Reduce the remainder (if any) to the next lower denomination and add to it the number expressed in this denomination in the given number. Divide as before, reducing the remainder to the next lower denomination.

Proceed in this manner till the division is complete.

Demonstration.—The general principle involved in this operation is the same as that involved in simple division, viz., the quotient is found by dividing the parts of the dividend separately and adding the quotients. (See rule for Simple Division and its demonstration.)

The relative position of dividend, divisor and quotient, is mainly matter of custom or convenience.

The division is commenced at the left hand in order that the several remainders which may arise may be reduced and combined with the lower denominations as the work proceeds.

4. Divide £25 10*s.* 8*d.* by 9. *Quot.*, £2 16*s.* 8$\frac{8}{9}$*d.*
5. Divide 28 *lb.* 14 *oz.* by 5.
6. Divide 3 *lb.* ℥ vij ʒ iv ℈ ij by 6.
7. Divide 20 *A.* 100 *sq. rd.* by 10. By 27. By 13.
8. Divide 6 *mi.* by 115. *Quot.*, 16 *rd.* 11 *ft.* 5$\frac{17}{23}$ *in.*
9. Divide 1 *bbl.* by 22. *Quot.*, 1 *gal.* 1 *qt.* 1$\frac{5}{11}$ *pt.*
10. Divide $\frac{1}{2}$ a bushel by 7. *Quot.*, 2$\frac{2}{7}$ *qt.*
11. Divide 7$\frac{3}{4}$ cords by 3. *Quot.*, 2 *cd.* 74$\frac{2}{3}$ *cu. ft.*
12. Divide 5 *yd.* 1 *ft.* 8 *in.* by 4.

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13. How many times is 2 *bu.* 3 *pk.* 7 *qt.* contained in 17 *bu.* 2 *pk.*? *Ans.*, 5$\frac{17}{19}$.

Suggestion.—Reduce both dividend and divisor to the lowest denomination in either, and then divide.

14. Divide 3 *bbl.* by 6 *qt.* By 20 *gal.* 2 *qt.* By 5 *gal.* 3 *qt.* 1 *pt.* *Last Quot.*, 16.08 +.

SECTION VII.

TWO PRACTICAL EXPEDIENTS.

233. **Making Change.**

1. Having bought 37 cents worth of goods I hand the clerk a $2 bill. How will he count the change due me?

Answer.—He will say "37," and laying out 3*c.* say "40;" then he will lay out a 10*c.* piece and say "50," then a 50*c.* piece and say "$1," and then a dollar bill and say "$2." Thus he counts on to the amount to be taken out by filling out the even parts of a dollar, and then counting on the remaining dollar.

2. Having Quarter Dollars and \$1 bills, how will the change for \$1.25 be counted out of \$5? How if I have only 5 *c.* 10 *c.* and \$1 pieces?

Answer to last. "\$1.25," "30," "40," "50," "60," "70," "80," "90," "\$2," "\$3," "\$4," "\$5." The pieces handed out being a 5 *c.* and 7 10 *c.* pieces and 3 \$1 bills.

3. Having 1*c.*, 2*c.*, 5*c.*, and Half Dollar pieces, how will the change for 27*c.* be counted out of \$1? What pieces will be given?

Answer.—The pieces given will be 1 1 *c.*, 1 2 *c.*, 4 5 *c.*, and 1 50 *c.* The counting will be "27, 28, 30, 35, 40, 45, 50, \$1."

4. Having \$5, \$2, and \$1 bills, and 1 *c.*, 5 *c.* and Quarter Dollar pieces, how will the change for \$2.13 be counted out of a \$10 bill?

5. Having 1 *c.*, 10 *c.*, and 25 *c.* pieces, how will the change for 7 *c.* be counted out of a 50 *c.* piece?

6. Having 2 *c.*, 3 *c.*, 10 *c.*, and 50 *c.* pieces, and \$1, \$2, and \$10 bills, how will the change for \$3.17 be counted out of a \$20 bill?

Answer. "\$3.17, 20, 30, 40, 50, \$4, \$6, \$8, \$10, \$20; the pieces given in change being 1 3 *c.*, 3 10 *c.*, 1 50 *c.*, 3 \$2 bills, and 1 \$10 bill.

7. With the same pieces as in the last, how will the change be made for \$11.26 out of a \$50 bill?

8. With 1 *c.*, 2 *c.*, and 10 *c.* pieces, how will the change for 15 *c.* be made out of 50 *c.*? How 45 *c.* out of \$1? How 8 *c.* out of 25 *c.*?

9. With 1 *c.*, 2 *c.*, 10 *c.*, and 25 *c.* pieces, and \$2 bills, how will the change for 62 *c.* be made out of a \$5 bill?

The counting is "62, 63, 65, 75, \$1, \$3, \$5."

The pieces used are 1 1 *c.*, 1 2 *c.*, 1 10 *c.*, 1 25 *c.*, and 2 \$2 bills.

10. With 1 *c.*, 10 *c.*, and 50 *c.* pieces, how will the change be made for 32 *c.* out of a \$2 bill? How for 37 *c.*? How for 12 *c.* out of a \$1 bill?

Aliquot Parts.

234. An **Aliquot Part** of a number is any number (integral or mixed) which will exactly divide it.

The common *Aliquot Parts* of \$1 are 50 *c.* = \$$\frac{1}{2}$; 25 *c.* = \$$\frac{1}{4}$; 20 *c.* = \$$\frac{1}{5}$; 10 *c.* = \$$\frac{1}{10}$; 12$\frac{1}{2}$ *c.* = \$$\frac{1}{8}$; 6$\frac{1}{4}$ *c.* = \$$\frac{1}{16}$; 33$\frac{1}{3}$ *c.* = \$$\frac{1}{3}$; and 16$\frac{2}{3}$ *c.* = \$$\frac{1}{6}$.

1. What cost 27 *yd.* of cloth at \$25 *c.* per yard?

Solution.—At \$1 per yard 27 *yd.* would cost \$27; hence at \$$\frac{1}{4}$ per yard 27 yards cost $\frac{1}{4}$ of \$27, or \$6$\frac{3}{4}$ = \$6.75.

Another Solution.—As every 4 *yd.* cost \$1, 27 *yd.* will cost as many dollars as 4 is contained times in 27; 27 ÷ 4 = 6$\frac{3}{4}$. Hence 27 *yd.* cost \$6.75.

Note.—The pupil should always be able to give some such analysis of the operation; *but for practical purposes he is to notice just what the numerical operation is; as, in this case, it is simply dividing by 4.*

2. What cost 42 *yd.* of calico at 12$\frac{1}{2}$ *c.* per yard? At 16$\frac{2}{3}$? At 6$\frac{1}{4}$?

Operation.—For the first, 42 ÷ 8 = 5$\frac{1}{4}$. ∴ The cost is \$5.25.

3. What cost 35 *lb.* of butter at 33$\frac{1}{3}$ *c.* per pound? At 25 *c.*? At 50 *c.*? At 20 *c.*?

4. What cost 15 *lb.* of tea at \$1.33$\frac{1}{3}$ per pound? At \$1.10? At \$1.50? At \$1.25? At \$1.12$\frac{1}{2}$? At \$1.20?

Operation.—For the first, 15 + 5 = 20. ∴ The tea cost \$20.

5. At 12$\frac{1}{2}$ *c.* per pound, how much sugar can be bought for \$1? For \$3? For \$1.25?

Operation. 8 + 2 = 10. ∴ 10 *lb.* can be bought for \$1.25.

6. At $33\frac{1}{3}$ *c.* per pound, how much coffee can be bought for \$1? For \$2? For \$5?

Operation. $2 \times 3 = 6$. $\therefore$ 6 *lb.* can be bought for \$2.

7. What cost 15 *yd.* of cloth at \$3.20 per yard? At \$2.10? At \$4.33$\frac{1}{3}$?

8. What cost 25 *yd.* of cloth at \$4.25 per yard? At \$3.40? At \$11.25?

Operation.—For the first, $425 \div 4 = 106.25$.

Explanation. 100 *yd.* would cost \$425, and 25 *yd.* would cost $\frac{1}{4}$ as much.

9. What cost $3\frac{1}{3}$ *lb.* of butter at 32 *c.* per pound? At 36 *c.*? At 45 *c.*? At 21 *c.*?

At 21 *c.* 10 *lb.* cost \$2.10, and $3\frac{1}{3}$ cost $\$2.10 \div 3 = 70$ *c.*

10. What cost $12\frac{1}{2}$ *yd.* of cloth at 40 *c.* per yard? At \$1.12? At 16 *c.*? At \$2.20?

11. What cost 48 *gal.* of molasses at $66\frac{2}{3}$ *c.* per gallon?

Operation. $48 - 16 = 32$. $\therefore$ \$32. Why?

12. What cost 120 *bu.* of potatoes at $37\frac{1}{2}$ *c.* per bushel?

13. What cost 256 *bu.* of onions at $87\frac{1}{2}$ *c.* per bushel?

\$256 — $\frac{1}{8}$ of \$256, or \$256 — \$32 = \$224. Why?

14. What cost 75 cords of wood at \$5.50 per cord?

\$550 — $\frac{1}{4}$ of \$550, or \$550 — \$137$\frac{1}{2}$ = \$412.50. Why?

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15. At \$360 per year what is the rent of a house for 2 *yr.* 7 *mo.* 25 *da.*?

Operation.

For 2 *yr.*,	2 times \$360,	or	\$720
For 6 *mo.*,	$\frac{1}{2}$ of \$360,	or	180
For 1 *mo.*,	$\frac{1}{6}$ of \$180,	or	30
For 15 *da.*,	$\frac{1}{2}$ of \$30,	or	15
For 10 *da.*,	$\frac{1}{3}$ of \$30,	or	10
For the required time, ...			\$955

16. At \$480 per year what is the rent of a house for 8 *mo.* 13 *da.?* *Ans.*, \$337.33$\frac{1}{3}$.

17. What cost 5 *bu.* 3 *pk.* 2 *qt.* of grain at \$1.20 per bushel? What 2 *bu.* 2 *pk.* 3 *qt.?*

$$\$6.00 + .60 + .30 + .07\tfrac{1}{2} = \$6.97\tfrac{1}{2}. \quad \text{Why?}$$

18. If John's salary is \$560 per year, how much does he receive for 5 *mo.* 15 *da.?* For 1 *yr.* 10 *mo.* 18 *da.?* For 2 *yr.* 6 *mo.* 10 *da.?* For 9 *mo.* 14 *da.?*

Operation for the last,		\$280
	\$46.666	\$140
14 *da.* = $\frac{1}{3}$ *mo.* + $\frac{2}{5}$ of $\frac{1}{3}$ *mo.*		15.555
		3.111
		3.111
		\$441.777

19. What cost 6 *lb.* 12 *oz.* of butter at 28 *c.* per pound? What 5 *lb.* 14 *oz.* at 30 *c.?* 4 *lb.* 10 *oz.* at 24 *c.?* 7 *lb.* 13 *oz.* at 40 *c.?* Same at 32 *c.* per pound?

$$\$2.80 + .20 + .10 + .02\tfrac{1}{2} = \$3.12\tfrac{1}{2}.$$

20. At 14 *c.* per pound what is the cost of a dressed turkey weighing 12 *lb.* 14 *oz.?* What one weighing 8 *lb.* 11 *oz.?* Same at 16 *c.* per pound?

21. At \$450 per year what is the rent of a house for 2 *yr.* 8 *mo.* 25 *da.?* For 1 *yr.* 5 *mo.* 11 *da.?* For 3 *yr.* 2 *mo.* 13 *da.?*

$$\$1350 + 75 + 12.50 + 3.75 = \$1441.25.$$

CHAPTER V.

BUSINESS RULES

SECTION I.

PERCENTAGE.

1. During a severe winter a farmer lost 5 sheep out of every 100 of his flock. What part of his flock did he lose? *Ans.*, .05, or $\frac{1}{20}$.

2. John's father agreed to give him $8 for every $100 he would earn for himself. To what part of his earnings was his father's gift equal? *Ans.*, .08.

235. Per Cent means *By the Hundred.*

To say that a man lost 5 per cent of his sheep is to say that he lost 5 out of every hundred of them, or .05 of them. Again, to say that a father gives his son a sum equal to 8 per cent of the son's earnings is to say that he gives the son $8 for every $100 he earns, or a sum equal to .08 of his earnings.

3. A nurseryman lost by drought 6 per cent of his trees. What part of his trees did he lose? If he had 2150 trees, how many did he lose, *i. e.*, .06 of 2150 = how many?

Answer. He lost .06 of his trees; and as he had 2150 he lost 129 trees.

4. A man's house was damaged by fire to an extent estimated at 15 per cent of its value. If the house was worth $6530, what was the amount of damage?

Ans., $979.50.

5. A speculator bought a car-load of wheat for $450 and sold it at a profit of 4 per cent. How much did he make by the speculation?

This means that he made .04 of $450, which is $18.

6. What is 9 per cent of 250? 10 per cent of 48? 17 per cent of 53? 11 per cent of 1437?

Answer to last, $1437 \times .11 = 158.07$.

236. Rate* is the number by which we multiply to obtain any required per cent of a given number.

Thus, to obtain 7 per cent of 250, we multiply 250 by .07. Hence .07 is the *Rate* (not *Rate per cent*).

237. The result obtained by taking a certain per cent of a number is called *the Percentage.* The term *Percentage* is also used as a general designation for all processes involving this method of reckoning by the hundred.

238. The **Base** is the number upon which the percentage is estimated.

Thus in the last illustration 250 is the *base.*

7. Mr. Smith having a flock of 340 sheep found that in 1 year they increased at the rate of 50 in a hundred. What was the per cent (or rate per cent) of increase? What was the *rate* of increase? How *much* was the increase?

The rate per cent, or per cent of increase was 50. The *rate* of increase was .50, or $\frac{1}{2}$. The percentage was 170.

239. The character % is used as a substitute for the words *per cent.*

Thus 4% means "4 per cent."

* It seems scarcely admissible to use the term *Rate per cent* in this sense, but we may so use *Rate;* in fact this is the common meaning of the word *rate* in mathematics. An allowance of 7 on a hundred is not at a rate of .07 *per cent, although* it is at a *rate* of .07; the *Rate per cent* is 7.

8. What part of a number is 10% of it? 20%? 50%? $12\frac{1}{2}$%? $16\frac{2}{3}$%? 40%? 11%? 15%? 7%? 6%? 4%? $\frac{1}{2}$%? $\frac{3}{4}$%? $\frac{5}{8}$%? 100%? 200%? 150%?

$\frac{1}{2}$%, or $\frac{1}{2}$ per cent, is $\frac{1}{2}$ on a hundred, just as 2% is 2 on a hundred. At $\frac{1}{2}$% the rate is .005.

9. What *rate* is $\frac{3}{5}$%? $\frac{3}{2}$%? $\frac{1}{3}$%? $1\frac{1}{2}$%? $\frac{3}{4}$%? $\frac{5}{8}$%?

$\frac{3}{5}$% is at the rate $.00\frac{3}{5}$, or .006. $\frac{3}{2}$% is at the rate $.01\frac{1}{2}$, or .015. The per cent being $\frac{1}{3}$, the rate is $.00\frac{1}{3}$. $\frac{3}{4}$% is at the rate .0075.

10. What is 5% of 146 *yd.*? Of 3470 *lb.*? Of 1 *mi.*?

11. What is 10% of $257? Of $75.40? Of $1? $100?

12. What is 7% of 258 apples? What 11%? 25%?

To Obtain any Required Per Cent of a Number.

240. Rule.—*Multiply the base by the rate.*

N. B.—The rate should always be expressed in the most convenient form, not necessarily in the form of a *decimal fraction.* Thus to find $33\frac{1}{3}$% of $360, we would not multiply by $.33\frac{1}{3}$, but by $\frac{1}{3}$, *i. e., divide* by 3. But to obtain 7% of any number, it is most convenient to multiply by .07. Again to find 25%, we would divide by 4, which is the same as multiplying by .25, or $\frac{1}{4}$.

Show the following to be true:

13. 5% of 780 = 39.
14. 12% of 475 *yd.* = 57 *yd.*
15. 10% of 860 trees = 86 trees.
16. 35% of 1840 = 644.
17. $33\frac{1}{3}$% of $234.54 = $78.18.
18. 45% of $18\frac{3}{4} = 8.43\frac{3}{4}$.
19. $\frac{3}{4}$% of $348 = $2.61.
20. $\frac{4}{5}$% of $\frac{15}{16}$ *lb.* = .12 *oz.*
21. 7% of $47 = $3.29.
22. Find 6% of $1.75.
23. Find 6% of $350.
24. Find 7% of $140.
25. Find 8% of $1.
26. Find 9% of $100.
27. Find $\frac{2}{3}$% of $0.75.
28. Find $1\frac{1}{2}$% of $\$\frac{5}{9}$.
29. Find $\frac{5}{8}$% of $1540.
30. Find $\frac{1}{2}$% of $2500.

31. Having $350, I used it in buying grain which I sold so as to gain $5\frac{1}{2}\%$. How much had I then?

$5\frac{1}{2}\%$ of $\$350 = \$19.25$. Hence I had $\$350 + \$19.25$, or $369.25.

241. The **Amount** is the sum of the base and percentage.

32. If the *base* is 784 and the *rate per cent* 6, what is the *amount?* *Ans.*, 831.04.

33. What is the amount of $1260 at $12\frac{1}{2}$ per cent? Of $347 at 10%? Of $4.86 at 7%? Of $125 at 6%?

34. Bought a flock of sheep numbering 350. In one season it increased 24%. How many had I then? Had it *decreased* 24%, how many would I have had?

35. Bought 12 crocks of butter weighing 35 *lb.* each, net,* for 19 *c.* per *lb.*, and sold it at a profit of 2%. What did I receive for the whole? How much did I *gain?*

36. My house which is valued at $5500, was damaged by fire 15%. What was the total damage?

37. Invested $3560 in town lots in a new village in Kansas. In the course of a year they increased in value $62\frac{1}{2}\%$. What were they worth then?

38. Bought 20 horses at an average price of $225 each. I lost 25% of them, and sold the remainder at an advance of 30% on the cost price. Did I gain or lose by the transaction? How much? *Ans.*, I lost $112.50.

39. What must be the selling price of cloth which cost $4 per yard, in order to realize a profit of 10%? Of $12\frac{1}{2}\%$? Of 25%? Of 30%? Of 15%? Of 20%?

40. At what must calico which cost 6 *c.* per yard be sold in order to realize a profit of 8%? Of $\frac{4}{5}\%$? Of 1%? Of $2\frac{1}{2}\%$? Of $\frac{1}{2}\%$? Of $1\frac{3}{4}\%$? Of 5%? Of $4\frac{1}{2}\%$?

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* This means exclusive of the crocks which contain the butter.

41. Bought a piece of cloth containing 30 *yd.*, at \$3 per yard. 10 *yd.* of it were damaged, so that I had to sell it at a loss of 50%. The remainder I sold at 20% profit. How much did I lose on the whole? *Ans.*, \$3.

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1. Out of a flock of 250 sheep 25 died. What per cent of the flock died, and what per cent remained?

The first inquiry is the same as "How many hundredths of 250 is 25?" Now 25 is $\frac{1}{10}$ of 250; hence $\frac{1}{10}$ of the flock died. But $\frac{1}{10}=\frac{10}{100}$, or .10, and 10 died out of a hundred, or 10% of the flock. Again, if 10 died out of 100, 90 survived; hence 90% (*i. e.*, 90 out of a hundred) remained.

2. What per cent of 520 is 130?

1. What part of 520 is 130 (139)?
2. $\frac{1}{4}$ of any number is how many hundredths of it?

3. What per cent of 9 is 3? *Ans.*, $33\frac{1}{3}$%.

4. What part of 600 is 42? Then what per cent of 600 is 42?

5. 12 is what part of 150? How many hundredths? What per cent?

How do you find what part one number is of another? (139.) How do you reduce a number to 100ths?

6. 28 is what part of 560? What per cent?

28 is $\frac{28}{560}$ of 560, or $\frac{1}{20}$. Give analysis as in (139).
Now $\frac{1}{20}$ of anything is 5 hundredths of it. Hence 28 is .05 of 560, or 5% of it.

7. 5 is what per cent of 11?

5 is $\frac{5}{11}$ of 11, and $\frac{5}{11}$ is $\frac{500}{11}$ hundredths, or $45\frac{5}{11}$ hundredths. Hence 5 is $.45\frac{5}{11}$ of 11, or $45\frac{5}{11}$% of it.

To find what per cent one number is of another.

242. Rule.—*Multiply the former by* 100 *and divide this product by the latter.*

8. 3 is what per cent of 8 ?

Operation. $\frac{\overset{25}{\cancel{100}} \times 3}{\underset{2}{\cancel{8}}} = 37\frac{1}{2}$. $\therefore$ 3 is $37\frac{1}{2}\%$ of 8.

Explanation. 3 is $\frac{3}{8}$ of 8 (**139**). $\frac{3}{8}$ is $\frac{100 \times 3}{8}$ hundredths, or $37\frac{1}{2}$ hundredths. $\therefore$ 3 is $37\frac{1}{2}\%$ of 8.

9. $\frac{1}{2}$ is what per cent of $2\frac{1}{2}$?

Operation. $\frac{100 \times \frac{1}{2}}{2\frac{1}{2}} = 20$. $\therefore$ $\frac{1}{2}$ is 20% of $2\frac{1}{2}$. (Explain.)

10. \$10.50 is what per cent of \$175 ?

$\frac{100 \times 10.50}{175} = \frac{1050}{175} = 6$. $\therefore$ \$10.50 is 6% of \$175.

11. \$21.40 is what % of \$428 ? Of \$700 ? Of \$87.50 ?

$\frac{100 \times 21.40}{87.50} = \frac{2140}{87.50} = 24\frac{16}{35}$.

12. 3 is what % of 12 ? Of 20 ? Of 17 ? Of 6 ? Of 30 ?

13. \$37 is what % of \$185 ? Of \$46.25 ? Of \$37 ?

243. Note.—*In such problems* the* BASE *and the* PERCENTAGE *are given. Hence to find the* RATE (*not the* RATE PER CENT) we simply have to *divide the Percentage by the Base, since by* (**240**) the percentage is the product of Base and Rate. The rate × 100 = the rate per cent.

14. What per cent of \$240 is \$16.80 ? \$27.60 ? \$13.80 ?
Ans., 7, $11\frac{1}{2}$ and $5\frac{3}{4}\%$.

15. What per cent of $\frac{2}{3}$ is $\frac{1}{5}$? $\frac{3}{7}$? $\frac{1}{12}$? $\frac{1}{9}$? $\frac{1}{3}$?
Ans., 30, $64\frac{2}{7}$, $12\frac{1}{2}$, $16\frac{2}{3}$, 50%.

16. What per cent of 125 is 125 ? 250 ? 375 ? 150 ?
Ans., 100, 200, 300, 120%.

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* Teacher explain what is meant by this word "problem."

17. The standard for gold coin in the U. S. is 9 parts pure gold and 1 part alloy. What % is alloy? What % is gold? *Ans.*, 10% alloy, 90% gold.

18. If I buy cloth at \$5 per yard and sell it at \$5.50, what % profit do I make? That is, \$.50 is what % of \$5?

$\frac{100 \times .50}{5} = 10$. I make 10% profit.

19. If I buy land at \$27 per acre and sell it at \$36, what % do I make? What if I sell it at \$30? At \$90?

Ans., $33\frac{1}{3}$, $11\frac{1}{9}$, $233\frac{1}{3}$.

20. A fruit grower shipped 300 baskets (pecks) of peaches to Chicago; but on the way 75 baskets spoiled. What % did he lose? What % was left?

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21. When I sell goods at $1\frac{1}{4}$ their cost what % profit do I make?

I make a profit of $\frac{1}{4}$ the cost, *i. e.* on every \$1 spent in buying I make $\frac{1}{4}$ of a dollar. The question then is, "$\frac{1}{4}$ is what % of 1?"

Ans., 25%.

22. When I sell goods at $\frac{7}{8}$ the cost what % do I lose?

The loss is $\frac{1}{8}$ of the cost. Then the question is, "$\frac{1}{8}$ is what % of 1?"

23. When I sell goods at twice the cost, what % do I make? When at $1\frac{1}{8}$ the cost? At $1\frac{1}{10}$ the cost? At $2\frac{1}{2}$ the cost? *Answer to last,* 150%.

24. What % is made by buying tea at 80 cents per pound and selling it at \$1? At 90 *c.*? At 85 *c.*? At \$1.10?

These questions are equivalent to, "What % of 80 is 20? 10? 5? 30?" Why?

25. Bought a span of horses for \$575 and sold them at \$650. What % did I make? *Ans.*, $13\frac{1}{23}$%.

26. Bought a house and lot for \$11500 and sold them at \$13640, after having expended \$350 in repairs. What % did I make?

Ans., $15\frac{25}{237}$%, or a little more than $15\frac{1}{10}$%.

27. A merchant marked prints which cost him 7 *c.* to be sold at 9 *c.* What per cent advance on cost was this?

Ans., $28\frac{4}{7}$%.

28. Bought 2560 *lb.* of coffee at 31 *c.* per pound, and paid \$1.50 per hundred for freight and \$1 for cartage. What % did I make by selling it at 45 *c.* per pound?

Ans., 38%+.

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1. By selling cloth at \$5.50 *per yd.* I make 10% on the cost. What was the cost?

\$5.50 is the cost + 10%, *i. e.*, it is $\frac{10}{100} = \frac{1}{10}$ more than the cost, or $1\frac{1}{10}$ times the cost. Now if \$5.50 is $1\frac{1}{10}$, or $\frac{11}{10}$ of the cost, $\frac{1}{10}$ of the cost is $\frac{1}{11}$ of \$5.50, or 50 *c.*, and $\frac{10}{10}$ is 10 times 50 *c.*, or \$5.00.

2. By selling a horse for \$230 I lost 8% on the cost. What was the cost?

\$230 is the cost − 8%, *i. e.*, it is $\frac{8}{100} = \frac{2}{25}$, less than the cost, or $\frac{25}{25} - \frac{2}{25} = \frac{23}{25}$ of the cost. Now if \$230 is $\frac{23}{25}$ of the cost, $\frac{1}{25}$ of the cost is $\frac{1}{23}$ of \$230, or \$10, and $\frac{25}{25}$ is 25 times \$10, or \$250.

NOTE.—If desired let the student write a rule for finding the *Base*, when the *Amount*, or the *Difference* between the percentage and the cost, and the % are given. It is the purpose of this treatise, however, to lead the pupil to depend upon his ability to analyze the problem, rather than upon a rule.

3. What was the cost of cloth marked \$3.50 *per yd.*, this being 15% advance on the cost?

4. A merchant having marked down his goods $33\frac{1}{3}$%

from his usual retail price, which was 20% advance on cost, what was the cost of an article now marked 20 *c.*?

20 *c.* was $33\frac{1}{3}\%$, or $\frac{1}{3}$ less than the regular retail price. Hence 20 *c.* was $\frac{2}{3}$ of the regular retail price. If 20 *c.* was $\frac{2}{3}$ of this price, $\frac{1}{3}$ was $\frac{1}{2}$ of 20 *c.*, or 10 *c.*, and $\frac{3}{3}$ was 3 times 10 *c.* or 30 *c.* Again, 30 *c.*, the regular retail price, was 20%, or $\frac{1}{5}$ advance on cost, and hence was $1\frac{1}{5}$, or $\frac{6}{5}$ times the cost, etc.

5. A merchant who had marked a certain lot of goods to sell at 15% advance on cost, in consequence of a rise in the market, marked them up 5% on the former retail price. At what % advance on cost were they now marked? *Ans.*, $20\frac{3}{4}\%$.

All questions in simple percentage can be solved by the three formulas $p = br$, $A = b + br = b(1 + r)$, and $D = b - br = b(1 - r)$ in which p is percentage, b base, r rate, A amount, and D difference, *i. e.*, the difference between the base and percentage when there is a loss.

$p = br$, means that Percentage = base × rate; hence base = Percentage ÷ rate, or $b = \frac{p}{r}$. So, also, $r = \frac{p}{b}$.

From $A = b(1 + r)$, we have $b = \frac{A}{1 + r}$, and $1 + r = \frac{A}{b}$, or $r = \frac{A}{b} - 1$.

So also $D = b(1 - r)$, gives $b = \frac{D}{1 - r}$, and $1 - r = \frac{D}{b}$, or $r = 1 - \frac{D}{b}$.

Teachers who prefer it can point out these *Nine Cases*, and show the pupil how the three formulas first given solve them all. The author's opinion is that for common arithmetical work the better way is to train the pupil to look at the nature of the problem,

and from a clear perception of the essential relations analyze it. This is what practical men call the *Common Sense Method*, and it is the one such men always use.

Miscellaneous Exercises.

1. From what price can I fall $33\frac{1}{3}\%$ on goods which cost \$3.20 per yard, and still make 20% ?

2. At what must I purchase nails by the keg (100 *lb.*) to sell them at 5 *c. per lb.* and make 15% ?

3. I have marked goods which cost me \$2.50 *per yd.* to sell at 25% advance. What % can I fall on this selling price and make 20% on the cost ?

4. A grocer by retailing sugar at $12\frac{1}{2}$ *c. per lb.* made 10% on the cost. What was the cost *per bbl.* of 200 *lb.* ?

5. If by selling nails at 6 *c. per lb.* I lose 4%, will I gain or lose by selling at 7 *c.*? What % on cost ?

6. A real estate dealer charges me 5% for selling my farm of 320 acres at \$58 per acre. How much do I receive for the farm ?

7. A grain dealer sells 2000 *bu.* of wheat for me, and pays me \$2450, his commission for selling being 2%. At what price per bushel did he sell it ?

8. At what must I buy boots by the case (1 *doz.* pairs) to make 15% and sell at \$4.60 per pair ?

9. Coffee which cost me 14 *c. per lb.* I sell at 6 *lb.* for \$1.00. What % do I make ?

10. After buying a bill of goods amounting to \$650

on 3 *mos.* credit, I find I am able to pay cash down, and my creditor takes \$617.50. What % does he abate (discount)?

11. If tea at 75 *c. per lb.* gives a profit of 20%, what would it yield at $56\frac{1}{4}$ *c.*? What at 50 *c.*? What was the cost?

12. Being in New York city and having a *Draft* from a bank in Omaha upon a bank in Chicago for \$2500, I present it at a New York bank, and they give me \$2493.75. What % does the bank in N. Y. deduct for collecting?

13. What % does a man make who sells a horse for \$100, which was given to him? What if he sells it for \$250?

14. Owning $\frac{3}{5}$ of a factory, I sold $16\frac{2}{3}$% of my interest for \$800, which was considered to be 10% less than its real value. What was the estimated value of the factory?

15. What % is made by buying berries by dry measure and selling at the same price per quart liquid measure? What % is lost if I buy by liquid measure and sell by dry, at the same rate per quart?

16. A druggist buys a certain drug at \$7.00 *per lb. Av.*, and sells it at \$1.00 *per oz.* Apothecaries Weight. What % profit does he make?

17. How must an article be sold by the dram (Apothecaries) which cost \$5.00 *per lb. Av.* to make 50% profit?

18. I made \$1750 in a certain business in 1876, which was 15% more than I made in 1875. How much more did I make in '76, than in '75?

19. A bankrupt's assets were found to be $33,000, and his liabilities $86,000. What % can he pay? What will a creditor receive whom he owes $650?

20. I receive $850 on a claim of $1250 against a bankrupt estate. What % does the estate pay? If the liabilities are $95,000, what are the assets?

[For further exercises in simple percentage, including the ordinary problems in Commission, Brokerage, Bankruptcy, Customs, etc., see the GENERAL REVIEW closing Sec. V., and the HAND-BOOK, pp. 130–136, and 156–164.]

SECTION II.

INTEREST.

1. Mr. Smith lends me $250 for a year, and I agree to pay him back the $250 at the close of the year, and 6% additional for the use of the money? How much do I pay for the use of the money? How much do I pay Mr. Smith in all at the end of the year?

Answer. For the *use* of the money I pay $15. In all I pay $265 at the end of the year.

244. Interest is money paid for the use of money.*

245. The **Principal** is the sum for the use of which *interest* is paid.

It will be seen that *Principal* corresponds to *Base*, as heretofore used, and *Interest* to Percentage. So also the *Amount* is the sum of principal and interest.

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* As the basis on which interest is computed is always money, it is not deemed best to cumber the definition with any allusion to anything else.

246. Simple Interest is interest which is considered as falling due only when the principal is paid, or when a partial payment is made. It is usually reckoned at a certain per cent per annum (year).

According to this principle, viz., that the interest does not fall due till a payment is made on the principal, no interest is allowed on accrued interest.

1. What is the simple interest on \$125 for 3 *yr.* at 7% per annum? What the amount?

Operation.

```
 $125
   .07
  8.75
     3
$ 26.25
 125.00
$151.25
```

Explanation.—Since 7% is .07 of the principal, the interest for 1 *yr.* is \$125 × .07, or \$8.75; and the interest for 3 *yr.* is 3 times the interest for 1 *yr.*, or \$8.75 × 3 = \$26.25. The amount being the sum of principal and interest is \$151.25.

2. What is the interest on \$250.60 for $2\frac{1}{2}$ *yr.* at 10% per annum? What the amount?

Interest, \$62.65; *Amount*, \$313.25.

3. If I borrow of Mr. White \$325 for 1 *yr.* 8 *mo.* 15 *da.* at 7%, what shall I have to pay him at the expiration of the time?

```
30 | 15.      da.
12 |  8.5     mo.
   |  1.708⅓  yr.
```

```
   $325
     .07
   22.75
   1.708⅓
     758
   18 200
 15 92 5
 22 75
 38.86 458
325.
$363.86
```

The interest for 1 year is \$22.75, and 1 *yr.* 8 *mo.* 15 *da.* = $1.708\frac{1}{3}$ *yr.* Hence the interest for 1 *yr.* 8 *mo.* and 15 *da.* is \$22.75 × $1.708\frac{1}{3}$.

To Compute Simple Interest.

247. Rule.—*Multiply the principal by the rate, and this product by the time in years.*

To find the amount, add the interest to the principal.

In multiplying by the time we may either reduce the months and days to decimals of a year, or take such aliquot parts of the interest for 1 *yr.* as the months and days are of 1 *yr.*

4. What is the amount of $325 at 7% per annum for 1 *yr.* 8 *mo.* 15 *da.?* Solve by taking aliquot parts of the interest for 1 *yr.*

$325	
.07	
$22.75	Int. for 1 *yr.*
11.375	Int. for 6 *mo.* = ½ int. for 1 *yr.*
3.792*	Int. for 2 *mo.* = ⅓ int. for 6 *mo.*
.948	Int. for 15 *da.* = ¼ int. for 2 *mo.*
$38.86	Int. for 1 *yr.* 8 *mo.* 15 *da.*
325.00	Principal.
$363.86	Amount.

5. What is the simple interest on $150 for 2 *yr.* 7 *mo.* 18 *da.* at 6% per annum?

By Aliquot Parts.

$150	
.06	
9.00	For 1 *yr.*
2	
18.00	For 2 *yr.*
4.50	For 6 *mo.* (½ *yr.*).
.75	For 1 *mo.*
.375	For 15 *da.* (½ *mo.*).
.075	For 3 *da.* (⅕ of 15 *da.*).
$23.70	For 2 *yr.* 7 *mo.* 18 *da.*

By Decimals.

3.0	18.	$150
12	7.6	.06
	2.63⅓	9.00
	9	
	$23.70	

The interest for 1 *yr.* is $9. And as the given time is 2.63⅓ *yr.*, we take 2.63⅓ times 9, using 9 as the multiplier, as it is more convenient.

* Nearer 2 than 1.

Note.—The method of computing interest by aliquot parts is in more general use, notwithstanding that the method by decimals usually requires less work.

Business men who have frequent occasion to compute interest—as bankers—*usually make use of Tables.* See next rule. See also **251**, **252**, **253**, and APPENDIX V.

6. What is the amount of \$350 for 3 *yr.* 10 *mo.* 19 *da.* at 8%?

By Aliquot Parts.

\$3 50	
.08	
28.00	
3	
84.00	
14.00	
7.00	
2.3333+	
1.1666+	
.2333+	
.0777+	
\$108.81	Interest.
350.00	
\$458.81	Amount.

By Decimals.

30	19.		\$350
12	10.633+		.08
	3.886+		\$28.00
	28		
	31 088		
	77 72		
	\$108.808	Interest.	
	350		
	\$458.81	Amount.	

Solve the following by each of the above methods, and observe which is the more expeditious. Find both interest and amount:

7. \$235.50 at 10% for 3 *yr.* 6 *mo.* 10 *da.* *Int.*, \$83.08.
8. \$245.60 at 8% for 2 *yr.* 7 *mo.* 21 *da.* *Int.*, \$51.90.
9. \$500 at 6% for 2 *yr.* 5 *mo.* 12 *da.* *Int.*, \$73.50.
10. \$750.50 at 7% for 1 *yr.* 8 *mo.* 20 *da.* *Amt.*, \$840.98—.
11. \$436.75 at 5% for 1 *yr.* 2 *mo.* 15 *da.* *Amt.*, \$463.14—.
12. \$230 at 6% for 11 *mo.* 15 *da.* *Int.*, \$13.22¼.

13. $1385.50 at 15% for 23 *da.* *Int.*, $13.28—.
14. $14.30 at 8% for 2 *yr.* 9 *mo.* *Int.*, $3.15—.
15. $325.25 at 6½% for 2 *yr.* 9 *mo.* 12 *da.*
16. $2360.25 at 8% for 7 *mo.* *Int.*, $110.14⅓.
17. $18.28 at 5% for 5 *yr.* 9 *da.* *Amt.*, $22.87+.
18. $87.50 at 7% for 3 *yr.* 3 *mo.* *Amt.*, $107.41—.
19. $480 at 15% for 6 *yr.* 3 *mo.* *Int.*, $450.
20. $18.20 at 5¾% for 9 *yr.* 9 *mo.* 9 *da.* *Int.*, $10.23.
21. $64.50 at 7% for 2 *yr.* 16 *da.*
22. $725 at 3⅓% for 5 *yr.* 2 *mo.* 18 *da.*
23. $5000 at 7% from May 6, 1875, to July 7, 1877.

Find the time by subtracting dates (**231**).

24. $81.25 at 6% from Aug. 6, 1873, to Nov. 4, 1876.
25. $105.23 at 10% from June 10, 1871, to Oct. 1, 1875.
26. $76.42 at 5% from May 9, 1874, to Aug. 9, 1874.
27. $18.00 at 8% from Aug. 8, 1875, to Aug. 30, 1875.
28. $5600 at 4½% from Apr. 1, 1876, to Apr. 1, 1878.
29. $43.60 at 3¾% from July 12, 1874, to June 1, 1877.
30. $150.30 at 10% from May 8, 1875, to Nov. 6, 1876.
31. $400 at 10% from Sept. 6, 1876, to Sept. 6, 1878.
32. $350 at 12% from Nov. 9, 1877, to Dec. 9, 1877.
33. $820 at 7% from Dec. 5, 1876, to July 24, 1877.
34. $1000 at 7% from Feb. 7, 1870, to Aug. 9, 1876.
35. $125.41 at 7% from July 10, 1873, to June 10, 1874.
36. $93.25 at 10% from Jan. 1, 1877, to July 1, 1877.
37. $48.50 at 5% from Oct. 3, 1877, to Jan. 3, 1878.
38. $150.40 at 7% from May 23, 1876, to Oct. 1, 1878.
39. $741.50 at 5½% from Nov. 29, 1875, to Aug. 30, 1877.
40. $13.50 at 10% from May 7, 1876, to Sept. 10, 1876.
41. $250 at 6% from July 1, 1873, to Apr. 1, 1874.
42. $450 at 5% from Aug. 7, 1875, to Aug. 7, 1877.
43. $158.23 at 8% from Dec. 25, 1877, to Sept. 23, 1878.

44. $354.40 at 8% from June 30, 1870, to Nov. 1, 1876.
45. $700 at 9% from May 20, 1873, to Mar. 6, 1875.
46. $60 at 12½% from Mar. 17, 1875, to Apr. 6, 1877.
47. $4000 at 5% from Jan. 25, 1872, to Feb. 18, 1874.
48. $250 at 10% from Mar. 6, 1872, to Apr. 30, 1873.
49. $175.50 at 7% from Feb. 7, 1876, to Aug. 11, 1878.
50. $300 at 8% from July 1, 1876, to Jan. 16, 1878.

[For further exercises and various forms of *notes*, see p. 263, *et seq.*; and for exercises for class drill see HAND-BOOK, pp. 137–142.]

To Find the Simple Interest on any Principal by means of Interest Tables.

There are several different volumes of such tables in use by bankers and accountants, but the general principle is the same. We have space to give only one page of such tables, and select that which gives the simple interest on $1 for any time less than 6 years, at 5%, 6%, 7%, 8%, 10%, and 12%. Such volumes always contain tables which enable us to take the interest on any sum directly from the table, usually requiring no arithmetical process but addition.

248. Rule.—*To find the interest on any sum from the following Table, take from the table the interest on $1 for the given number of years, months, and days, and add these results. Multiply this sum by the given principal.*

1. Find from the table the simple interest on $143.25 for 3 *yr.* 7 *mo.* 22 *da.*, at 7% per annum.

Interest on $1.		The Interest on $143.25 is 143¼ times the interest on $1; hence
.21	For 3 *yr.*	.255
.0408	For 7 *mo.*	143¼
.0042	For 22 *da.*	64
.255	For 3 *yr.* 7 *mo.* 22 *da.*	765
		1020
		255
		$36.53 *Int.* required.

12%.	6%.	7%.		10%.	5%.	8%.
			YEARS.			
.12	.06	,07	1	.10	.05	.08
.24	.12	.14	2	.20	.10	.16
.36	.18	.21	3	.30	.15	.24
.48	.24	.28	4	.40	.20	.32
.60	.30	.35	5	.50	.25	.40
			MONTHS.			
.01	.005	.00583	1	.00833	.00416	.00666
.02	.01	.01166	2	.01666	.00833	.01333
.03	,015	.01750	3	.02500	.01250	.02000
.04	.02	.02333	4	.03333	.01666	.02666
.05	.025	.02916	5	.04166	.02083	.03333
.06	.03	.03500	6	.05000	.02500	.04000
.07	.035	.04083	7	.05833	.02916	.04666
.08	.04	.04666	8	.06666	.03333	.05333
.09	.045	05250	9	.07500	.03750	.06000
.10	.05	.05833	10	.08333	.04166	.06666
.11	.055	.06416	11	.09166	.04583	.07333
			DAYS.			
.00033	.00016	.00019	1	.00027	.00013	.00022
.00066	.00033	.00038	2	.00055	.00027	.00044
.00100	.00050	.00058	3	.00083	.00041	.00066
.00133	.00066	.00077	4	.00111	.00055	.00088
.00166	.00083	.00097	5	.00138	.00069	.00111
.00200	.00100	.00116	6	.00166	.00083	.00133
.00233	.00116	.00136	7	.00194	.00097	.00155
.00266	.00133	.00155	8	.00222	.00111	.00177
.00300	.00150	.00175	9	.00250	.00125	.00200
.00333	.00166	.00194	10	.00277	.00138	.00222
.00366	.00183	.00213	11	.00305	.00152	.00244
.00400	.00200	.00233	12	.00333	.00166	.00266
.00433	.00216	.00252	13	.00361	.00180	.00288
.00466	.00233	.00272	14	.00388	.00194	.00311
.00500	.00250	.00291	15	.00416	.00208	.00333
.00533	.00266	.00311	16	.00444	.00222	.00355
.00566	.00283	.00330	17	.00472	.00236	.00377
.00600	.00300	.00350	18	.00500	.00250	.00400
.00633	.00316	.00369	19	.00527	.00263	.00422
.00666	.00333	.00388	20	.00555	.00277	.00444
.00700	.00350	.00408	21	.00583	.00291	.00466
.00733	.00366	.00427	22	.00611	.00305	.00488
.00766	.00383	.00447	23	.00638	.00319	.00511
.00800	.00400	.00466	24	.00666	.00333	.00533
.00833	.00416	.00486	25	.00694	.00347	.00555
.00866	.00433	.00505	26	.00722	.00361	.00577
.00900	.00450	.00525	27	.00750	.00375	.00600
.00933	.00466	.00544	28	.00777	.00388	.00622
.00966	.00483	.00563	29	.00805	.00402	.00644

Solve by the Table the following for both Int. and Amt.:

2. $340 at 5% for 2 *yr.* 5 *mo.* 11 *da.* *Int.*, $41.60.
3. $28 at 10% for 93 *da.* (3 *mo.* 2 *da.*). *Amt.*, $28.72.
4. $12.50 at 8% for 63 *da.* *Amt.*, $12.68.
5. $135.37 at 7% for 5 *mo.* 13 *da.*
6. $81.40 at 8% for 1 *yr.* 17 *da.*
7. $471 at 10% for 2 *yr.* 6 *mo.* 5 *da.*
8. $251.13 at 7% for 30 *da.* For 1 *yr.* 3 *mo.*
9. $125.10 at 12% for 340 *da.* For 3 *yr.*
10. $2000 at 10% for 2 *yr.* For 3 *yr.* 6 *mo.* 10 *da.*
11. $57.35 at 7% for 2 *yr.* 8 *mo.* 10 *da.*
12. $145 at 8% for 3 *yr.* 11 *mo.* 5 *da.*
13. $280 at 6% for 7 *mo.* 16 *da.*
14. A note* of $65.80, dated Feb. 20, 1868, and bearing interest at 7%, was paid June 25, 1870; what was the amount paid? *Ans.*, $76.61.

Find the time by subtracting dates (**231**).

15. On the 21st day of January, 1874, for value received, I promise to pay to John Jones, or order,† $350, with interest at 7% per annum.

AUBURN, *Dec.* 5, 1869. HENRY FISH.

What was the amount of this note when it became due? *Ans.*, $451.13.

16. One day after date, for value received, I promise to

---

* A *Note* is a written contract by which one party agrees to pay another party a specified sum.

† The words "or order" in this connection prevent John Jones from selling the note, without putting his name on it, *i. e.*, endorsing it. It is then said to be "negotiable," and John Jones can be made to pay it, if Henry Fish does not.

pay John Smith, or bearer,* One hundred and twenty-five and $\frac{25}{100}$ dollars, with interest at 10%.

ROCHESTER, MICH., *May* 6, 1875. HENRY HOYT.

What was the amount of this note March 5, 1877 ?

17. January 6, 1877, for value received, I promise to pay Enos Ames,† Five hundred and fifty dollars with interest at 7%.

PERRYSBURG, OHIO, *May* 7, 1875. M. C. PETERS.

What is the amount of this note when due?

18. Due Charles Minton, or order, Thirty-six dollars, with interest at 6%, value received.

WESTON, OHIO, *Apr.* 6, 1874. JOHN PIPER.

What was the amount of this DUE BILL, Jan. 15, 1876.

19. One day after date, for value received, we jointly and severally agree to pay Sarah Miner, or order, seven hundred dollars, with interest at 7%.

CHICAGO, ILL., *June* 6, 1875. SOLOMON PIKE.
JAMES NOAH.

Wnat was the amount of this note Mar. 29, 1877.

Such a note as the above is called a "*Joint and Several*" note, and either signer is equally liable for the entire amount. The holder may take his choice as to which he will collect it from, or he may proceed against both signers.

20. Two years from date, for value received, I promise

*** This note is negotiable without being endorsed. Anybody can collect it who may chance to have it. But if John Smith or anybody else does endorse it, the endorser becomes liable for it.**

† As this note is payable to nobody but Enos Ames, no one else can collect it. It is not "negotiable," and Enos Ames cannot sell it even by endorsing it. This is the common law. There are statutes in some States, as in Illinois, making such paper negotiable by endorsement.

to pay Stephen Ely, or order, Three hundred and seventy-five dollars, with interest?

PLYMOUTH, MICH., *June* 7, 1875. SMITH PHILLIPS.

What was the amount of this note when due?

What would the amount have been if the note had been dated Plymouth, Mass.? If in Wisconsin? If in Ohio? In Illinois? In Minnesota? In Iowa? (See **260.**)

21. Sold my house and lot, Aug. 21, 1875, for $5500, receiving 2500 cash, and a 7% note for 3 *yr.*, secured by mortgage for the balance. I immediately let the $2500 at 10% for 3 *yr.* When both became due I bought a house and lot for $8560. How much money besides the avails of the house and lot sold did I have to raise?

22. Bought a bill of goods amounting to $750, $\frac{1}{3}$ payable in 30 *da.*, $\frac{1}{3}$ in 60 *da.*, and $\frac{1}{3}$ in 90 *da.*, at 6%. What was the entire cost of the goods?

23. What is the amount of $83.25 at 8% from May 6, 1861, to Nov. 10, 1870?

At 10% from July 8, 1871, to Apr. 17, 1873?

At 6$\frac{1}{2}$% from Sept. 13, 1870, to Feb. 13, 1875?

EXACT INTEREST.

249. The above method of reckoning time, *i. e.*, calling 12 *mo.* a year and 30 *da.* a month, does not usually get the *exact* interest when months and days are involved, since it does not get the *exact time.* (See APPENDIX V., **25.**)

1. What is the exact interest on $450 at 10%, from May 25, 1868, to Jan. 8, 1871?

Suggestion.—The exact time (**231**) is 2 *yr.* and 228 *da.* The interest on $450 for 2 *yr.* at 10% is $90. For 1 *da.* it is $\frac{1}{365}$ of $45, and for 228 *da.* it is $\frac{228}{365}$ of $45, or $28.11–. Hence for the exact time the interest is $118.11–.

2. What is the exact interest on $140.40 from Aug. 29, 1864, to Nov. 29, 1865, at $6\frac{1}{2}$%? *Ans.*, $11.43–.

3. What is the exact interest on $1580 from June 10, 1874, to Feb. 17, 1875, at 10%? *Ans.*, 109.08+.

4. What is the exact interest on a $1000 U. S. Bond, at 5%, from Oct. 1 to May 6 following? From March 13 to Dec. 12 following?

In all transactions with the U. S. Government exact interest is to be computed. Several States have statutes requiring that a day be reckoned $\frac{1}{365}$, not $\frac{1}{360}$ of a year. The only safe way to reckon interest, if the case is likely to come to legal test, is to reckon the entire number of years and the exact number of days over, calling the latter 365ths of a year.

Compute the exact interest on the following and find the amounts:

	Date.	Principal.	%.	When Due.
5.	May 10, 1876,	$45.25,	7,	Aug. 8, 1877.
6.	Sept. 20, 1876,	$82.10,	8,	June 5, 1877.
7.	Feb. 10, 1876,	$125.80,	5,	May 11, 1877.
8.	Jan. 1, 1871,	$530.00,	$4\frac{1}{2}$,	Nov. 10, 1873.
9.	April 7, 1874,	$1000.00,	6,	July 17, 1876.
10.	Aug. 13, 1876,	$250.00,	10,	Mar. 19, 1877.
11.	May 1, 1876,	$125.00,	7,	Sept. 6, 1876.
12.	Aug. 17, 1875,	$35.50,	10,	Sept. 21, 1875.

NOTE.—It is the custom of some to reckon the entire calendar

months as so many 12ths of a year, and the odd days as 365ths. The above examples can be used as an exercise in this method if desired, comparing the results with those obtained in the former way. For further class drill see HAND-BOOK, pp. 142, 143.

BANKER'S METHOD.

250. Bankers, and sometimes other business men, reckon interest on short time paper by counting the exact number of days, adding 3 days grace (See **259**), and calling these days 360ths of a year. This is done by taking the number of days + 3 and using the Tables,—the tables in most common use being computed on the basis of a day as $\frac{1}{360}$ of a year.*

1. By the banker's method, what is the amount of a 7% $350 note dated May 11, 1876, and *nominally* payable Sept. 10, 1876? What by the common method? What by the method of Exact Interest?

The interest for 1 *yr.* is $24.50.

By the *Banker's Method* the time is 125 *da.*, or $\frac{125}{360}$ of a year, and the amount is $358.51−.

By the *Common Method* the time is 3 *mo.* 29 *da.*, and the amount is $358.10+.

By the *Exact Method* the time is 125 *da.*, or $\frac{125}{365}$ of a year, and the amount is $358.39−.

2. For value received, I promise to pay George Van Horn, or order, $500, Nov. 6, 1877, with interest at 10%.

PONTIAC, MICH., *June* 1, 1877. AMOS WHITE.

What was the amount of this note at maturity, by the Banker's Method?

Date of Maturity, Nov. 9, 1877, *Amount,* $522.36.

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* The General Government and the State of New York require that a day be considered $\frac{1}{365}$ of a year. See APPENDIX V., 24–27.

3. Date of note July 25, 1876, principal $250, rate per cent 6, nominally due Feb. 15, 1877. Find the amount by the banker's method.

4. Same as above, with date Feb. 6, 1877, principal $180.50, 7%, nominally due June 21, 1877.

5. Same as above, with date Oct. 18, 1876, principal $600, rate 8%, nominally due March 1, 1877.

6. Same, with date Nov. 10, 1876, principal $425.75, rate 5%, nominally due May 25, 1877.

7. Same, with date Aug. 28, 1875, principal $850, rate 10%, nominally due Feb. 3, 1878.

Amount, $1058.25.

When the time is more than 1 *yr.* the entire years are treated in the ordinary way.

8. Same, with date July 27, 1874, principal $382.40, rate 7%, nominally due Sept. 17, 1876.

9. Same, with date Aug. 12, 1876, principal $730, rate 8%, nominally due Nov. 21, 1877.

10. Same, with date Jan. 1, 1877, principal $450, rate 7%, nominally due, Aug. 15, 1878.

OTHER METHODS.

251. THE SIX PER CENT METHOD.—When the time is to be reckoned in the common way (*i. e.*, 12 *mo.* = 1 *yr.*, and 30 *da.* = 1 *mo.*), *call ½ the number of months* CENTS, *and ⅙ the number of days* MILLS, and the sum will be the interest on $1 for the given time at 6%.

The reason for this is evident, since at 6% the interest on $1 for 1 *yr.* is 6 *c.*, or ½ *c. per mo.* Again, as the interest on $1 for 1 *mo.* is 5 *mills*, it is 1 *mill* for every 6 days?

Ex. What is the interest on \$245.50 at 6% for 2 *yr.* 7 *mo.* 21 *da.*?

$\frac{1}{2}$ the months is 15.5, and $\frac{1}{6}$ the days 3.5. Hence the interest on \$1 for the time at 6% is \$0.1585. Multiplying this by $245\frac{1}{2}$ gives \$38.91, the interest required.

NOTE.—Having the interest at 6%, that at 5% can be obtained by *deducting* $\frac{1}{6}$ of the interest at 6%, at 7% by *adding* $\frac{1}{6}$, at 4% by *deducting* $\frac{1}{3}$, at 8% by *adding* $\frac{1}{3}$, etc.

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252. THE ONE PER CENT METHOD.—*Remove the decimal point in the principal 2 places to the left. Multiply this result by the rate per cent, and the time.*

Moving the decimal point 2 places to the left gives the interest on the principle for 1 *yr.* at 1%. Multiplying this by 6 gives it for 6%, etc.

Ex. Solve the above example in this way:

Interest for 1 *yr.* at 1%,	\$2.455
	6
" " 1 *yr.* at 6%,	\$14.73
" " 2 *yr.*	29.46
" " 6 *mo.*	7.365
" " 1 *mo.*	1.227
" " 10 *da.*	.409
" " 10 *da.*	.409
" " 1 *da.*	.041
" " 2 *yr.* 7 *mo.* 21 *da.*	\$38.91

BY DECIMALS.—The time is $2.64\frac{1}{6}$ *yr.* Hence we multiply the interest for 1 *yr.* by the number of years.

$$\begin{array}{r} \$14.73 \\ 2.64\frac{1}{6} \\ \hline 245 \\ 5892 \\ 8838 \\ 2946 \\ \hline \$38.9117 \end{array}$$

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253. BY CANCELLATION.—*Write the continued product of the principal, the rate per cent, and the time in*

days, for the numerator of a fraction, and 100 × 360, or 100 × 365, *for the denominator.* In performing the operation, cancel as much as practicable.

For $245.50, at 6%, for 7 *mo.*, containing 212 days, the solution by this method would be $\frac{245.5 \times 6 \times 212}{100 \times 360} = \frac{245.5 \times .106}{3} = \$26.02.$

N. B.—For special, expeditious methods obtained in this way, see APPENDIX V.

COMPOUND INTEREST.

254. Compound Interest is interest considered as falling due at regular intervals of time, and to be reckoned as increasing the interest-bearing debt from such times.

This method of reckoning interest allows interest on interest accrued, and hence the term *compound,* meaning *interest on interest.*

1. What is the amount of $250 at annual compound interest for 3 years, at 7% ?

For EXPLANATION see next page.

Operation.

$350	1st Prin.
.07	
24.50	*Int.* for 1 *yr.* on 1st Prin.
350.00	1st Prin.
$374.50	*Amt.* for 1st *yr.*, or 2d Prin.
.07	
26.2150	*Int.* on 2d Prin.
374.50	2d Prin.
$400.715	*Amt.* of 2d Prin. for 1 *yr.*, or 3d Prin
.07	
28.05005	*Int.* on 3d Prin.
400.715	3d Prin.
$428.76	*Amt.* at end of 3d year.

Explanation.—As the interest is considered as falling due at the end of each year, at the end of the first year the debt is $374.50. This is, therefore, to be on interest for the next year. Again, as the interest on this for a year, $26.215, falls due at the end of the year, it is added to the principal for this year, and makes the interest-bearing sum for the 3d year $400.715. This sum on interest for a year amounts to $428.77, which is therefore the amount of $250 on compound interest for 3 *yr.* at 7%.

2. What is the amount of $152 at semi-annual compound interest for 2 years at 6% per annum?

Operation.

```
   $152
    .03
   ----
   4.56
   152
 ------
 156.56
    .03
 ------
 4.6968
156.56
-------
161.257
    .03
-------
4.83771
161.257
-------
166.095
    .03
-------
4.98285
166.095
-------
$171.08
```

Explanation.—As the interest is considered as falling due at the end of each ½ year we compute the interest for ½ a year and then add it to the principal, thus making a new principal for the next ½ year. Instead of multiplying by .06 in this instance, and dividing the product by 2 to get the interest for ½ a year, we simply multiply by .03, which gives the same result.

255. Rule.—*To compute Compound Interest, reckon the interest on the principal for the first interval of time, add it to the principal, and consider this as a new principal for the next interval, etc.*

Or, *Find from the interest Tables the amount of $1 for the given rate and time and multiply this by the given principal.*

The result thus found is the AMOUNT. *The Compound Interest is the remainder after the first Principal is subtracted from this amount.*

Compound Interest Table.

yr.	3%.	4%.	4½%.	5%.	6%.	7%.
1	1.030000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.060900	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.092727	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.125509	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.159274	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.194052	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.229874	1.315932	1.360862	1.407100	1.503630	1.605781
8	1.266770	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.304773	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.343916	1.480244	1.552969	1.628898	1.790848	1.967151

3. Find the amount of $243.12 at annual compound interest for 3 *yr.* at 4%, both with and without the use of the table. Also the interest.

By the Table.—Amount of $1 at 4% for 3 *yr.* Now $243.12 amounts to 243.12 times as much as $1, or $273.48.

Hence the interest is $273.48 − $243.12 = $30.36.

```
    $1.12486
      243.12
    --------
      224972
     112486
    337458
   449944
  224972
 -----------
$273.4759632
```

Find the compound interest of the following sums for the respective times and rates, both by the use of the table and without it.

4. $340 for 2 *yr.* compounded semi-annually at 6%.*
5. $100 for 7 *yr.* at 4½%. *Amt.*, $136.09.
6. $230 for 6 *yr.* at 8%. At 5%. At 4%.
7. $125 for 3 *yr.*, compounded quarterly, at 12%.
8. $270 for 4 *yr.*, compounded semi-annually, at 8%.
9. $250 for 3½ *yr.*, compounded semi-annually, at 10%.

* This means, "at 6% per annum," but compounded (*i. e.* interest added to principal) every 6 *mo.*; hence it is the same as 3% for 4 years, or $42.67

10. What is due on a note of $200 bearing semi-annual compound interest at 9%, 2 *yr.* 10 *mo.* from date?

Suggestion.—For 2½ *yr.* the amount is $249.2364. This is then on interest for 4 *mo.*, which makes the whole amount $256.71.

11. What is the difference between the simple interest of $500, at 10% for 3 years, and the compound interest on the same sum for the same rate and time?

12. What is the amount of $325 at quarterly compound interest, at 2% per quarter, for 2 *yr.* 5 *mo.* 10 *da.*?

13. What is the interest of $540.20, interest compounded semi-annually, at 5% per semi-annum, for 4 years? What is the difference between this and the interest compounded annually at 10%?

14. What is the difference between the interest of $100, compounded quarterly at 6% per annum, for 2 *yr.*, and the simple interest of the same sum for the same time at 7%?

15. What is the compound interest of $480 at 5% per annum, from May 6, 1873, to July 13, 1875?

16. What is the amount of a note for $500 Jan. 15, 1877, which draws 8% semi-annual compound interest, and is dated Aug. 18, 1874?

The laws of the States usually do not allow the collection of compound interest. In some States such notes as the above would be collectible with simple interest, while in others the taking of such a note would forfeit all interest, and in other States it would entail a still heavier loss, in some, even the entire debt.

ANNUAL, SEMI-ANNUAL, AND QUARTERLY INTEREST.

256. Contracts are often made in which it is agreed that the interest shall be paid annually, semi-annually, or even quarterly. This is, in fact, compounding the interest thus often; but if the payments of interest are not made as they fall due, the general rule is that only simple interest can be collected, although the statutes of some of the States allow simple interest on *the deferred payments of interest.* See APPENDIX V., 26.

1. On a note for $150 bearing annual interest at 7%, the debtor had neglected to pay the interest for 3 *yr.* Allowing simple interest on the deferred payments, what was then due on the note?

Suggestion.—At the end of 1 *yr.* $10.50 of interest fell due. This being deferred 2 *yr.*, at 7%, would amount to $11.97. At the end of the 2d year another $10.50 of interest fell due, which would be on interest 1 *yr.*, and would amount to $11.24. At the end of the 3d year another $10.50 of interest fell due. Hence at this time there was due in all, $150 + $11.97 + $11.24 + $10.50, or $183.71.

2. On the same principle as in the last, what is due on a note for $525, bearing 6% annual interest, the interest payments having been deferred 4 years?

Ans. $662.34.

3. On the same principle, what is due on a $275 10% note, interest payable semi-annually, but deferred 3 *yr.* 8 *mo.* 17 *da.*?

4. As above, what is due on a $100 8% note, interest payable quarterly, but deferred 1 *yr.*?

5. As above, what is due on a note for $200, annual interest at 10%, deferred 10 years?

Observe that the *interest upon the interest deferred* is 9 *yr.* on the first year's interest, 8 *yr.* on the second, 7 *yr.* on the third, etc.; that is, $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ years interest on 1 year's interest, or \$20. This is \$90. To this add the simple interest \$200, and we have the amount, \$200 + \$200 + \$90 = \$490.

Find the amount due on the following sums at the respective rates and times, the interest considered as deferred:

6. \$350, 7%, annual, for 3 *yr.* 5 *mo.* 10 *da.*
7. \$820, 5%, semi-annual, for 3 *yr.* 9 *mo.* (See below.)
8. \$85.30, 6%, semi-annual, for 1 *yr.* 10 *mo.*
9. \$250, $4\frac{1}{2}$%, annual, for 5 *yr.* 8 *mo.* 12 *da.*
10. \$500, 10%, semi-annual, for 2 *yr.* 7 *mo.*

The 7th gives $2\frac{1}{2}$% interest on \$20.50 for $6\frac{1}{2} + 5\frac{1}{2} + 4\frac{1}{2} + 3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2} = 24\frac{1}{2}$ periods of 6 *mo.* each, *i. e.*, \$12.56 as the *interest on the interest*. To this add the amount of the \$820 at simple interest for the entire time and we have \$973.75 + \$12.56 = \$986.31.

SAVINGS BANK INTEREST.

257. Savings Banks are banks organized for the purpose of receiving deposits in small sums, and allowing the depositors interest on the same. Interest is usually reckoned on the following principles:

1. Some savings banks compound the interest on deposits, *i. e.*, give the depositor credit for it, quarterly, some semi-annually, and some annually.

2. Some banks reckon the interest for each *month* in one of the above periods on the smallest balance on

deposit during that month, and call the sum of these monthly interests the interest for that period. Others reckon the interest *quarterly* on the smallest balance on deposit during the quarter, and call the sum of these interests the interest for the period.

3. No interest is allowed on money drawn out during a period for the antecedent part of that period.

1. A sewing woman opened a savings account on Jan. 1, 1875, and deposited \$15 Jan. 1, 1875, and a like amount on the first day of each month during the year. The bank allows 5% *per ann.*, reckoning the interest on monthly balances and compounding it quarterly. How does the account stand at the close of the year if the woman has drawn nothing out?

At the end of the 1st quarter she would have to her credit \$45.375, the .375 being 6 *mo.* interest on \$15, *i. e.*, one \$15 the first *mo.*, two the second, and three the 3d. And this would be the addition she would make to her deposits by new payments each quarter. Hence at the end of the 2d quarter she would have to her credit \$45.375 with 3 *mo.* int. + \$45.375 = \$91.32. At the end of the 3d quarter her credit would be \$91.32 + the int. for 3 *mo.* + \$45.375 = \$137.84. At the close of the year the account would stand \$137.84 + the int. for 3 *mo.* + \$45.375 = \$184.94.

2. What would have been the balance of the above account if the rule of the bank had been to allow interest on the least amount during each quarter?

Ans., \$184.17.

3. What would be the answer to the 1st example if the woman had drawn out \$20 the 1st of April, and \$25 the first of October? *Ans.*, \$138.86?

4. What is the balance of the following account

July 1, 1876, interest at 5%, compounded Jan. 1, Apr. 1, July 1, and Oct. 1, on the least balance for the quarter?

Dr. HOWARD SAVINGS BANK in Acct. with E. SMITH. *Cr.*

1875.				1875.			
Mar. 11.	To cash,	$195	00	May 1.	By Draft,	$25	00
July 10.	" "	$75	00	June 7.	" "	$50	00
Aug. 25.	" "	$45	50	Sept. 9.	" "	$18	45
Nov. 1.	" "	$62	50				
				1876.			
1876.				Jan. 1.	" check,	$100	00
Apr. 10.	" "	$43	50	May 10.	" draft,	$27	50

There was no interest the 1st quarter; int. on $120, $1.50, the 2d quarter; int. on $121.50, $1.52, the 3d quarter; int. on $225.07, $2.81, the 4th quarter; int. on $190.38, $2.38, the 5th quarter; int. on $192.76, $2.41, the 6th quarter.

Ans. $211.17.

5. What would be the balance of the last account if the interest of each quarter was taken as the aggregate of the monthly interests on least balances for the month? *Ans.*, $213.13.

The following form will facilitate such solution:

DATE.		DR.	LST. MO. BAL.	MO. INT.	DATE.		CR.
1875. Mar. 11.	Cash.	$195.00	0.00	0.00	....	....	....
April.	....	....	$195.00	0.81	....	....	....
May.	....	....	$170.00	0.71	May 1.	Draft.	$25.00
June.	....	....	$120.00	0.50	June 7.	"	$50.00
July 1.	Qr. Int.	$2.02	....	....	....	....	....
July 10.	Cash.	$75.00	$122.02	0.51	....	....	....
Aug. 25.	"	$45.50	$197.02	0.82	....	....	....
Sept.	....	....	$224.07	0.93	Sept. 9.	Draft.	$18.45
Oct. 1.	Qr. Int.	$2.26	$226.33	0.94	....	....	....
Nov. 1.	Cash.	$62.50	$288.83	1.20	....	....	....

6. What would be the balance of the above account if the interest on the monthly balances were credited July 1, and Jan. 1? *Ans.*, $213.07.

PARTIAL PAYMENTS.

258. It frequently happens that a debtor does not pay his note all at one time. In such a case, whatever is paid at any time is endorsed (credited) on the back of the note, and is called a **Partial Payment** (or simply, a payment).

There are several methods in more or less general use for computing interest on such notes; but the one adopted by the U. S. Court and by most of the States, is the only one for which we have space in this treatise.

This rule is based upon these two principles:

1. *The principal can not be diminished until the accrued interest is paid;*

2. *Interest shall not draw interest.*

U. S. Court Rule for Computing Interest on Notes on which Partial Payments have been made.

259. Rule.—*I. Compute the interest on the Principal from the date of the note to the time of the first payment. If this payment equals or exceeds this interest, find the amount and subtract the payment. Treat this remainder as a New Principal, and proceed to the next payment. Continue the process till the time of settlement is reached.*

II. If any payment is less than the accrued interest, add such payment to the next, and treat the sum as one payment made at the latter date.

$350.

One day after date, for value received, I promise to pay John Jay, or bearer, Three hundred and fifty dollars ($350), with interest at 7% per annum.

ROCHESTER, N. Y., *May* 7, 1868. AMOS AMES.

On this note there were the following endorsements:

September 17, 1870, \$100;

February 10, 1872, \$50.

How much was due on the note October 25, 1872?

Interest at time of 1st Payment	\$57.85
As the payment (\$100) exceeds this,	350.00
we find the amount	\$407.85
and subtract the Payment	100.00
New Principal	\$307.85
Interest on New Principal at the time of the 2d Payment	30.11
As the payment (\$50) exceeds this, we find the amount .	\$337.96
and subtract the payment	50.00
2d *New Principal*	\$287.96
Amount of this 2d New Principal from date of last payment to Oct. 25, 1872	\$302.24

2. \$475. CHICAGO, ILL., *Sept.* 14, 1869.

On July 12, 1875, for value received, I promise to pay Peter Price, or order, Four hundred and seventy-five dollars (\$475), with interest at 6% per annum.

JAMES WHITE.

On this note were the following endorsements:

April 12, 1871, \$25; August 20, 1873, \$150;

November 27, 1874, \$100; May 1, 1875, \$5.

What remained due on this note at the date of payment?

Suggestions.—At the time of the 1st payment the accrued interest was \$44.97. If this be added to the principal, and the payment, \$25, subtracted, *part* of this interest would be included in the *New Principal*, and hence interest would draw interest, and the 2d principle be violated. Hence we consider the 2d payment, made Aug. 20, 1873, as \$175, and compute the interest on the face of the note, \$475, up to this date.

This is $112.10, and the payments ($175) being greater than the accrued interest, we find the amount, $587.10, and deduct these payments, leaving as a New Principal $412.10. [N. B.—Each New Principal must be less than the preceding, otherwise there will be interest on interest.]

The amount of this New Principal at the time of the 3d payment was $443.49, and as this payment, $100, was more than the accrued interest, $31.39, our 2d New Principal is $343.49.

Finally, it is evident, without careful computation, that the next payment, $5, over 5 *mo.* from the last, did not equal the interest then accrued. So we compute the interest on this 2d New Principal, $343.49, to the time of settlement, and finding the amount, deduct the $5. This leaves the amount due on the note $351.37.

3. $504. CLEVELAND, O., *June* 10, 1869.

On demand, for value received, I promise to pay Zenas White, or order, Five hundred and four dollars ($504), with interest at 6% per annum.

J. A. KING.

On this note were the following endorsements:

Jan. 25, 1870, $84; May 15, 1870, $100;
Feb. 20, 1871, $200.

What was due July 5, 1871? *Ans.*, $166.51.

4. $450. LOUISVILLE, KY., *Jan.* 1, 1865.

Two years after date, for value received, I promise to pay to the order of James Jones, Four hundred and fifty dollars ($450), with interest at 8% per annum.

RANDALL WRIGHT.

On the back of this note were the following endorsements:

March 16, 1865, $75; Jan. 1, 1866, $100;
Apr. 4, 1866, $200.

What was due on the note at maturity (*i. e.*, at the time it should be paid? *Ans.*, $119.77.

5. Date of note, March 11, 1870; face, $58.50; rate per cent, 10; payments, June 5, 1871, $12; Nov. 23, 1873, $6; Aug. 7, 1874, $5; Dec. 18, 1874, $20; May 10, 1876, $5; how much remained due July 1, 1876? *Ans.*, $43.12.

6. On a note of $400, at 7%, there was paid $100 annually for 3 years. How much remained due 3 *yr.* 4 *mo.* from the date of the note?

260. Legal Interest.—*Legal Interest* is the rate per cent established by law as that which is to be implied in an interest-bearing obligation in which the rate is not specified.

In Louisiana the legal rate is	5%.
In the N. E. States, except Conn., N. C., Penn., Del., Md., Va., W. Va., Tenn., Ky., O., Mo., Miss., Ark., Ia., Ill., Ind., the Dist. of Columbia, and debts due the United States. .	6%.
In N. Y., Conn., N. J., S. C., Ga., Mich., Minn., Kan., and Wis.	7%.
In Alabama Flor and Texas	8%.
In Col., Neb., Nev., Or., Cal., and Washington Territory. .	10%.
In England and France	5%.
In Canada, Nova Scotia, and Ireland	6%.

Usury is a higher rate of interest than is lawful. Most of the States allow interest above the legal rate, when it is agreed upon between the parties and specified in the contract. Thus O. and La. allow any rate up to 8%; Ill., Ia., Mich., Miss., Wis., Mo., and Tenn., up to 10%; Minn. and Tex. up to 12%; Neb., 15%; Kan. 20%; Mass., R. I., Flor., Ark., Cal., Nev., Col., any rate agreed upon.

Of course, in those States where parties are prohibited from making a contract for more than a certain rate, *Compound Interest* is illegal, and can not be collected at law.

Some of the States have special statutes allowing simple interest on accrued payments of *Annual Interest* which have been deferred. Such is the case in Michigan.

MERCHANT'S RULE.

261. It is a common practice with business men to treat obligations maturing and settled in a year or less, and upon which payments have been made, according to the following

Rule.—*Find the amount of the principal from the date of the note to the time of settlement; find the amount of each payment from the time it was made to the time of settlement, and subtract their sum from the first result.*

1. $250.60. ANN ARBOR, *July* 7, 1876.

For value received, I promise to pay Stephen Beckwith, or order, Two hundred and fifty, and $\frac{60}{100}$ dollars, April 15, 1877, with interest at 7%.

EDWARD SNOW.

Endorsed, Sept. 20, 1876, $80.00.
" Jan. 1, 1877, $50.00.
" Mar. 13, 1877, $50.00.

What was the amount due at maturity?

Computing the interest by the Banker's Method, *i. e.*, using exact time and the common Interest Tables, we have:

Amount of note Apr. 18, 1877,	$264.49
Amt. of 1st Pay't, from Sept. 20 to Apr. 18,	$83.27
" " 2d " " Jan. 1 " "	51.04
" " 3d " " Mar. 13 " "	50.35
Total amount of payments.	$184.66
Balance due Apr. 18, 1877.	$79.83

Calling a day $\frac{1}{365}$ of a year, instead of $\frac{1}{360}$, makes the *interest* $\frac{1}{73}$ less, or in this case 13 cents less. Hence the amount thus reckoned is $79.70.

2. Date of note Aug. 23, 1874, principal $420, rate 10%, nominal maturity May 1, 1875; endorsements $100, Oct. 15, 1874, $200, Jan. 1, 1875. What was due at maturity, reckoning by Banker's Method? What by the common method (no grace)? What by the exact method, with grace?

Ans., $137.22, $136.82, $136.98.

3. $500. RICHMOND, *Jan.* 1, 1875.

Ninety days after date, for value received, I promise to pay to the order of Frank H. Ransom, Five hundred dollars, with interest at 6%.

JOHN M. SABIN.

Endorsements: January 20, $100; February 10, $50; February 25, $100; March 1, $150.

What was due at maturity, Banker's Method?

Note matures April 4, *Amt.*, $104.59.

4. $400. BUFFALO, *Jan.* 1, 1874.

One year after date, for value received, I promise to pay N. Stacy, or order, Four hundred dollars, with interest at 7%.

M. M. DEYOUNG.

Endorsements: March 16, 1874, $200; July 1, $100.

What was due at maturity? *Ans.*, $113.43.

This result is obtained by reckoning calendar months, and the odd days 360ths of a year. The last payment is on interest 6 *mo.* 3 *da.* This is a common method when there is a considerable number of even months.

5. $700. DANBURY, *Feb.* 17, 1874.

Six months after date, for value received, I promise to

pay John Gordon, or bearer, Seven hundred dollars, with interest at 6%.

Received on the above, May 10, 1874, $350.
" " " " June 25, 1874, $200.

When did the note mature, and what was due?

Note matured, Aug. 20, 1874; *Due*, $163.65.

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6. A buys a house and lot of B for $10,000. He pays cash $2000, and gives his note, secured by mortgage, for the balance, payable in 4 equal annual installments, with annual interest at 7%. At the end of the 1st year A is able and willing to pay $3000 besides the interest then due, provided B will allow him 10% annual interest on what he pays above what is due, as money is worth 10% in the market. To this B assents. How is the endorsement to be made, and what is due at the end of the 2d year?

There should be two endorsements, one of $2560, specifying that it is to cover payment and interest then due, and one of $1000 with the statement that 10% annual interest is to be allowed on it till the next payment falls due. At the end of the 2d year there would then be due $1320.

7. If at the end of the 2d year A pays $4000 on the same terms as above, and then pays no more till the note matures, what is then due?

At the end of the 2d year B is to make an endorsement of $1320 to balance payment then due, and $2680 on 10% till payments falling due absorb it. At the end of the 3d year there falls due $2280, and the amount of the payment is $2948. This leaves a balance $668 of payment unabsorbed, on which 10% interest is to be

allowed for the next year. This leaves the amount due and unpaid at the end of the 4th year $1405.20.

8. $4000. DETROIT, *Jan.* 1, 1874.

For value received, I promise to pay C. D., or order, $4000 in four equal annual installments, with annual interest at 8%.

M. NIL.

July 1, 1874, Received on the above $500, on which I am to allow 10% interest till the amount is absorbed by payments falling due. C. D.

Jan. 1, 1875, Received on the above $1000, part of which is to apply to cancel the balance of payment and interest now falling due, and the remainder to bear 10% interest till absorbed by payments falling due.

Nothing more being paid on this note till its maturity, what is due Jan. 1, 1878, allowing simple interest on deferred payments of annual interest?

Ans., $3516.02.

9. An $8000 7% note with interest payable annually, and principal in 4 equal annual installments, is dated July 10, 1873, and has endorsements as follows:

Feb. 12, 1874, $1000,
July 10, 1874, $2500,
Jan. 23, 1875, $1000,
May 18, 1875, $1800,
Oct. 17, 1875, $1500.

Interest at 10% being allowed on all payments made in advance, and simple interest on all deferred payments of annual interest, what was due July 10, 1877?

SECTION III.

DISCOUNT.

262. Discount is a general term used by business men to signify any deduction made from a *nominal* price, or value. There are *three* principal uses of the term as illustrated by the three following examples:

1. I asked a bookseller the price of a certain book. He answered: "The list * price is $15; but I can allow you a *discount* of 30%." What did he ask me for the book?

By a "*discount* of 30%," he meant 30% less than $15. 30% of $15 is $4.50. Hence he proposed to sell me the book for $15 − $4.50, or $10.50. The $4.50 may be called the *Commercial Discount.*

263. Commercial Discount is a deduction from the *nominal* price, or value, of an article.

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2. Desiring to borrow some money *at a bank,* they tell me that they are *discounting* at 1% a month. How much money shall I receive on my note for $200 made payable in 2 months?

The custom of banks is to require the payment of interest in *advance, i. e.,* when the note is given, and for *Three Days* more than the nominal time. Thus, in this case, 1% per month on $200 for 63 days is $4.20. Hence they will give me $200 − $4.20, or $195.80, on my note of $200 for 60 days. Of course I shall have no *interest* to pay when my note falls due. Only the face of the note, $200, will be required then. The $4.20 is called the *Bank Discount.*

264. Bank Discount is interest paid in advance, and

* Regular retail price fixed by the publisher.

for 3 days more than the nominal time. These 3 days are called *Days of Grace.* See APPENDIX V., **28.**

This custom of allowing days of grace has become well nigh universal with reference to *Business Paper* (obligations for the payment of money). The general rule is that a suit at law cannot be instituted for the collection of any such paper until 3 days after its nominal maturity. Hence, in discounting such paper, it has become customary to compute the amount including these 3 days.

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3. I take a note due 2 years hence which will bring me at that time $228. I wish to obtain the money on it *now.* What is it worth, the use of money being worth 7% per annum?

The supposition is that the use of $1 for a year is worth to me $.07, and for 2 years $.14. Hence every $1.14 of the $228 due 2 years hence is worth to me $1 *now.* $228 \div 1.14 = 200$. Therefore the note is worth $200 *now.* The $200 is called the *Present Worth.* $228 - $200, or $28, is the *Discount.* To distinguish this from *Bank Discount* it is usually called *True Discount.*

265. True Discount is a deduction made for the present payment of a sum of money due at some future time.

266. The Present Worth of a sum of money due at some future time, is a sum which, put at interest at a rate agreed upon, will in the given time amount to the sum due.

4. I take a note for $300, bearing interest at 7% and due $3\frac{1}{2}$ years hence. What is its present worth, money being worth 10%?

Solution.—The *amount* of the note at maturity will be $373.50. Now $1 at 10% will amount to $1.35 in the given time. Hence the *Present Worth* of said note is $373.50 \div 1.35 = 276.66\frac{2}{3}$, or $\$276.66\frac{2}{3}$.

Proof.—That this is just appears from the fact that if I retain the

note I shall get \$373.50 at the expiration of $3\frac{1}{2}$ years; while if I sell it for \$276.66$\frac{2}{3}$ and the money is worth 10% to me, I shall realize the *amount* of \$276.66$\frac{2}{3}$ at 10% for $3\frac{1}{2}$ years, or \$373.50.

5. I take a note for \$300, bearing interest at 10%, due $3\frac{1}{2}$ years hence. What is its present worth, money being worth 7%? *Ans.* \$325.30.

The *amount* due on the note at the end of the time will be \$405. But as money is now worth only 7%, \$1 in hand now will amount to \$1.245 in the $3\frac{1}{2}$ years. Hence the present worth is $405 \div 1.245$.

That this note is worth *more than its face* (\$300), is evident, since it is drawing a *higher rate of interest* than money is *now* worth.

The face of the note is its *nominal* present worth. The difference between this and its *true* present worth is called *Premium.*

267. When the *True Present Worth* of a note exceeds the *Face* of the note this excess is called **Premium.**

To find the Present Worth of a Sum of Money due at some future time.

268. Rule.—*Divide the sum due at the future date by the amount of \$1 at the rate agreed upon for the time from which it is proposed to discount the sum till the time said sum is due. The quotient is the Present Worth.*

269. The difference between the nominal present value (as the *face* of a note) and the *True Present Worth,* is the **True Discount,** or **Premium,** as the case may be.

6. What is the present worth of a note of \$125.50, due 2 *yr.* 3 *mo.* 15 *da.* hence, without interest, money being worth 7%? What the discount?

$125.50 \div 1.1604 = 108.15+$, the present worth. Hence the discount is $125.50 - 108.15 = \$17.35$.

7. How much should be discounted for the present payment of a note of \$1174.32, due in 3 *yr.* 3 *mo.*, without interest, money being worth 8%? *Ans.* \$242.32.

8. January 14th, 1875, a speculator offered me $300 for a note of $350, dated May 7, 1874, payable Oct. 21, 1876, and bearing 6% interest, money being worth 10%. Did he offer me the full value of the note? *Ans.*, $41.194+ less.

What amount of money would this note yield Oct. 21, 1876? What is the present worth of this amount Jan. 14, 1875, at 10%?

9. A merchant bought goods amounting to $4200, on a credit of 6 months, without interest. What sum in ready money would discharge the debt, money being worth 8%?

10. A note, $52.25, dated Sept. 12, 1869, and bearing 10% interest, will be paid Jan. 1, 1876. What is it worth July 1, 1874, money being worth 6%? *Ans.* $78.15.

11. Having bought a bill of goods amounting to $250 on 90 days credit, the tradesman says to me, "For cash I could discount you 10% on this bill." What amount of money would pay the bill *now?* *Ans.* $225.

12. If you get your note for $50 discounted at a bank for 2 months at 10% per annum, what are the *proceeds, i. e.,* how much money will you receive on the note?

13. On a note for $150 given at a bank for $^{90}/_{93}$* days, at 8% per annum, what are the proceeds? *Ans.* $146.90.

14. In many banks, as in those of New York city, bank discount is reckoned as *Exact Interest* (**249**) in advance. In such a bank, what is the discount on a note of $5000 for $^{60}/_{63}$ days at 10% per annum? What if reckoned as common interest in advance? *Answers,* $86.30, $87.50.

Note.—Let the student show that the difference between the bank discount on a note for $^{60}/_{63}$ days computed as *Common Interest*

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* A note given at bank for 90 days is considered as due any time from 90 to 93 days from date (though legally only at 93 days) and the time is written thus, $^{90}/_{93}$. So the time on a note for 30 days is written $^{30}/_{33}$.

in advance, and as *Exact Interest* in advance is nearly $\frac{24}{10000}$ of the interest on the sum for 1 year. For $^{30}/_{33}$ days it is a little more than $\frac{185}{100000}$; for $^{90}/_{93}$ days a little more than $\frac{85}{10000}$.

15. What is the Bank Discount on a note for $250 (*Ex. Int.*) at 7% for $^{60}/_{63}$ *da.*? For $^{30}/_{33}$ *da.*? At 10% for $^{90}/_{93}$ *da.*? For $^{30}/_{33}$ *da.*?

Observe that taking exact interest gives for $^{30}/_{33}$ $\frac{33}{365}$ of the interest for 1 *yr.*, while common interest gives $\frac{33}{360}$.

16. A note for $6000 was made May 10, 1868, payable in six months with interest at 9%, and discounted at a bank in Michigan, Oct. 3, 1868, at 7%; what were the proceeds? *Ans.*, $6225.164+.

The *Amount* due Nov. 13, 1868 (allowing grace), is $6274.50 This is discounted at bank for $^{38}/_{41}$ days, exact interest.

To find the Face of a Note to be made at Bank in order to obtain a given sum as Proceeds.

Ex. 1. I wish to obtain $500 at bank for $^{60}/_{63}$, and they are discounting at 10%. For what amount must I draw my note?

$1 face of note gives $1.00 − $.01726 = $0.98274 proceeds, since the interest on $1 for $^{60}/_{63}$ at 10% is $0.01726. Hence to obtain $500 I must make my note for $\frac{\$500}{.98274} = \$508.78$.

Proof.—If I make a note for $508.78 for $^{60}/_{63}$, the bank will deduct from the face of the note the *Bank Discount*, which is the interest in advance. Now the interest for $508.78 for $^{60}/_{63}$ *da.* at 10% is $\$508.78 \times \frac{1}{10} \times \frac{63}{365} = \$8.78$. Hence the proceeds of such a note are $500.

270. Rule.—*Find the interest of* $1 *for the given rate and time* (*including* 3 *da. grace*), *and deducting this interest from* $1, *divide the sum desired by the remainder. The quotient is the face of the note.* (See APPENDIX V.)

2. For what must I draw my note in order to obtain \$50 at bank for ${}^{30}/_{33}$ *da.*, when they are discounting at 8%? Give proof. *Ans.* \$50.36.

3. For what must I draw my note at bank for ${}^{90}/_{93}$ *da.*, in order to obtain \$1000, when they are discounting at 7%? Give proof. *Ans.* \$1018.16.

4. What is the bank discount on a note for ${}^{45}/_{48}$ *da.*, which yields \$2500 proceeds, at 9%? *Ans.* \$29.94.

5. A merchant buys a bill of goods which he can have at ${}^{60}/_{63}$ *da.* credit, for \$2850, or for \$2800 cash. He can borrow at bank for 8%; would it be better to do so? What would be the difference?

Ans., He would make \$10.80 by borrowing at bank.

Miscellaneous Exercises.

1. I paid \$2.20 for a book on which the bookseller allowed me 20% discount from the retail price. What was the retail price?

2. A certain article is marked to sell at 25% advance on cost, and the dealer gives me 10% off from retail price, and I pay \$6.75 for it. What was the cost?

3. I have a 7% note for \$360, dated July 7, 1875, and due March 1, 1877. What is it worth Sept. 15, 1876 allowing the purchaser 10% on his investment? (Allow grace and reckon the time by calendar months, calling the exact odd days 360ths of a year.)

4. June 6, 1878, for value received, I promise to pay A. B., or order, \$730, with 6% interest.

GRAND RAPIDS, *Oct.* 10, 1875. C. D.

Endorsed, $300, Feb. 20, 1877.

What was this note worth Sept. 1, 1877, allowing 10% discount? (Grace and time as in the 3d.)

Ans., $490.12.

Find what the note will yield June 9, 1878, by the U. S. Court rule, and discount this amount at 10%.

5. What would the note in the last example yield if discounted at bank, Sept. 1, 1877?

Ans., $487.20.

6. What is the present worth of $5000, due 3 *yr.* hence without interest, discounting at 10% *compound interest?*

Discounting at *compound* interest is the same as common discount, except that the amount of $1 for the given rate and time is taken at *compound* instead of at *simple* interest.

7. I have a $750 note bearing 7% simple interest, dated July 3d, 1875, and due March 10, 1878. Feb. 6, 1876, Mr. A. proposes to buy it if I will allow him 10% compound interest discount. What does he offer me?

8. A note for $500, bearing 7% interest, dated June 20, 1874, and due Jan. 1, 1878, was sold Aug. 15, 1875, allowing the purchaser 10% *annual* interest. What did the note sell for?

Strictly speaking, the purchaser can not receive *annual* interest, as the note does not yield anything annually. He must, therefore, receive the equivalent of annual interest. This is properly *compound* interest, yet it might be construed in the light of statutes, to be simple interest on the deferred payments of annual interest.

On the former hypothesis we divide the amount of the note at maturity, ($623.96, calendar *mo.* and exact odd days as 360ths), by

the amount of \$1 at 10% compound interest, from Aug. 15, 1875, to Jan. 4, 1878, or $2\frac{7}{18}$ *yr.*, \$1.257. $\frac{\$623.96}{1.257} = \$496.38$.

On the hypothesis simple interest on deferred payments of annual interest, we have $\frac{\$623.96}{1.25\frac{2}{3}} = \$496.51$.

It will be observed that the difference between the two results is quite unimportant, unless the time be long, or the amount large.

9. What is the difference between the present worth of \$10000, discounted for 12 years, when 10% compound interest is allowed for use of money, and when 10% annual interest deferred is allowed?

Present worth with comp. int., \$3186.31; *with annual interest deferred*, \$3496.50; *a difference of* \$310.19.

10. C has a 7% note for \$500, bearing annual interest, dated May 30, 1874, and due July 1, 1880. What is it worth Jan. 1, 1877, discounting at 10% annual interest, and assuming all payments of annual interest as deferred?

Ans. \$538.37.

11. What was the worth of the above note Jan. 1, 1877, discounting at 10% simple interest?

Ans., \$556.32.

12. A 7% note for \$360, dated Jan. 15, 1877, and payable (nominally) Nov. 6, 1877, has one endorsement of \$80 March 15, and one of \$100 Aug. 1. It is discounted at bank Sept. 20, at 10%. What are the avails?

Ans., \$192.38.

Find the amount due Nov. 9 by the Merchant's rule. Discount this for 50 *da.* as $\frac{5}{36}$ of a year.

13. A 6% note for \$300, dated May 1, 1873, and due in 3 equal annual installments, with annual interest, was

discounted at bank discount* by an outside party at 10% on July 1, 1874. What were the avails? (No grace.)

The avails of the $118	due May 1, 1875,	were	$108.17
" " " " $112	" May 1, 1876,	"	91.47
" " " " $100	" May 1, 1877,	"	71.67
Total avails,			$271.31

N.B.—A single principle covers all such problems as the above, viz.: Find what the paper will yield, and when, and discount these amounts for their respective times.

14. An 8% note for $3000 with annual interest is dated Aug. 10, 1876, and the principal payable $1500 in 2 years, and $1500 in 4 years. What is the worth of the note Feb. 1, 1878, assuming all payments made as agreed, and discounting at 10% by bank discount, allowing grace on all payments?

NOTE YIELDS,

Aug. 13, 1877,	$240,	Paid before the note was sold.		
Aug. 13, 1878.	$1740,	Discounted for	6 *mo.* 12 *da.* is	$1647.20
Aug. 13, 1879,	$120,	"	" 1 *yr.* 6 *mo.* 12 *da.* is	101.60
Aug. 13, 1880,	$1620,	"	" 2 *yr.* 6 *mo.* 12 *da.* is	1209.60
Total avails,				$2958.40

15. Jan. 1, 1877, I took a note for $4000, payable in 4 equal annual installments with annual interest at 6%. The same day I got it discounted at 10% annual interest. What were the proceeds (common discount without grace)?

* It is quite the custom in many parts of the country for "out-door" parties to reckon discount by multiplying the amount to be discounted by $1 — the interest of $1 for the time, instead of dividing $1 + this interest. This is applying the method of banks to ordinary outside transactions.

TRADE DISCOUNT.

271. Instead of the old method of averaging accounts it is now the custom of wholesale dealers allowing their customers time, to give them specific times of credit on their various bills, these times being 30, 60, or 90 days. Then each house has its rules of discounting for payment before due. This may be called *Trade Discount*, and makes it for the interest of the customer to pay as soon as practicable. The following examples illustrate the practice.

1. A western shoe merchant buys of a Boston dealer, on 60 days time, the following bill, the understanding being that if payment is made in 30 *da.* he shall have "2% off," and if in 10 *da.*, 3%:

2 cases boots	$30 . . .	$60.00
3 cases "	$96 . . .	$288.00
½ case shoes	$90 . . .	$45.00

What amount will pay the bill in 10 *da.*? What in 30 *da.*?

2. I buy of A. T. Stewart & Co., on 4 *mo.* time, a bill of goods amounting to $500, the rule of the house being to allow 6% off if payment is made in 10 *da.*, and 5% if made in 30 *da.* What amount will pay the bill in 10 *da.*? What in 30 *da.*?

3. I buy $350 on 30 *da.*, and $500 on 60 *da.*, and pay the former in 10 *da.* with 3½% discount, and the latter in 20 *da.* with 5% discount. How much better is this than 10% *per ann.* for my money for the time I anticipate the payments?

4. What is saved by paying a 60 *da.* bill for $1200, ½ in 10 *da.* at 5% discount, and ½ in 20 *da.* at 4% discount?

SECTION IV.

INSURANCE AND TAXES.

[The arithmetical principles involved in the ordinary problems under these heads are simple applications of percentage, and all the special instruction the pupil needs is in reference to the nature of the business, and to the technical terms used.]

272. Insurance is a branch of business in which companies called *Insurance Companies,* make contracts to pay specified sums of money to other parties, in the event of certain losses to which the latter may be liable, the company receiving a percentage on the sum guaranteed.

273. The contract is called a **Policy.** The sum which the party insured pays to the company is called the **Premium.**

274. There are two principal departments of the Insurance Business; viz., *Fire Insurance,* and *Life Insurance.*

1. An insurance company gives me a contract agreeing to pay me $\frac{2}{3}$ the value of my house in the event of its being burned during the year; for which security I pay the company $\frac{5}{8}\%$ on the amount insured. What is the written contract called? My house being worth \$3000, how much do I pay for the insurance? What is this called? If my house is burned during the time, how much do I receive? The premium is \$12$\frac{1}{2}$.

2. What is the annual premium on a policy which insures a house worth \$12000 for $\frac{5}{6}$ its value at $\frac{1}{2}\%$?

Ans. \$50.

3. My house is worth \$6000. I take out a fire insurance policy for $\frac{2}{3}$ of its value, paying $\frac{3}{8}\%$ annual premium. At the expiration of $7\frac{1}{2}$ years the house is burned. What is my actual loss, reckoning money worth 7%.

I receive from the insurance company \$4000. I have paid 8 annual premiums of \$15 each. On the first I compute $7\frac{1}{2}$ years interest at 7%, on the second $6\frac{1}{2}$, on the third $3\frac{1}{2}$, etc., and on the last, $\frac{1}{2}$ year's interest. These amounts are \$22.875, \$21.825, \$20.775, \$19.725, \$18.675, \$17.625, \$16.575, \$15.525, or in all \$153.60. Hence my actual loss is \$2153.60. Hence I have saved \$4000 – \$153.60 = \$3846.40 by insuring.

4. A life insurance company gives me a contract in which they agree to pay my wife \$4000 at my death, in consideration of my paying them an annual premium of 3%. What is my annual payment? *Ans.*, \$120.

5. What is the annual premium on a life policy for \$3500 at $2\frac{3}{4}\%$? *Ans.*, \$96.25.

6. A man takes out a life policy in favor of his wife for \$6000 at 2%. He dies at the end of 5 *yr.* 3 *mo.* 15 *da.* How much more does his wife receive than the amount of his payments at compound interest at 7%?

There were 6 premiums paid, of \$120 each. Had these been invested at compound interest at 7%, the first, which would have been on interest 5 *yr.* 3 *mo.* 15 *da.* would have amounted to \$171.74; the second, on interest 4 *yr.* 3 *mo.* 15 *da.*, would have amounted to \$160.51, the 3d to \$150.01, the 4th to \$140.19, the 5th to \$131.02, and the 6th to \$122.45. Hence the entire amount of the payments had they been put at interest would have been \$875.92; and the widow has \$6000 – \$875.92 = \$5124.08 more than she would have realized from the investment of the premiums.

7. A man takes out a life policy in favor of his wife for \$5000 at $2\frac{1}{2}\%$. He carries this till his death, which

occurs $20\frac{1}{3}$ years after he was insured. Does this prove better for his wife than an investment of the annual premiums at 10% simple interest would have been?

Ans., No: she receives $337.50 less.

8. My house is worth $1200. I have it insured at $\frac{2}{5}$% so as to cover $\frac{4}{5}$ of its value and the premium, if it chances to burn within the year. What is the sum named in the policy? What the premium?

Of every $1 premium paid how much applies on the property, and how much to refund premium?

9. Same as the above, save that the amount specified in the policy is sufficient to reimburse me for $\frac{4}{5}$ the value of the house, and all the premiums paid with 7% interest on them, if the house burns during the 3d year, and the company pay at the close of that year.

10. I insure $6000 worth of property for 3 years, paying $42, which includes $1 for policy and $1 for the survey. What must be the amount named in the policy, and what the %, in order that in case of loss during the 2d year I may receive $\frac{4}{5}$ of the value of the property, and 80% of the premium paid?

Ans., $4832; .83% nearly.

11. A man at 45 takes out a life policy in favor of his wife for $5000 at $2\frac{3}{4}$%. He carries this till his death, which occurs 10 years after. How much better is this for his wife than would have been an annual investment of the premium at 6% compound interest?

Ans., $3078.90.

[For a clear and simple exposition of the principles on which Life Insurance calculations are made, see the author's SCIENCE OF ARITHMETIC.

TAXES.

275. A **Tax** is money required by the government to be paid by the people of the country for the support of government, or for public enterprises.

The general theory of taxes is that they are laid upon *property*, and not upon persons. So it is designed that every dollar's worth of property shall bear an equal part of the tax to be raised. Apportioning the tax to be raised, and determining the value of each person's property, is called *Assessing*.

There are, however, in some of the States, what are called *Poll Taxes*. A poll tax is usually a small sum (75 *c.* or $1) required of every male over 21 years of age.

1. In a certain school district the entire property of which is valued * at $125,000, a school-house costing $5000 is to be built by public tax. How much is assessed on a dollar? What is Mr. Smith's school-house tax, his property being assessed at $3000? Mr. Jones's, whose property is assessed at $4500? Mr. White's, whose property is assessed at $10000?

If $125000 worth of property is taxed $5000, $1 bears $\frac{1}{125000}$ of the tax. Hence the tax on $1 is $\frac{5000}{125000}$ of a dollar, or 4 cents. Mr. Smith's tax would therefore be 3000 times 4 *c.*, or $120, Mr. Jones's $180, and Mr. White's $400.

2. The total State tax of Michigan for 1875 was $903,434.50.† The total valuation of property in the

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* Such evaluation for the purpose of assessing taxes is usually made at half or less than half the actual value of property.

† This was for such objects as the support of the State government, providing for interest and payments on the State debt, for the various State institutions, as the University, Agricultural College, Normal School, State Public School, the various asylums and prisons, and for carrying forward the building of the State Capitol.

State was \$630,000,000. What was the State tax on \$1 valuation? What would a man's State tax be whose property was valued at \$2000?

Ans., Tax on \$1 about $1\frac{1}{2}$ mills; on \$2000, \$3.00.

3. In a certain village the taxes were, for State purposes $1\frac{1}{2}$ mills on \$1, for county purposes $\frac{1}{2}$ mill on \$1, for school 3 mills, for township purposes 2 mills, for corporation (village expenses) 2 *c.* What were a man's taxes whose property was listed (put on the tax list by the assessor) at \$3000? One whose property was listed at \$600? What was the school tax of each? What the village tax?

Answers in order, \$81; \$16.20; \$9; \$1.80; \$60; \$12.

4. A certain State Legislature levies $\frac{1}{20}$ of a mill on a dollar as a tax for a particular interest. The valuation being \$630,000,000, what will this yield?

Ans., \$31,500.

5. In a State in which the entire valuation is \$2,500,000,000, what amount will a tax of .01 of a mill on a dollar raise?

6. In a certain school district the people desire to raise \$5000 for the purpose of building a school-house. It is estimated that 6% of the amount levied will not be collected, and 5% is to be allowed for collecting. What amount must be levied?

Ans., \$5599.10.

What will \$1 tax levied net for the building of the house? The collector has 5% on all he collects, including his own percentage.

7. Verify the above. That is, in a certain school district there was levied \$5599.10 tax for building a

school-house. Of this 6% was uncollectible, and the collector was allowed 5% for collecting. What did the tax net for the building?

8. A bridge costing $250,000 is to be built between two cities. The larger city is to bear $\frac{3}{5}$ of the expense. The taxable property of this city being $80,000,000, what will be a man's bridge tax who is taxed on $15,000, assuming that the levy is made to include $3\frac{1}{2}$% for collecting, and 10% as uncollectible?

Ans., $32.38.

9. Verify the last. That is, having all the facts given, including the answer, except the cost of the bridge, to find the cost of the bridge.

10. The valuation of the taxable property of a town being $2,100,000, what will a levy of $1\frac{1}{2}$ mills on a dollar net, 8% being uncollectible, and 3% being paid for collecting?

11. In a certain city a man pays $35.22 tax on a valuation of $1050. The entire tax levied being $425,000, what is the total valuation of the city property? What is the actual value of the city property, if this man's property is worth $4500?

12. The total valuation in a certain city is $1,500,000, and a man taxed on $1050 valuation pays $35.22. What is the total tax levied?

13. The net proceeds of a certain assessment was $150,000. Allowing $4\frac{1}{2}$% of the levy as having proved uncollectible, and $3\frac{1}{2}$% commission for collection, what was the total tax levied?

SECTION V.

*GENERAL PROBLEMS IN PERCENTAGE AND PRACTICAL EXERCISES.**

1. Mr. Smith left with merchant Jones 5 doz. pr. gloves to be sold at $1.50 per pair, agreeing to allow Mr. Jones 10% for selling. What was Mr. Jones's commission (percentage) on 4 doz. which he sold? How much would he pay over to Mr. Smith?

276. *Commission* is a sum allowed for selling or buying property, collecting debts, or doing other similar business. It is usually reckoned at a certain % on the amount involved.

Analysis.—The receipts for the gloves sold were $18 × 4 = $72. 10% of $72 is .10 of it; hence the commission was .10 of $72, or $7.20. Mr. Smith receives $72 – $7.20, or $64.80.

2. Mr. Smith left with merchant Jones a certain number of doz. pr. gloves, to be sold at $1.50 per pair, agreeing to allow him 10% for selling. On settlement Mr. Jones's commission was $12.60. How many dozens were sold?

Analysis.—If $12.60 is .10 of the amount received, .01 of it is $\frac{1}{10}$ of $12.60, or $1.26, and 100-hundredths, or the whole amount received, was 100 times $1.26, or $126. Now as 1 doz. was sold for $18, $126 would be the price of 126 ÷ 18, or 7 dozens.

3. $13.86 commission for selling $198 worth of goods is what % commission?

* Instead of dividing this subject into "*Cases*," and thus presenting a multiplicity of rules, it is thought better to train the pupil to look at the nature of each particular problem, as he will have to do in actual business life, and from an intelligent analysis discern the operations. See the next page for the suggestion of a general principle and the method of applying it to all such problems. In any event this method can scarcely produce worse practical results than the old mechanical method of "*Cases*" and special rules.

Analysis. 1% of $198 is $1.98. Hence $13.86 is as many per cent of $198 as $1.98 is contained times in $13.86, or 7%.

4. At what must a merchant sell cloth which cost $4.50 per *yd.* to make 15%?

5. What % is made by selling calico at $12\frac{1}{2}$ *c.* which cost 9 *c.* per *yd.*? What by selling it at 12 c.? At 11 *c.*? At 10 *c.*?

6. Sold flour at $8 per *bbl.*, thereby making 10% on cost. What was the cost?

Analysis.—As I made 10%, each $1 of cost was sold at $1.10. Hence what sold for $8 cost as many dollars as $1.10 is contained times in $8. $8 \div 1.10 = 7.27\frac{3}{11}$. Therefore the flour cost $$7.27\frac{3}{11}$.

7. By selling sugar at $10\frac{1}{2}$ *c.* per *lb.* I made 5%. What did it cost me?

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8. Principal $576, rate per cent 6, time 2 *yr.* 3 *mo.* What is the interest?

9. Principal $576, interest $77.76, time 2 *yr.* 3 *mo.* What is the rate per cent?

Analysis.—At 1% for 2 *yr.* 3 *mo.* the interest on $576 is $12.96. But $77.76 is 6 times $12.96, which we learn by dividing $77.76 by $12.96. Hence, as the principal yields 6 times the interest it would at 1%, the rate per cent is 6.

10. Principal $576, interest $77.76, rate per cent 6, what is the time?

Analysis.—In 1 *yr.* at 6% $576 yields $34.56 interest. Hence it requires as many years to yield $77.76 as $34.56 is contained times in $77.76, *i. e.*, $2\frac{1}{4}$ years, or 2 *yr.* 3 *mo.*

11. Interest $77.76, % 6, time 2 *yr.* 3 *mo.* What is the principal?

Analysis. $1 principal yields $0.135 interest in 2 *yr.* 3 *mo.* at 6%

Now to yield \$77.76 requires as many dollars principal as \$0.135 is contained times in \$77.76, which is 576. Hence the principal is \$576.

12. Amount \$102.81 at 10% for 3 *yr.* 9 *mo.* 18 *da.* What is the principal?

Analysis. \$1 principal at 10% for $3\frac{4}{5}$ years yields \$1.38 amount. Now to yield \$102.81 requires as many dollars principal as \$1.38 is contained times in \$102.81. $102.81 \div 1.38 = 74.50$. Hence the principal is \$74.50.

Note.—The student should not fail to notice that the general argument is the same in all the cases, viz., *Find the effect produced by* 1 *of the thing required, under the given circumstances, and compare this with the given effect.*

13. Amount \$102.81, on \$74.50 at 10%. What is the time?

The interest is \$102.81 − \$74.50 = \$28.31, the *given effect.* The thing required is *time.* The interest on the principal, \$74.50, for 1 *yr.* at 10%, is the effect produced by 1 of this kind.

14. Amount \$102.81, on \$74.50 for 3 *yr.* 9 *mo.* 18 *da* What is the rate per cent? Same as *Ex.* 9.

15. At what per cent will \$75 yield \$28.125 in 6 *yr.* 3 *mo.*? At what % will it yield \$15.30 in 2 *yr.*?

16. How long does it take \$750 to amount to \$942, at 6%? How long at 5%? At 3%?

17. What principal yields \$150 at 4% in 7 *yr.* 2 *mo.* 15 *da.*? In 3 *yr.*? In $5\frac{1}{2}$ *yr.*?

18. In order to secure an annual income of \$3500, what amount must I invest at $4\frac{1}{2}$%? At 6%? At $7\frac{3}{10}$%?

19. How long does it take \$100 to double itself at 5%? At 6%? At 7%? At 10%? How long \$300?

20. A man borrowed a sum of money at 15%, and it ran

till the amount was 3 times as much as he borrowed. How long did it run? *Ans.*, 13 *yr.* 4 *mo.*

21. What % of $\frac{1}{3}$ is $\frac{2}{5}$? \$10.50 of \$50? 5 of 11?

22. I made \$235 by selling goods at 5% commission What amount did I sell?

23. What amount of commission business must I do per year to give me an income of \$4500, if the commission on half the business is 5%, and on the other half 7%?

Ans., \$75000.

24. Borrowed \$100 at 7%, and it ran till it amounted to \$182.50. How long did it run?

25. At 10% per annum in what time will 1*c.* amount to \$1?

26. One class of U. S. bonds bore annual interest at $7\frac{3}{10}$%. What was the daily interest on a \$100 bond of this class?

27. Gold to-day is quoted at $113\frac{1}{2}$,* *i. e.*, at $13\frac{1}{2}$% premium. What then is a \$100 U. S. bank-note worth in gold?

Ans., \$88.10$\frac{1}{2}$+.

28. When gold is 112, what is a \$50 National bank-note worth? What when gold is 110? When 100?

29. I have a 4% U. S. bond which yields me annually an amount in gold which at $113\frac{1}{2}$ is equivalent to \$22.70 in National bank notes (called "currency" at the banks). What is the face of the bond? *Ans.*, \$500.

The interest on such a U. S. bond is payable quarterly, but no allowance is made for this fact in the above, and the following:

30. Gold being 112, what amount of currency must I invest in U. S. 4% bonds at 111, to yield me an annual income of \$4000 in currency? *Ans.*, \$99107.14+.

* The meaning of this is that it takes \$1.13$\frac{1}{2}$ of paper currency to buy \$1 gold.

31. When gold is quoted 105, what is the value of a \$1 greenback?

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32. New York *Exchange* being quoted at $\frac{1}{10}\%$ premium in St. Louis, what will a firm in the latter city have to pay for a draft of \$2500 on a N. Y. bank?

Ans., \$2502.50.

277. A *Draft* is an order drawn in one place (city) and payable in another. Drafts are usually issued by banks. When the two places are in different countries the order is called a *Bill of Exchange.*

33. Living in Detroit I owe a firm in St. Louis \$6,250. I propose to pay them by a draft on a N. Y. bank, which will be worth $\frac{1}{10}\%$ premium in St. Louis. What must be the face of the draft? How much will the draft cost me in Detroit if N. Y. Exchange is quoted at $\frac{1}{20}\%$ premium in Detroit?

Ans., \$6243.76; \$6246.88.

278. Drafts which are to be paid when they are presented are called *Sight Drafts. Time Drafts* are such as are to be paid a certain specified time after date.

34. What is a \$1200 St. Louis draft at $^{30}/_{33}$ *da.*, on New York worth, N. Y. exchange being at 101, and the time discount being at 3%?

The nature of this transaction is that a man in St. Louis buys at a bank there a a draft on N. Y. for \$1200. Since N. Y. exchange is at 101, *i. e.*, at 1% premium, if the draft were a *sight* draft it would cost him $\$1200 \times 1.01 = \$1212$. But inasmuch as the bank in N. Y. will not have to pay the draft till $^{30}/_{33}$ *da.* after its date, they will not charge the St. Louis bank with it till they pay it. Hence the

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* That is, a draft nominally payable 30 *da.* after date; grace is always reckoned.

St. Louis bank will have the use of the money $^{30}/_{33}$ *da.* before it will be charged to them in N. Y., *i. e.*, before they have to pay it. Therefore they allow 3% discount on *the face of the draft* for the use of the money. 3% of $1200 for $^{30}/_{33}$ *da.* is $3.25.* Hence the purchaser of the draft pays $1212—$3.25, or $1208.75, or $1208.77 including stamp.

35. A merchant in Boston wishes to pay $980 in Milwaukee. Required the cost of a draft, payable in 60 days, exchange being at 1¾% discount, the bank, or time discount being 4%? *Ans.*, $956.10+.

The nature of this transaction is as follows:

The Boston merchant steps into a bank and buys a draft on a Milwaukee bank. Milwaukee exchange being at discount in Boston, a sight draft for $980 could be bought for $980 less 1¾% of $980, or for $962.85. But as the Boston bank will have the money $^{60}/_{63}$ *da.* before it will be charged to them by the Milwaukee bank, and as the Boston merchant will have paid the money $^{60}/_{63}$ *da.* before his creditor in Milwaukee receives it, it is but right that the Boston bank should allow the merchant for the use of the money. Hence they allow him 4% on $980 for the $^{60}/_{63}$ *da.*, *i. e.*, $6.766. This deducted from $962.85 leaves $956.084, and adding the 2c. for the stamp, the cost of the draft is $956.10+.

The justice of such an arrangement will appear very clearly if we suppose that the debt in Milwaukee is drawing interest. Now no interest will be stopped until $^{60}/_{63}$ *da.* after the debtor has paid his money, hence the party which has the money these 63 days should pay interest on it.

36. Exchange being at 1½% premium, and time discount at 5%, what is the cost of $1 exchange on draft for $^{60}/_{63}$ *da.*? What of a draft for $500? Of a draft of $450 for $^{30}/_{33}$ *da.*? Of a draft of $1000 for $^{90}/_{93}$ *da.*?

37. What will it cost a merchant in New York to pay $1000 in Fort Dodge, Iowa, if he buys a N. Y. draft for

* Reckoning 365 *da.* to a year.

$^{90}/_{93}$ *da.*, at 5%, N. Y. exchange being at $\frac{1}{5}$% premium in Ft. Dodge? What will be the face of the draft?

Ans., Cost of draft $985.29; Face, $998.00.

The *face of the draft* must be such that $\frac{1}{5}$% premium on it will make $1000. On this, *that is, on the face of the draft*, the N. Y. bank will allow bank discount at 5% for $^{90}/_{93}$ *da.*

38. Exchange $\frac{3}{8}$% premium, as above, time discount 6%, what is the cost of a 90 *da.* draft which will pay $1000? What if the exchange be at $\frac{3}{8}$% discount instead of premium? What the face of the draft in each case?

Ans., Cost $981.03, Face 996.26; Cost $988.42, Face $1003.76.

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39. I find in my daily newspaper that the N. Y. Central R. R. stock is selling at $89\frac{3}{4}$, and that the road is paying 2% quarterly dividends. What % annual interest is this on an investment, allowing the use of money to be 5%?

279. *Stocks* are certificates of ownership in some property or business. They are usually reckoned by *Shares* of $100 each. Thus if the capital of a bank is $200,000, and I own 50 shares of the stock, I own $\frac{5,000}{200,000}$, or $\frac{1}{40}$ of the capital, and hence am entitled to $\frac{1}{40}$ of the net profits of the business.

When stock is quoted at $89\frac{3}{4}$, it means that you can buy it for $\$89\frac{3}{4}$ per share, and it is then said to be *below par*. The *Dividends* are the profits paid to the stockholders.

In our example the meaning is that I can buy a share (nominally $100) for $\$89\frac{3}{4}$, and that this will yield me $2 at the close of each quarter during the year. Thus, allowing 5% on the first three payments from the time they are made to the end of the year, an investment of $\$89\frac{3}{4}$ yields me $8.15 for the year, *i. e.*, I obtain a little over 9% on my money.

40. If I buy stocks at 120 which pay 6% semi-annual dividends, what % does my investment yield annually, allowing money to be worth 10% interest?

Ans., $10\frac{1}{4}$%.

41. Wishing to invest in bank stock I send $10,000 to a broker in New York, which he invests for me in "Pacific" (*i. e.*, stock of the Pacific Bank), at 130, charging me $\frac{1}{4}$% brokerage* (commission). How many entire shares do I get, how much money is returned to me, and what % annual interest does my investment yield if the bank pays 4% quarterly dividends, allowing 5% for the use of money?

Ans., 76 shares, yielding $12\frac{1}{2}$%+, and $101 returned.

42. Wishing to make a permanent investment in stocks which are quoted at 120, and which pay 3% quarterly dividends, so that I shall have a quarterly income of $600, how much currency will it cost me if I remit to a broker in New York, N. Y., exchange being $\frac{1}{10}$% premium where I live, and the brokerage being $\frac{1}{8}$%?

Ans., $24064.04.

43. I find that U. S. 5–20 coupon bonds of 1868† are quoted to-day as selling in N. Y. at $113\frac{1}{4}$. I have $10,000 to invest; N. Y. exchange is $\frac{1}{10}$% premium, and brokerage $\frac{1}{4}$%. What amount of bonds can I secure, remembering that the smallest bond is $50? What will my bonds cost me in currency?

Ans., $8750 in bonds, costing $9963.08.

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* The main distinction between the words "brokerage" and "commission" is that the latter term is general, while the former is applied to commissions on stocks or monetary transactions.

† Such bonds are also referred to as 5-20's of '88, thus distinguishing them by the date of their maturity, instead of their issue.

280. U. S. *Bonds* are the government certificates of indebtedness (like notes). *Coupons* are certificates of interest attached to the bonds, to be cut off and presented for payment as the interest falls due.

5–20's of 1868 are bonds dated in 1868 and payable any time between 5 years and 20 years after date, at the option of the government. These bonds bear 6% interest, payable semi-annually in gold.

44. What % annual interest in currency would such an investment as that in Ex. 42 yield me, gold being quoted at 105, allowing 10% on the mid-year payments till the end of the year? *Ans.*, About $5\frac{7}{10}$%.

The $8750 in bonds would yield $262.50 in gold, or $275.625 in currency semi-annually. Allowing 10% on the mid-year payment makes the annual receipt from the bonds $565.03.

45. What % on my money do I get if I invest in 8% city bonds, *interest to seller*,* at 107?

Ans., $\frac{800}{107}$%, or a little less than $7\frac{1}{2}$%.

In strict propriety the interest on the interest allowed the seller (see foot-note), from the time of sale to the time the interest on the bond falls due, should be added to the quotation (107).

46. If I buy 10% city bonds dated May 1, on Nov. 1, *interest to buyer*, at 113, what % do I make on my investment?

At the time of purchase there is 5% of interest due on the bonds. As this comes to me it refunds me $5 on $100 of bonds, of the $113 which I pay (saying nothing about the interest on this $5 for the 6 *mo.* before I receive it). Hence the bonds

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* This means that the interest due on the bond at the time of the sale is to go to the seller. Thus, if the bond is dated Jan. 1, and I buy it June 30, I allow the seller 4c. on a dollar for interest then due. This I get back at the end of the year, when the interest is paid to me.

really cost me 108, and my per cent. is $\frac{1000}{108}$, or a trifle over $9\frac{1}{4}\%$.

47. Same questions as the last, considering the interest on the interest as indicated in the suggestions at 10%.

Ans., $\frac{1000}{108.2381}$, or a little less than $9\frac{1}{4}\%$.

This interest on interest accrued is such a trifling element in its influence on the % which an investment yields that it is not necessary to consider it.

48. If I buy a $500 $7\frac{1}{2}\%$ county bond, 9 *mos.* after its date, at 105, interest to seller, what do I pay for the bond? What % does my investment yield? What if I buy at 110, interest to buyer?

Ans. $553.13, $\frac{3750}{525.00}\%$, or a little more than 7%; $550 $\frac{750}{104.375}\%$, or nearly $7\frac{1}{5}\%$.

Government bonds are always sold "interest to buyer." Other bonds are made subjects of special contract in this respect, but are usually quoted "interest to seller."

Find the % yielded by investments as follows, and the amount paid for a $1000 bond:

49. U. S. 5–20's,* @ 113, gold at $104\frac{3}{4}$.

50. U. S. 5–20's, @ 115, gold at 106.

51. State 7%, at 98, interest annually in currency.

52. City 8%, @ 106, interest to seller, at 3 *mo.* after date. At 108, 6 *mo.* after date. At 109, 9 *mo.* after date.

53. Township 10%, at 109, interest to seller, 6 *mo.* after date. At 110, 9 *mo.* after date.

For examples illustrating Custom House work, see pp. 252, 253.

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* Interest 6%, payable semi-annually, in gold.

SECTION VI.

AVERAGING ACCOUNTS.

1. C. H. Millen & Co., of Ann Arbor, Mich., dry goods merchants, have the following account with a merchant in New York:

Dr. C. H. MILLEN & Co. *Cr.*

1875.				1875.			
June 5.	Mdse.	$400	00	July 1.	Cash.	$350	00
Aug. 12.	" 3 *mo.*	$600	00	Aug. 25.	Draft at 60 *da.*	$500	00
Sept. 8.	" 6 *mo.*	$500	00	Sept. 10.	Cash.	$600	00
Sept. 20.	" 4 *mo.*	$1000	00	Sept. 15.	Draft at 30 *da.*	$250	00
		$2500	00			$1700	00
		$1700	00				
	Balance.	$800	00				

When should Millen & Co. pay this $800 balance in justice to both parties?

First, observe the latest date at which any item in this account falls due. This is Mar. 8, the time when the 3d item on the debtor side falls due.

Second, notice when the Debit items fall due, and how many days before Mar. 8, and the interest which would accrue on them up to that date, at *any convenient per cent.**

June 5, 1875, $400, 276 *da.*, . . . before Mar. 8.
Nov. 12, " $600, 116 *da.*, . . . " "
Jan. 20, 1876, $1000, 47 *da.*, . . . " "
Mar. 8, " $500, 0 *da.*, hence due Mar. 8.

The interest which would accrue on these up to Mar. 8, 1876, at 10%, is $62.19.

* This does not imply that the items are on interest; the fact is that they are not—but it is a convenient way of adjusting the matter so that the amounts and times of credit shall be equalized for both parties.

Third, we treat the credit side of the account in a like manner. Thus there were payments as follows:

July 1, 1875,	$350,	250 *da.*,	. . .	before	Mar. 8.
Sept. 10, "	$600,	179 *da.*,	. . .	"	"
Oct. 18, "	$250,	141 *da.*,	. . .	"	"
Oct. 27, "	$500,	132 *da.*,	. . .	"	"

The interest which would accrue on these sums up to March 8, at 10%, is $81.135.

This leaves a balance of $81.135 — $62.19, or $18.945 *in favor* of the debtor, Millen & Co.

Hence, finally, the question becomes, How long can the balance of the account, $800, be retained by the debtor in order to offset this interest, *i. e.*, how long will it take $800 at 10% to yield $18.945? This is 86 *da.*; and the equated time is June 2, 1876.

281. The **Equated Time** is the equitable date for the payment of several obligations maturing at different dates.

282. Averaging an Account is finding the equated time for the payment of the balance, or finding the cash balance to be paid at some given time.

283. The following rule covers all cases of Equation of Payments and Averaging Accounts:

Rule.—*Find the interest which would accrue on each obligation (at any rate per cent, when none is named), from its maturity to the most remote maturity. Then ascertain how long it would take the sum of the obligations, or the unpaid balance, to produce the sum of these interests, or the balance of interest, at the same rate per cent. Subtract this time from the date of the most remote maturity, or add it as the case may require, and the result will give the Equated Time.*

Average the following accounts:

2. *Dr.* PHILIP BACH. *Cr.*

1876.				1876.			
May 6.	Mdse.	$340	00	June 1.	Cash,	$250	00
July 10.	"	$500	00	Aug. 10.	"	$400	00
Nov. 16.	"	$800	00				

If the computer has tables from which to take the interest he may take any rate he chooses. If he has not tables, the simplest way is to multiply each item by the number of days, which is equivalent to reckoning 100% per day.

In this manner the computation of the last becomes

$$\begin{array}{r} 340 \times 194 = 65960 \\ 500 \times 129 = 64500 \\ \underline{800} \times \quad 0 = \underline{00000} \\ \overline{1640} \qquad 130460 \\ \underline{650} \qquad \underline{81200} \\ 990 \qquad 49260 \end{array} \qquad \begin{array}{r} 250 \times 168 = 42000 \\ \underline{400} \times \ \ 98 = \underline{39200} \\ 650 \qquad 81200 \end{array}$$

$49260 \div 990 = 50$ nearly. Hence the equated time is 50 *da.* *before* Nov. 16, *i. e.*, Sept. 27.

NOTE.—As the balance of this account, $990, was due Sept. 27, but was not then paid, legal interest should be reckoned upon this balance from this time to the time of the settlement. This date may be called the *Average Date.*

3. *Dr.* J. SMITH in Acct. with M. BROWN. *Cr.*

1876.					1876.			
Jan. 10	Mdse.		$650	00	Feb. 1.	Draft at 30 *da.**	$1400	00
Feb. 20.	"	30 *da.**	$240	00	Mar. 10.	" " 60 *da.*	$200	00
Mar. 15.	"	60 *da.*	$320	00	May. 1.	Mdse.	$1650	00
Apr. 21.	"	30 *da.*	$180	00				

Ans., Apr. 30.

* Reckon *grace* on checks, but not on the Mdse. items.

4. What would be the equated time for the payment of the debtor items of the last account were there no credit items? *Ans.*, Mar. 9.

5. What would be the cash balance of the account in Ex. 1, if paid by C. H. Millen & Co. July 1st? What if paid April 1st? *Ans.*, $804.45; $790.49.

Instead of common discount in such cases, the custom is to deduct the interest for the time. This is the same as Bank Discount without the grace.

6. Suppose Mr. Bach's account, Ex. 2, to be in Boston, what balance would he pay Nov. 16? What amount of cash would have made the account balance had it been paid Aug. 15? *Ans.*, $998.14; $983.

7. What was the cash balance of the account in Ex. 3, June 1? What Mar. 15? (Interest 6%.)

Ans., $1869.78; $1846.04.

NOTE.—This old method of Averaging Accounts is little used at the present time. Some wholesale dealers will average an account if the equated time is kept within certain limits, say 4 *mo.*; but an unlimited average is never practiced. By such a process it is easy to see that a remote country dealer could "gain time," and then carry a debt of a thousand or two dollars for a year or more. The examples on page 295 illustrate the more common practice of large dealers.

8. A farmer purchases of a commission merchant 20 tons of guano at $60 a ton, and is to be allowed a credit of 90 days. On condition of paying ¼ of the money at present, the merchant proposes to grant an adequate extension of time for the payment of the remainder; at what time should the remainder be paid? *Ans.*, 120 days.

9. B borrowed of W $675 for 4 months; what sum of money ought B to lend W for 3 months as an equivalent?

Ans., $900.

CHAPTER VI.

RATIO AND PROPORTION.

RATIO.

284. Ratio is the quotient of one number divided by another.

Thus the ratio of 12 to 4 is 12 ÷ 4, or 3. The ratio of 5 to 7 is 5 ÷ 7, or $\frac{5}{7}$.

Note.—If the numbers are concrete they must be of the same kind, since we cannot divide one concrete number by another of a different kind. Thus the ratio of $10 to $5 is 2; but to ask "What is the ratio of $10 to 5 miles," is absurd.

285. The first number named is called the **Antecedent,** and the second the **Consequent.** The two together constitute the *Terms* of the ratio, or a *Couplet.*

286. The ratio between two numbers is indicated by writing the antecedent before the consequent and the colon (:) between them; or by writing the *antecedent* as the *numerator* of a fraction and the *consequent* as the *denominator.*

Thus 8 : 4 is read "the ratio of 8 to 4;" so also $\frac{8}{4}$ may be read "the ratio of 8 to 4," both forms meaning exactly the same thing.

287. The term *Ratio* is also applied to such forms as 6 : 2, $\frac{4}{5}$, etc., that is, to the indicated operation of division, the sign : being an equivalent of ÷.

Thus we speak of the ratio 6 : 2 (not the ratio *of* 6 : 2), the ratio $\frac{4}{5}$ reading "the ratio 6 to 2," "the ratio 4 to 5." The ratio *of* 6 to 2 is

3, or the *value* of the ratio 6 : 2 is 3. So the ratio *of* 4 to 5 is $\frac{4}{5}$ (4-fifths), or the *value* of the ratio $\frac{4}{5}$ (read "4 to 5") is 4-fifths.*

1. What is the ratio of 15 to 3? 8 : 2? 9 : 3? 10 : 2? 5 : 7? 4 : 8? 1 : 3? 3 : 1? 7 : 11? 11 : 7?

2. Which is greater, 12 : 3, or 8 : 4? 6 : 3, or 9 : 3? 5 : 6, or 7 : 8? 2 : 3, or 5 : 4? 10 : 5, or 6 : 3? 5 : 8, or 15 : 24? $\frac{4}{8}$, or $\frac{3}{6}$? $\frac{7}{11}$, or $\frac{3}{5}$?

3. Mention several ratios which are each equal to 15 : 3. Several which are equal to $\frac{6}{8}$. To $\frac{2}{3}$. To 3 : 5.

Principle.

288. *A ratio has all the properties of a common fraction with the antecedent for its numerator and the consequent for its denominator.*

4. What effect does it produce on a ratio to *multiply* the antecedent by 2? by 3? by any number? Try it on 2 : 4. What effect to *divide* the antecedent by any number? Try it.

5. What effect does it produce on a ratio to multiply its consequent? To divide its consequent? Try it.

6. How do you compare two common fractions to ascertain which is the greater? (p. 152). How then do you compare two ratios?

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7. If 24 is the antecedent and 4 the ratio, what is the consequent?

Having the antecedent and ratio given, how do you find the consequent?

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* This double use of the word ratio has given no little trouble to students. That the word is habitually used by mathematicians in both of these ways no one at all conversant with mathematical writing can doubt. Thus when we ask "What is the ratio of 12 to 4?" all answer "3;" and all with equal unanimity speak of the ratio a : b—"the ratio *a* to *b*."

8. If 3 is the consequent and 7 the ratio, what is the antecedent? If 45 is the consequent and the ratio $\frac{1}{3}$?

9. If 7 is the antecedent, what is the consequent when the ratio is $\frac{7}{3}$? When it is $\frac{2}{3}$? When it is 6?

10. If 28 is the antecedent, what is the consequent when the ratio is 7? When it is 4? When it is 14? When it is $\frac{2}{3}$? When it is $\frac{4}{3}$?

11. Antecedent 10, ratio 2, what is the consequent? Antecedent 27, ratio 9? 3? $\frac{1}{2}$?

PROPORTION.

289. Proportion is an equality of ratios, the terms of the ratios being expressed. The equality is indicated by the ordinary sign of equality (=), or by the double colon (: :).

Thus 8 : 4 = 12 : 6, or 8 : 4 : : 12 : 6 is a proportion. It is read "8 is to 4 as 12 is to 6." The expression $\frac{8}{4} = \frac{12}{6}$ may be read in the same way, and means the same thing.

290. Two ratios, at least, are required for a proportion; hence we have two antecedents and two consequents. Of four terms which constitute a proportion, the 1st and 4th are called **Extremes**, and the 2d and 3d **Means**.

1. Is 15 : 3 : : 10 : 2 a true proportion? What is the ratio of 15 to 3? What of 10 to 2? Are they equal?

2. Show which of the following are true proportions:

1. 20 : 5 : : 8 : 2	7. 10 : 7 : : 20 : 14
2. 2 : 12 : : 5 : 30	8. 3 : 7 : : 12 : 26
3. 5 : 35 : : 8 : 64	9. 13 : 27 : : 117 : 243
4. 2 : 3 : : 14 : 21	10. $2\frac{1}{2}$: 5 : : $3\frac{1}{3}$: $6\frac{2}{3}$
5. 7 : 11 : : 35 : 55	11. 1.05 : 8.4 : : 1 : 8
6. 8 : 3 : : 16 : 9	12. .05 : 7 : : .3 : 42

3. If the first 3 terms of a proportion are 18 : 6 : : 21 what is the 4th term?

The ratio of 18 to 6 is 3; hence the 4th term must be $\frac{1}{3}$ of 21, so that the ratios may be equal. Is 18 : 6 : : 21 : 7 a true proportion? Why?

4. Find the 4th term of 7 : 3 : : 5 : —.

What is the ratio of 7 : 3? Then if 5 is the antecedent and $\frac{3}{7}$ the ratio, what is the consequent? *Ans.*, $\frac{15}{7}$ or $2\frac{1}{7}$. Is 7 : 3 : : 5 : $2\frac{1}{7}$ a true proportion? Why?

5. Find the lacking term of 12 : — : : 8 : 6.

Which ratio is given? What is the ratio of 8 : 6? If 12 is the antecedent and $\frac{3}{4}$ the ratio, what is the consequent?

6. Find the lacking term of 13 : 7 : : — : 11.

We have the ratio of 13 : 7, $\frac{13}{7}$. Hence the lacking term is $11 \times \frac{13}{7}$, or $\frac{143}{7} = 20\frac{3}{7}$.

7. Find the lacking term of — : 43 : : 5 : 17.

The given ratio is $\frac{5}{17}$. Hence we have $43 \times \frac{5}{17} = \frac{215}{17} = 12\frac{11}{17}$. Is $12\frac{11}{17}$: 43 : : 5 : 17 true?

Principle.

291. *The product of the means of a proportion is equal to the product of the extremes.*

This is evident, since the 1st *mean* is the 1st *extreme divided* by the ratio, and the 2d *mean* is the 2d *extreme multiplied* by the ratio. Hence the product of the means is $\frac{\text{1st } Extreme}{Ratio} \times$ 2d *Extreme* $\times$ *Ratio*. In this the Ratio cancels and leaves the product of the Extremes.

8. Find by means of this principle the lacking term in 13 : 5 : : 12 : —.

The two means being given, we know their product, 60. But this is also the product of the extremes. Now if 60 is the product of the extremes and 13 is one of the extremes, the other is $\frac{60}{13}$, or $4\frac{8}{13}$.

9. In like manner find the lacking term in each of the following, giving the explanation:

1. 2 : 7 : : 5 : —	7. 23.05 : 4.5 : : 7.1 : —
2. — : 4 : : 11 : 6	8. 1.05 : 342 : : 100 : —
3. 5 : 12 : : — : 8	9. 42 : 6 : : — : 30
4. 34 : — : : 17 : 16	10. $\frac{2}{3}$: $\frac{4}{5}$: : $\frac{5}{8}$: —
5. 131 : 47 : : 1.5 : —	11. 112 : 16 : : 49 : —
6. $12\frac{1}{2}$: $6\frac{2}{3}$: : 8 : —	12. $11\frac{3}{5}$: $2\frac{1}{7}$: : 4 : —

RULE OF THREE.

292. The **Rule of Three** is an old term applied to the method of solving problems in which *Three Terms* of a proportion are given and the fourth is to be found.

1. If 8 yards of a certain kind of cloth cost $35, how much will 42 yards of the same cloth cost?

It is evident that the same ratio exists between the cost of the two quantities, as between the quantities, since the price per yard is the same. Hence the ratio of 8 *yd.* to 42 *yd.* is the same as the ratio of the cost of 8 *yd.*, $35, is to the cost of 42 *yd.* Stated as a proportion this is 8 *yd.* : 42 *yd.* : : $35 : the cost of 42 *yd.*

We have therefore to find the 4th term of the proportion 8 : 42 : : 35 : —. This is $\frac{35 \times 42}{8}$ = $183.75. Hence 42 *yd.* will cost $183.75.

2. If it require 12 *bbl.* of flour per year for a family of 10, how many barrels will it require for a family of 6?

The proportion is 10 : 6 : : 12 : —. Hence we have $\frac{6 \times 12}{10} = 7\frac{1}{5}$. Let the pupil give the reasons: 1st, for the statement of the proportion; 2d, for the method of finding the 4th term.

3. If 14 cords of wood cost $98, what will 32 cords cost?

Ans., $224.

Note.—In such questions there are always 3 quantities given, and *two of these are of the same kind.* Now it is necessary that we determine from the nature of the case, whether the same ratio (relation) exists between the other two, one of which is *not* given, as exists between the two which are given. Not every problem in which three terms are given and a 4th required can be solved by proportion.

4. If 12 acres yield 384 *bu.* of wheat, what will 36 acres yield at the same rate?

What are the two quantities of the same kind? Does the same relation (ratio) exist between the quantities of wheat produced as between the quantities of land?

5. If the interest on $350 at a certain rate and for a certain time is $65, what is the interest on $700, for the same time and rate?

Will *twice* as great a principal give *twice* as much interest at the same rate and for the same time? What are the two like terms? Are they not all alike? They are all *dollars.*

6. If $100 principal amounts to $122.50 for a certain rate and time, what principal does it require to amount to $422.62½ for the same rate and time?

Amount.	*Amount.*	*Principal.*	*Principal.*
$122.50 :	$422.62½ : :	$100 :	$\frac{42262.5}{122.5}$ (= $345).

Note.—This is the question in True Discount.

7. What is the present worth of a note $422.62½, due 2 *yr.* 3 *mo.* hence, without interest, money being worth 10%?

Same as Ex. 6. $100 *now* amounts to $122.50 at the given rate and time. Hence the question is, "How much *now* will it take to amount to $422.62½ in the given time at the given rate.

8. What is the present worth of a note which amounts to $350, 1 *yr.* 6 *mo.* 15 *da.* hence, money being worth 7%?

Find the amount of *any sum* for the given time and rate ($100 is convenient). Then state and work the proportion.

9. What is the present worth of $725.50, due in 6 *yr.* at 8%? *Ans.*, $490.20 +.

10. What is the present worth of $2000, due in 3 *yr.* 6 *mo.*, interest at 7%? *Ans.*, $1606.43.

The examples in p. 302, *et seq.*, can be solved by proportion, and should be reviewed and solved in this way.

11. A bankrupt paid 43 cents on every dollar of his debts. How much did he pay on a debt of $569.31?

12. A difference of 15° in longitude, makes an hour's difference in time. What is the difference in time between Boston, which is 71° 4′ 20″ W. long., and Washington, which is 77° 1′ 30″ W. long.?

Ans., 23 *min.* 48⅔ *sec.*

13. Murray's Cough Mixture is composed of

Paregoric . . .	ʒ iv,
Sulphuric Ether .	ʒ ij,
Tinc. Tolu . . .	ʒ ij.

How much paregoric in a dose of 1 teaspoonful (*i. e.* ʒ j)?

14. Reece's Chirayta Pills are composed of

Carbonate of Soda .	℈j,
Ginger	*gr.* xv,
Extract of Chirayta .	ʒ ij.

This amount makes 36 pills, of which 2 is a dose. How much Chirayta in a dose?

15. Allowing a person to perform a certain journey in 13½ days, by traveling 10 hours a day, in what time ought he to perform the journey if he travel 11¼ hours per day?

Ans., 12 days.

16. Moll and Van Beek, in 1823, found that sound travels 332.05 meters (1 meter = 39.37 *in.*) in a second. What is the velocity per second in feet?

17. How far off is a battery when the flash precedes the report 15 *sec.*, no allowance being made for the progressive motion of light?

18. It is found that the eclipses of Jupiter's moons occur 16 *min.* 26.6 *sec.* sooner when the earth is on the side of her orbit nearest Jupiter than when she is on the opposite side. The diameter of the earth's orbit being 183,000,000 miles, what is the velocity of light per second?

19. How many times would light go round the earth in a second, the earth's circumference being called 24,000 miles?

20. The nearest of the fixed stars are probably 100,000,000,000,000 miles from the earth. How long would one of them continue to be seen on the earth after it was annihilated, were such a thing possible?

21. Divide 384 into two parts which shall be to each other as 15 to 17. As 5 to 8. As 2 to 5.

22. Divide 135 into 3 parts which shall be to each other as 3, 7, and 5. As 2, 3, and 4.

23. If a staff 5 *ft.* long casts a shadow 3 *ft.*, how high is a steeple whose shadow at the same time is 90 *ft.*?

The examples in p. 302, *et seq.*, afford excellent exercise in the Rule of Three, many preferring this method of solving them. They can be reviewed in this manner, if desired. See also TEACHER'S HAND BOOK.

CHAPTER VII.

POWERS AND ROOTS.

POWERS.

293. A **Power** is the product arising from multiplying a number by itself a certain number of times.

Thus $3 \times 3 \times 3 = 27$; and 27 is called the 3d power of 3. $5 \times 5 = 25$; and 25 is the 2d power of 5. So $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$; and $\frac{8}{27}$ is the 3d power of $\frac{2}{3}$. The 2d power of .4 is .16. Of 1.2 is 1.44.

294. The **Square** of a number is its 2d *power;* and the **Cube** of a number is its 3d *power.* The number itself is called the first power, or the *Root.*

1. What is the square of 4? Of 7? Of $\frac{1}{2}$? Of $2\frac{1}{3}$? Of .35? The cube of 2? Of 10? Of 6? Of $1\frac{2}{3}$?

2. Write the squares and cubes of the 9 digits, and commit them to memory thoroughly.

295. A *Whole Number* written at the right and a little above another number indicates the power of that number. It is one form of what is called an *Exponent.*

Thus $4^2 = 4 \times 4$, or 16. $2^3 = 2 \times 2 \times 2$, or 8. $3^4 = 3 \times 3 \times 3 \times 3$, or 81. $.3^5 = .00243$. $1.1^3 = 1.331$.

3. What is the 4th power of 6? Of 10? Of 1.5? Of $\frac{1}{2}$? Of .2? Of 9? Of $\frac{2}{3}$? Of $2\frac{2}{3}$? Of 1.07?

4. What is the value of these expressions: 2.1^2? 12^3? 7^4? $(\frac{2}{3})^2$? $.051^2$? $.01^3$? 1.01^2? 246^2? 10^5? 100^4?

5. How many figures in the square of 1? Of 10? Of 100? Of 1000? Of 9? Of 99? Of 999? Of 458?

Principle I.

296. *The square of any number contains twice as many figures as the number itself, or 1 less than twice as many.*

Principle II.

297. *The cube of any number contains three times as many figures as the number itself, or 1, or 2, less.* Try it.

6. What is the *least* number that can be represented by 2 figures? How many figures in its square? In its cube? What is the *greatest* number that can be represented by 2 figures? How many figures in its square? In its cube?

In this way show the truth of the two preceding principles.

7. In what order or orders does the square of units fall? The square of tens? Of hundreds? Of thousands?

The square of units will either occupy units place or units and tens places. So the square of tens (as of 10, or 90) will fall in hundreds order, or in hundreds and thousands orders. The square of hundreds falls either in ten-thousands order, or in ten-thousands and hundred-thousands orders.

8. Show as above in what orders the *cube* of any order, as of tens, hundreds, or thousands, falls?

Principle III.

298. 1. *By separating a number into periods by placing a point over the units, and then one over each alternate figure therefrom, we mark the periods in which the* SQUARES *of the successive orders of the root fall.*

2. *By separating a number into periods by placing a point over the units figure, and then one over each 3d*

figure therefrom, we mark the periods in which the CUBES *of the successive orders of the root fall.*

The number of periods indicates the number of figures in the corresponding root, if the number is an exact power, or in the highest exact power in it, if it is not.

Thus the *square* root of 5873246 has 4 figures, and the square of the highest order in the root is in 5. So the *cube* root of 48265432 has 3 figures in it, and the cube of the highest order in the root is in 48.

Principle IV.

299. *The square of any number made up of tens and units is the square of the tens, + twice the product of the tens by the units, + the square of the units.*

Let us show this by squaring 68. Multiplying in the ordinary way, but writing each product by itself, we see at once that the square of 68 is $(6 \text{ tens})^2 + 2\,(6 \text{ tens} \times 8) + 8^2 = 4624$.

$$\begin{array}{rr} & 68 \\ & \underline{68} \\ 8^2 = & 64 \\ 6 \text{ tens} \times 8 = & 48 \\ 8 \times 6 \text{ tens} = & 48 \\ (6 \text{ tens})^2 = & \underline{36} \\ & 4624 \end{array}$$

But it is necessary to prove this truth in a more general way, as it is the foundation of the very important *Rule for the Square Root*, which we are soon to learn.

For this purpose, instead of using 8 for the units and 6 for the tens of the number we wish to square,

let u stand for *any number* of units,
and t for *any number* of tens.

Then as 68 is 6 tens + 3 units, our number will now be represented thus, $t + u$.

We will now square $t + u$, observing, first, that tu means the same as $t \times u$, and that $2tu$ means twice tu. We multiply first by u, saying "u times u is u^2, and u times t is tu." Hence $t + u$ multiplied by u is $tu + u^2$ (see

$$\begin{array}{r} t + u \\ \underline{t + u} \\ tu + u^2 \\ \underline{t^2 + tu } \\ t^2 + 2tu + u^2 \end{array}$$

58). So t times $t+u$ is t^2+tu. Adding these partial products (as in common multiplication), we have $t^2+2tu+u^2$. Which corresponds with the principle.

9. Find the square of 56 according to Principle IV. Of 87. Of 243. Of 3469.

$87^2 = (8 \text{ tens})^2 + 2\,(8 \text{ tens} \times 7) + (7^2) = 6400 + 1120 + 49 = 7569$

We may regard 243 as 24 tens and 3 units, and thus have $243^2 = (24 \text{ tens})^2 + 2\,(24 \text{ tens} \times 3) + 3^2$. Pupil complete the work.

3469 may be regarded as 346 tens and 9 units, etc.

ROOTS.

300. A **Root** is one of the equal factors into which a number is conceived to be resolved. The *Square Root* of a number is one of *two* equal factors into which the number is conceived to be resolved. The *Cube Root* is one of three equal factors.

301. The *Radical* or *Root Sign* is $\sqrt{\ }$. When written thus $\sqrt{25}$, it indicates that the square root of 25 is to be taken; that is, that 25 is to be resolved into 2 equal factors, and one of them taken. To indicate the cube root, 3 is written in the sign. Thus $\sqrt[3]{125}$ means the cube root of 125. It is 5.

$\sqrt{9}$ is 3, because 3 is one of the 2 equal factors which compose 9.

$\sqrt[3]{343}$ is 7, because $7 \times 7 \times 7 = 343$.

1. What is the square root of 16? Of 36? Of 144? Of 81? Of 49? Of 1? Of 4? Of 9? Of 25? Of 121? Of 100? Tell why in each case.

2. What is $\sqrt[3]{8}$? $\sqrt[3]{27}$? $\sqrt[3]{1}$? $\sqrt[3]{1728}$? $\sqrt[3]{64}$? $\sqrt[3]{125}$? $\sqrt[3]{343}$? $\sqrt[3]{729}$? $\sqrt[3]{1000}$? $\sqrt[3]{216}$?

302. Finding the root of a number is called *Extracting the Root*.

To Extract the Square Root.

1. Extract the square root of 4624.

Operation.

$$
\begin{array}{rl}
(t+u)^2 = t^2 + 2tu + u^2 = & \dot{4}6\dot{2}4 \ (68 = t + u. \\
t^2 = & 36 \\
\hline
2t = 120 & |1024 = 2tu + u^2 = (2\,t + u)\,u.^* \\
u = \underline{\quad 8} & | \\
2t + u = 128 & |\underline{1024} = (2t + u)\,u.
\end{array}
$$

Explanation.—By pointing off we find that the root will consist of two figures, a *tens* and a *units*, and that the square of the tens figure is in 46 (**298**). Hence, as 36 is the greatest square in 46, 6 is the tens figure of the root. Letting $t + u$ represent the root, we have $$(t + u)^2 = t^2 + 2tu + u^2 = 4624.$$

But knowing t to be 6 (tens), we subtract its square from 4624, and have $$2tu + u^2 = (2t + u)\,u = 1024 \text{ remaining.}$$

Now, as u is small with reference to $2t$, we may, *for a trial*, put $(2t) \times u$, or (12 tens) $\times\, u = 1024$. Hence $1024 \div 12$ (tens) will give the units figure of the root, approximately, at least. (In this case it gives it exactly.) Now completing the divisor by adding the units figure, we find the *True Divisor* to be 128. This multiplied by 8 gives 1024. Hence 68 is the exact square root of 4624.

2–6. Extract the square root of each of the following: 5776, 6889, 1225, 784, 529.

General Rule for Extracting the Square Root.

303. *I. Separate the number into periods by placing a point over the units figure, and one over each alternate figure to the left (and also to the right, if decimals occur in the number). Write as the highest order in the root the root of the greatest square contained in the left-hand period, and subtract its square from that period.*

---

* This means $2t + u$ multiplied by u, which makes $2tu + u^2$.

II. To the remainder annex the second period, and divide the tens of the number thus formed by twice the first root figure, placing the result in the root, and also at the right of the Trial Divisor. Multiply the True Divisor by the new root figure, subtract the product from the dividend, and annex the third period to the remainder.

III. Double the root figures already found for a new trial divisor, and proceed as before until all the periods are brought down.

When any trial divisor is not contained in the tens of the dividend, place a zero in the root, and also at the right of the divisor, and bring down the next period.

If any figure obtained for the root proves too large, diminish it by 1 and repeat the work.

Approximate roots may be obtained by annexing decimal periods of two zeros each. Decimal periods must always be full, since the square of any decimal has an even number of figures. Why?

7–12. Extract the square root of each of the following: 2209, 361, 9216, 2601, 4900.

13–20. Also of the following: 120409, 74529, 412164, 123201, 6718464, 2125764, 966289.

Operation.—When there are more than two figures in the root,

```
      74529 ( 273                 2125764 ( 1458
      4                           1
 47 ) 345                    24 ) 112
      329                         96
543 ) 1629                  285 ) 1657
      1629                        1425
                           2908 ) 23264
                                  23264
```

21–28. Find the square root of 78564, 43, 7856.42, 87.512, 2, 3.476, .4, 23.5. (See next page.)

87.5120 (9.354+	2 (1.4142+	.40 (.632+
81	1	36
183) 651	24) 100	123) 400
549	96	369
1865) 10220	281) 400	1262) 3100
9315	281	2524
18704) 90500	2824) 11900	576
74816	11296	
15684	28282) 60400	

Note.—The same explanation which has been given when the root consists of two figures, can be readily extended to any number. Thus in the 1st example solved under 13–20, the square of the first two figures 27 (tens) falls in orders from hundreds upward. We may therefore proceed to find these two figures exactly as if we were extracting the root of 745. Having found these, we may take them as tens, and consider the root as made up of 27 (tens) and some number of units, etc.

To Extract the Cube Root.

As a basis for extracting the cube root we need to prove the following

Principle.

304. *The cube of any number made up of tens and units is the cube of the tens, + 3 times the square of the tens multiplied by the units, + 3 times the tens multiplied by the square of the units, + the cube of the units.*

Demonstration.—Let the number we propose to cube be represented by $t + u$, as in (**299**). Now the square of $t + u$ is . . . . $t^2+2tu+u^2$

Multiplying this by $t + u$, we have the cube.

The multiplication is explained thus: multiplying by u, we have u^2 multiplied

$$\begin{array}{r} t^2+2tu+u^2 \\ t+u \\ \hline t^2u+2tu^2+u^3 \\ t^3+2t^2u+\ \ tu^2\qquad \\ \hline t^3+3t^2u+3tu^2+u^3 \end{array}$$

by u, which makes u^3, just as 2 squared (2^2) multiplied by 2 makes 2 cubed (2^3). $2tu$ multiplied by u makes $2tu^2$, for $2tu$ is $2 \times t \times u$, and putting in another factor of u, we have $2 \times t \times u \times u$, or $2tu^2$. In like manner the other terms are multiplied. In adding the partial products, we notice that 2 times tu^2 and 1 time tu^2 make 3 times tu^2, or $3tu^2$. So 2 times t^2u, and 1 time t^2u make $3t^2u$. Finally, we observe that this result, $t^3 + 3t^2u + 3tu^2 + u^3$, agrees with our principle.

1. Extract the cube root of 262144.

Operation.

$$(t+u)^3 = t^3 + 3t^2u + 3tu^2 + u^3 = 26\dot{2}14\dot{4}\ (\ 64 = t + u.$$

$$\begin{array}{rr|l}
 & t^3 = 216 & \\
3t^2 = & 10800 & 46144 = 3t^2u + 3tu^2 + u^3, \text{ or} \\
3tu = & 720 & \qquad (3t^2 + 3tu + u)\ u.^* \\
u^2 = & 16 & \\
3t^2 + 3tu + u^2 = & 11536 & 46144 = (3t^2 + 3tu + u^2)\ u, \text{ or} \\
 & & \qquad 3t^2u + 3tu^2 + u^3.
\end{array}$$

Explanation.—By pointing off (**298**) we find that the root will consist of 2 figures, a *tens* and a *units*, and that the cube of the tens figure is in 262. Hence, as 216 is the greatest cube in 262, 6 is the tens figure of the root.

Letting $t + u$ represent the root, we have $(t + u)^3 = t^3 + 3t^2u + 3tu^2 + u^3 = 262144$.

But, knowing t to be 6 (tens), we subtract its cube from 262144, and have $3t^2u + 3tu^2 + u^3 = (3t^2 + 3tu + u^2)\ u = 46144$ remaining.

Now, as u is small with reference to $3t^2$, we may, *for a trial*, put $(3t^2) \times u = 46144$, and as we know t this becomes $10800 \times u = 46144$. Hence u is, approximately, $46144 \div 10800$.

We thus find that the units figure is probably 4. Now completing the divisor by adding to it $3t^2$, *i. e.*, 3 times the product of the root already found (remembering that it is tens) by the units, and the square of the units, we find that the *True Divisor*, $3t^2 + 3tu + u^2$, is 11536. This multiplied by 4 gives 46144. Hence 64 is the exact cube root of 262144.

---

* This means $3t^2 + 3tu + u^2$ multiplied by u, which makes $3t^2u + 3tu^2 + u^3$.

305. Note.—Let the pupil write the rule. **He must tell,**

1. *How to point off.*
2. *How to find the first figure in the root.*
3. *What to do with this figure of the root.*
4. *How to form the Trial Divisor.*
5. *How to find the probable next figure of the root.*
6. *How to complete the divisor and form the True Divisor.*

Finally, the reason for each step should be given.

2 to 5. Extract the cube root of 54872; of 41063625; of 354894912; of 3416.53.

```
          54872 (38
          27
   2700 | 27872
    720 |
     64 |
   3484 | 27872
```

```
          41063625 (345
          27
   2700 | 14063
    360 |
     16 |
   3076 | 12304
 346800 | 1759625
   5100 |
     25 |
 351925 | 1759625
```

```
           354894912 (708
           343
  *14700 | 11894912
 1470000 |
   16800 |
      64 |
 1486864 | 11894912
```

```
            3416.530 (15.06
            1
     300 | 2416
     150 |
      25 |
     475 | 2375
 6750000 | 41530000
   27000 |
     216 |
 6777216 | 40663296
             866704
```

6 to 13. Extract the cube root of the following. 157464; 74088; 571787; 15625; 2744; 1124864; 2571353; 651714363.

* As this is not contained in 11894, we write 0 in the root and bring down the next period.

14 to 22. Extract the cube root of 34285.7; 34.3472; 5; 48; 2; .4932; .8; .343; .27; $\frac{125}{256}$; $\frac{343}{729}$; $\frac{28}{125}$; $\frac{14}{33}$.

Note.—The root of a common fraction may be extracted by extracting the root of the numerator and denominator separately, or by reducing it to a decimal and then extracting the root.

Applications.

1. What is the area in acres of a square field 35 rods on a side?

2. What is the length of a side of a square field containing 40 acres? 100 acres? 10 acres? 20 acres?

20 acres = 3200 square rods. $\sqrt{3200} = ?$

3. If a square orchard contains 2916 trees, how many are in a row on each side?

4. A man has a rectangular board 128 in. long and 32 in. wide, from which he makes a square table as large as possible; required its length, no allowance being made for sawing. *Ans.*, 64 in.

5. What would it cost to enclose a square lot, containing 160 acres, with a fence costing at the rate of $4 per rod? *Ans.*, $2560.

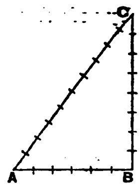
Note.—The areas of plane figures of the same shape (similar), are in the same ratio as the squares of their like sides, or lines.

6. There are two fields of the same shape (similar); one is 10 rods on a certain side, and the other is 15 rods on the corresponding side. What is the ratio of their areas? *Ans.*, As 100 : 225, or 4 : 9

7. Two circles have their diameters respectively 6 and 8. What is the ratio of their areas? *Ans.*, 9 : 16.

8. What is the radius* of a circle whose area is 2 acres? 50 *sq. ft.*? 1 *sq. mi.*? (See **201**).

306. **A Triangle is a figure with only 3 sides: when one of its angles is a right angle, it is called a *Right Angled Triangle*. If you make a right-angled triangle whose *Base*, AB, is 6, and whose *Perpendicular*, CB, is 8, and then measure the *Hypotenuse*, AC, you will find the latter to be just 10. Now, notice that $6^2 + 8^2 = 10^2$, and in Geometry this is found true of all right-angled triangles; viz., *The Square of the hypotenuse equals the sum of the squares of the other sides.***



9. If the base and perpendicular of a right-angled triangle are respectively 14 and 24, what is the hypotenuse? If the hypotenuse and perpendicular are respectively 35 and 18, what is the base? If the base and hypotenuse are respectively 12 and 20, what is the perpendicular?

For the last, $20^2 = 400$, and $12^2 = 144$. Thus $400 - 144 = 256$, and $\sqrt{256} = 16$. The other answers are 27.78 +, and 30.01 +.

10. What must be the length of a brace to run from corner to corner of a gate 4 *ft.* high and 10 *ft.* long?

Ans., 10 *ft.* 9¼ *in.* very nearly.

11. What is the distance from one corner on the floor of a room to the diagonally opposite corner in the ceiling, the room being 18 *ft.* by 20 *ft.*, and 12 *ft.* high?

Ans., 29 *ft.* 5¼ *in.* +.

---

12. What are the contents in cubic feet of a cubical box 8 *ft.* on each edge? How many bushels does it contain?

*** Distance from the center to the circumference, *i. e.* ½ the diameter.**

13. What is the edge of a cube whose contents are 512 *cu. ft.*? What of one whose contents are 215040 *cu. in.*?

$\sqrt[3]{215040} = 59.9 +$, or within .01 of an inch of 5 *ft.*

Having the edge of a cube given, how are the contents found? Then, having the contents given, how is an edge found?

14. How large a cube does it take for a cord?

Ans., 5.04 *ft.* on an edge, very nearly.

15. How large must a cubical bin be to hold 100 *bu.* of wheat? What is the difference in volume between a cord and an hundred bushels?

Note.—*Similar Solids, i. e.* those of the same shape, have their volumes in the ratio of the cubes of their corresponding dimensions,* or edges.

16. I have two rectangular boxes, one is 8 inches long, 4 inches deep, and 5 inches wide; and the other, 16 inches long, 8 inches deep, and 10 inches wide. Are they similar? What is the ratio of their contents?

Answer. They are; and their contents are as 1 : 8, *i. e.* the second contains 8 times as much as the first.

17. If a ball 1 foot in diameter weighs 100 *lb.*, what is the weight of one 3 feet in diameter, made of the same material? *Ans.*, 1 Ton and 700 *lb.*

18. What must be the depth of a cubical vat to contain 100 barrels?

19. A horse is tied by a rope to a stake in a meadow The rope being attached to his head, how long must it be so that he can graze over an acre. *Ans.*, 117.7 *ft.*+.

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* That is, their corresponding lines,—in cubes their edges, in spheres their radii, or diameters.

20. How many cubic feet of square timber can be hewn from 3 logs 2 *ft.* in diameter and 25 *ft.* long, 30 *in.* and 25 *ft.*, and $1\frac{1}{2}$ *ft.* and 40 *ft.*, respectively?

Ans., $173\frac{1}{8}$ *cu. ft.*

Scribner's Log Book, which may be considered the standard among lumbermen, so universally is it used, makes the above 154 *cu. ft.* The rule there used is *arbitrary*, but is practically more just than the exact scientific method, which assumes the log a true cylinder. No lot of logs will hew as much square timber as this latter method indicates, and no intelligent lumber dealer will buy on such reckoning. The lumbermen's (or Scribner's) rule is as follows:

307. RULE FOR COMPUTING THE AMOUNT OF SQUARE TIMBER YIELDED BY A GIVEN LOG.—Call $\frac{1}{2}$ the sum of the extreme diameters the average diameter, and $\frac{2}{3}$ of this diameter the side of the log when hewn square.

(For DOYLE'S RULE for reducing logs to board measure see Art. **195.**)

21. How many cubic feet of square timber can be cut from a log 30 *in.* in diameter at top, and 36 at butt, and 48 *ft.* long?

$\frac{30+36}{2} = 33$, the av. diameter, $\frac{2}{3}$ of $33 = 22$, side of square stick.

$\frac{22\times22\times\overset{4}{\cancel{48}}}{\cancel{12}\times\underset{3}{\cancel{12}}} = \frac{484}{3} = 161.4$, nearly. This is the result given in *Scribner*. He usually "saves" the fractions in this table by reckoning the next unit.

22. What is the greatest amount of *absolutely* square timber that can be cut from the log in Ex. 21, remembering that it will make a square stick only as large as the top end will allow? *Ans.*, 150 *cu. ft.*

In hewing timber for ordinary purposes, the top end is not reduced to an exact square.

23. Compute the following by Scribner's Rule: Diameters 24 and 29 *in.*, length 35 *ft.* Diameters 20 and 32 *in.*, length 50 *ft.* Diameters 36 and 48 *in.*, length 42 *ft.*

24. What must have been the diameter of a log from which a square stick of timber 30 *ft.* long, and measuring 100 *cu. ft.*, was cut?

Ans., 31 *in.* nearly, or 33 *in.*

25. Which is the stouter,* a man 5 *ft.* 10 *in.* in height who weighs 175 *lbs.*, or one 6 *ft.* who weighs 180 *lbs.*? *Ans.*, The former.

26. In the last example, what must be the weight of the 6-foot man in order that he may have the same proportions as the other?

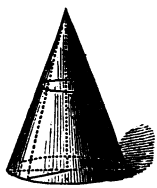
Ans., 190 *lb.* +.

27. What must be the length of the rafter for a *quarter-pitch* roof on a house 32 *ft.* wide? What for a third-pitch roof? Half-pitch? Whole-pitch?

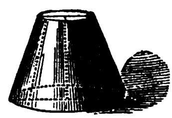
Ans., 17 *ft.* 10.66 *in.*, 19 *ft.* 2¼ *in.*, 22 *ft.* 7½ *in.*, 35 *ft.* 9.3 *in.*

Quarter-pitch means that the height of the ridge above the plates is ¼ the span, etc.

28. Knowing that the volume of a cone is ⅓ the product of the area of its base into its altitude, what is the volume of a cone the radius of whose base is 6 *ft.* and whose altitude is 4 *ft.*?



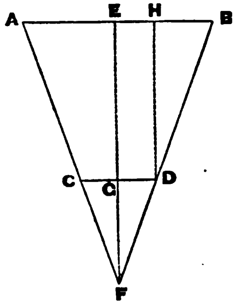
CONE.



FRUSTUM OF CONE.

* Bulkier, stouter built, "fleshier."

29. What are the contents in barrels of a cistern in the form of an inverted frustum of a cone, the diameter of the bottom being 4 *ft.*, that of the top 10 *ft.*, and the depth 8 *ft.*?



The first thing is to find the entire altitude of the complete cone, that is, EF. This is done on the principle that the coresponding sides of similar triangles (those of the same shape) are proportional. Now BHD and BEF are similar triangles; and as HB = 3 *ft.*, EB = 5 *ft.*, and HD = 8 *ft.*, we have EF = $\frac{5}{3}$HD, or 13$\frac{1}{3}$ *ft.* Hence GF = 13$\frac{1}{3}$ − 8 = 5$\frac{1}{3}$ *ft.*

30. What are the contents of a cistern of similar form to the above, the diameter of the bottom being 6 *ft.*, that of the top 15 *ft.*, and the depth 12 *ft.*?

31. Common rocks and earths average 2$\frac{1}{2}$ times as heavy as water (*i. e.*, their *specific gravity* being 2.5), what would be the weight of the earth in tons, if all of its material were of this character?

The mean radius of the earth is about 3960 *mi.*, and 27.69 *cu. in.* of water weighs 1 *lb.* Av. (207).

The area of the surface of a sphere is 4 times the area of a circle of the same radius, and the volume is $\frac{1}{3}$ the product of the area of the surface into the radius.

CHAPTER VIII.

THE METRIC,* OR DECIMAL SYSTEM OF WEIGHTS AND MEASURES.

Diameter 2 Centims.  *Weight 5 Grams.*

308. The metric system can be readily learned if the student will first fix in mind a definite conception of

The Units.

The **Meter** (*mee'-ter*) is the unit of *Length*, and is the basis from which all the rest are deduced.

A **Meter** = 39.37 *inches.*

The **Liter** (*lee'-ter*) is the unit of *Measures of Capacity.*

$$\text{A } \textbf{Liter} = \begin{cases} 1.0567 \textit{ Liquid Quarts}, \text{ or} \\ .908 \textit{ Dry Quarts.} \end{cases}$$

The **Gram** is the unit of weight.

A **Gram** = 15.432 *grains.*

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* This system takes its name *metric* from the *meter*, the unit of linear measure established by the French government, and made the basis of all the others. See Appendix III.

Subdivisions and Multiples of the Units.

309. These units are divided and subdivided into 10ths, 100ths, and 1000ths, and multiplied by 10, 100, 1000, and 10,000, to make the other denominations; hence the system is a *Decimal System*.

The names of the denominations *lower* than the unit are formed by prefixing the Latin syllables *Deci* ($\frac{1}{10}$), *Centi* ($\frac{1}{100}$), and *Milli* ($\frac{1}{1000}$), to the name of the unit.

The names of the denominations *higher* than the unit are formed in like manner by prefixing the Greek syllables *Deka* (dek'a) (10), *Hekto* (100), *Kilo* (1000), *Myria* (10,000), to the name of the unit.

Complete Table of the Metric System.

Relative Values.	Length.	Weight.	Capacity.	Surface.	Solidity.
10,000	Myriam (Mm.)	....	....	....	....
1,000	Kilōm (Km.)	Kilōg (Kg.)	Kilōl (Kl.)	....	....
100	Hektōm (Hm.)	Hektōg (Hg.)	Hektōl (Hl.)	Hektar (Ha.)	....
10	Dekam (Dm.)	Dekag (Dg.)	Dekal (Dl.)	....	Dekaster (Ds.)
Unit.	*Meter* (m.)	*Gram* (g.)	*Liter* (l.)	*Ar* (a.)*	*Stere* (s.)*
.1	Decim (dm.)	Decig (dg.)	Decil (dl.)	Deciar (da.)	Decister (ds.)
.01	Centim (cm.)	Centig (cg.)	Centil (cl.)	Centiar (ca.)	....
.001	Millim (mm.)	Millig (mg.)	Millil (ml.)	....	....

This table contains all the denominations in use, with the spelling and abbreviations approved by the Metric Bureau, Boston, and the American Metrological Society, New York. The abbreviated names will be seen to consist of the prefixes with the first letter of the principal word, or name of the unit. Thus we have decim for decimeter, kilōg for kilogram, centil for centiliter, etc.

* 1 *ar* = 1 square dekam; 1 stere = 1 *cu.* meter.

The *accent* is always put on the first syllable, c is soft (s), e in the prefixes short, and o long (ō). Thus decim is dĕcĭm, centig is sĕntĭg, kilol kĭlōl, etc. Ar is like *are*.

MEASURES OF LENGTH.

310. THE INSTRUMENTS USED in place of our common 2 *ft.* ruler or carpenter's square, and the yard-stick or measure, are the *meter*, a rule 39.37 *in.* long, and divided into 10ths (decims), and 100ths (centims); and a short ruler 2 *dm.* in length, graduated into centims, and these again divided into 10ths, making millims. The meter is folded into 4 parts, or 10 parts, for a pocket-measure, and the 2 *dm.* ruler into 2 parts.

Such lengths as we usually indicate by yards, feet, and inches, are indicated by meters and centims.* Such as we indicate by miles are indicated by kilōms (kilometers). Very small dimensions, as those used in microscopy, are indicated in millims.

1. To which of our common measures is a meter nearest equal? How many meters in a rod? What is the length of a 12 *ft.* board expressed in meters? Express the dimensions of a room 20 *ft.* by 24 *ft.* in meters.

2. What is the stature of a 6 *ft.* man expressed in the metric system? What of one 1.8 *m.* expressed in feet and inches? *Ans.*, 1.83 *m.*, 5 *ft.* 10.866 *in.*

Is a man's stature any more likely to be conveniently expressed in feet and inches than in meters and decimals? Is a man any more likely to be just 6 *ft.* in height than 1.8 *m.*?

3. How does the ten-folded pocket-meter compare in length (when folded) with the common four-folded

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* There seems to be a well-defined tendency to use the decim as a common unit for smaller measures. The use of the *double-decim* ruler, and the near commensurability of the decim with one foot will facilitate this.

2 *ft.* pocket-ruler? How does the four-folded meter compare with the two-folded 2 *ft.* pocket-ruler?

4. When the metric system comes into common use, what lengths of boards will probably take the place of our 12 *ft.*? 14 *ft.*? 16 *ft.*? What thicknesses will probably take the place of our 1 *in.*, 2 *in.*, and 3 *in.* stuffs, respectively? What dimensions of scantling our 2 *in.* by 3 *in.*? Our 3×4? Our 2×8 joist? Our 2×12?

As an inch is very nearly $\frac{1}{4}$ of a decim, we shall probably speak of "quarter-decim" stuff, or simply "quarter-stuff," instead of "inch-stuff," "half-decim" stuff, or "half-stuff," for "2 *in.* stuff," etc. Of course positive answers can not be given to such questions as the above; nevertheless the student will get a better appreciation of the relation of the metric to the common system by exercising his judgment on such questions than by any mere reductions.

5. With what in our common measure will a 2 *dm.* by 3 *dm.* timber most nearly correspond? What a 3 *dm.* by 4 *dm.*? What a 3 *dm.* square?

6. What will 12 *in.* by 16 *in.* glass be in the metric system? That is, what size will be likely to replace this? What 18 *in.* by 24 *in.*?

7. What is the distance from Albany to New York, expressed metrically, it being 145 miles? What the distance from Detroit to Chicago, *via* the M. C. R. R. (288 *mi.*)?

8. What simple fraction of a mile is a kilōm (approximately.)

9. A R. R. train running 40 *Km.* per hour, runs how many miles?

10. How will the rate 1 mile in 2 *min.* 4 *sec.* be expressed metrically? One mile in 5 *min.*?

11. Which is the faster rate, 1 *km.* in 2 *min.* 20 *sec.* or 1 *mi.* in 2 *min.* 40 *sec.*?

12. What part of an inch is a millim? What is the approximate value in hundredths of an inch?

13. Glass is ruled for microscopic measurements in parallel lines from $\frac{1}{100}$ to $\frac{1}{1000}$ *mm.* apart. What are these distances in inches?

14. Animal cells vary in diameter from $\frac{1}{40}$ *mm.* to $\frac{1}{80}$ *mm.*, and vegetable from $\frac{1}{8}$ *mm.* to $\frac{1}{200}$ *mm.* Express these facts in inches, calling a millim .04 of an inch.

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*MEASURES OF WEIGHT.**

311. For the ordinary purposes of the grocery and market, the *kilōg* (called kilō in Europe) is used. For jeweller's and apothecaries' purposes, and for the chemical laboratory, the *gram* and *millig* are the units used.

The standard Government weights at Washington are of brass and platinum. The brass weights are a five-kilōg, double-kilōg, kilōg, demi-kilōg, double-hektōg, hektōg, demi-hektōg, double-dekag, dekag, demi-dekag, double-gram, and gram. The platinum are a demi-gram, double-decig, decig, demi-decig, double-centig, centig, demi-centig, double-millig, and millig.

1. When steak is 14 *c. per lb.*, what is it *per Kg.*? Sugar at 30 *c. per Kg.* is what *per lb.*? At 25 *c. per kg.*?

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* The measures of *length* and *weight* are the two of the metric system of the most practical importance in our country at present. Hence this arrangement and the fuller attention given them.

2. What is the weight of a bushel of wheat in *Kg's*? Of oats? Of corn? Of a barrel of flour?

3. One *lb.* Av. equals how many kilōg's? One kilōg is how many pounds?

4. How many grams in an ounce of gold? In a pennyweight? How many milligs make a grain Troy or Apothecaries? *Ans.*, 31.1 *g.*, 1.55 *g.*, 64.8 *mg.*

For jeweller's and apothecaries' purposes, and for the chemical laboratory, the *gram* and *millig* are the units used.

5. The U. S. post-office allows 15 *g.* as the weight of a single letter, or $\frac{1}{2}$ *oz.* Troy. Which is the greater?

6. What would be the *practical* equivalent for the Apothecaries grain, scruple, and dram in metric weights?

7. How many grams in an ounce Avoirdupois? What would be the practical equivalent in grams for $\frac{1}{4}$ *lb.* Avoirdupois?

8. One-eighth of a grain is the common dose of of morphine. What would be the prescription in the metric system? *Ans.*, 8 *mg.*

9. Quinine is frequently given in 4 or 5 grain doses. What would be the prescription in the metric system?

Ans., $\frac{1}{4}$ *g.*, or $\frac{1}{3}$ *g.*

10. In weighing a quantity of sugar I find it is balanced by a double-kilog, a demi-hektōg, and a dekag weights. What is the weight in kilōgs?

N.B.—It is one purpose of a number of the preceding exercises to suggest that when the metric system comes into use, most of our common specifications of quantities for practical purposes will undergo slight changes to conform to the units of the new

system, so as not to involve troublesome fractions. Thus, instead of a *rod* we shall speak of 5 meters, and instead of laying out village lots 4 rods by 8, we shall lay them out 20 *m.* by 40. Instead of 12 by 16 *in.* glass, we shall have 3 by 4 *dm.* glass. Instead of prescribing ℥ ij, the physician will write 8 *g.*, etc.

11. Instead of 1 ℥ what amount will probably be substituted in the metric system?

Ans., Doubtless 4 *g.* in ordinary cases.

312. The *Tonneau*, or *ton*, of the metric system is 2,204.6 *lbs.*, and is consequently so nearly equivalent to our *long ton* (2,240 *lbs.*), as to take its place without difficulty. The name *Ton* will doubtless be used instead of the French *tonneau*.

12. Coal at $9 per common ton (2,000 *lbs.*) would be how much per metric ton? Hay at $12 per common ton would be how much per metric ton?

MEASURES OF CAPACITY.

313. For such quantities of liquids or dry substances as we usually designate by the pint, quart, or gallon, the *liter* is used, as 7.5 *l.*, 15 *l.*, $\frac{1}{2}$ *l.*, etc.; but for larger quantities the *hektol* is used, as 8.2 *hl.*, 10.5 *hl.*, etc.

The U. S. Government standards at Washington are a double-liter, liter, demi-liter, double-decil, decil, demi-decil, double-centil, and centil.

The double-liter, liter, and demi-liter, are so nearly equivalent to our $\frac{1}{2}$ *gal.*, quart, and pint measures for liquids, respectively, as to take their places without embarrassment.

A double-dekal, and a dekal, would take the place of our $\frac{1}{4}$ *bush.* and peck measures for grain, very readily.

1. To what common measure is a liter nearly equal? How many liters in a gallon? In a barrel of

$31\frac{1}{2}$ gallons? How many does a common pail ($2\frac{1}{2}$ *gal.*) hold?

2. How many liters in a peck? In a half-bushel?

3. If a bushel of wheat is to weigh 60 *lb.*, what should be the weight of a hektol?

4. Wheat at $3.50 *per Hl.* is what *per bush.*?

5. Molasses at $1.25 *per gal.* is how much *per Dl.*?

314. One of the principal advantages which the metric system offers for scientific purposes is the facility which it affords for passing from measures of capacity to those of weight, and *vice versa.* Thus a *liter* is a *cubic decim*, and a *gram* is a *cubic centim* of pure water at the temperature of melting ice. Hence, knowing the specific gravity of any substance (*i. e.*, its weight as compared with water), we can readily pass from weight to volume, and *vice versa.*

6. What is the weight of a liter of distilled water at the temperature of melting ice?

Ans., 1000 *g.*, or 1 *Kg.*

7. The specific gravity of linseed oil is .94. How much would a cask of 2 *Hl.* weigh? *Ans.*, 188 *Kg.*

8. I find that a liter of alcohol weighs 8 *Hg.* What is its specific gravity?

The weight of a liter of any liquid expressed in kilogs, is its specific gravity, or the weight of a millil expressed in grams, etc.

9. The specific gravity of milk is 1.032. What does 1 *Dl.* weigh?

10. A pail containing 1 *Dl.* of cider is filled and the cider found to weigh 10.18 *Kg.* What is the specific gravity of cider?

11. One millil of sulphuric acid is found to weigh 1.842 *g.* What is its specific gravity?

MEASURES OF AREA.

315. For measuring surfaces the square dekam is used and is called the AR. The *Hektar* (2.471 acres) is the more convenient unit for land measure.

1. How many hektars in a section of land?
2. Land at $250 a hektar is how much per acre?
3. How many square feet in a square meter?
4. Land at 8 Nap. a hektar is how many dollars an acre?
5. How many hektars in a rectangular piece of ground 1000 *m.* by 400?
6. How many square meters in 6 boards, 4 *m.* in length and 5 *dm.* in width?

MEASURES OF VOLUME.

316. The *Stere*, which is a cubic meter, is the proposed unit of volume, but it has fallen into general disuse.—President *Barnard*, in Johnson's Cyclopœdia.

The chief interest, therefore, which attaches to the metric measurement of volume, at present, is as a means of defining the measures of weight and capacity. See (**314**), and also *Appendix* III.

1. How many hektols of water does a cylindrical cistern contain which is 2 meters in diameter and 2.5 meters deep?
2. In 200 steres how many cords?
3. 500 *cu. yd.* are how many steres?
4. How many cubic meters (steres) of earth are removed in digging a ditch 2 *m.* wide, 1.5 deep, and 4 *Km.* long?

CHAPTER IX.

REVIEW.

ONE HUNDRED TEST EXERCISES IN FRACTIONS.

1. $3\frac{1}{2} - \frac{\frac{2}{3}}{4} + \frac{4}{\frac{1}{5}}$.

2. $\frac{3}{5}$ of $4\frac{2}{5} \times .25$.

3. $1.33\frac{1}{3} \times 4\frac{1}{5} \div \frac{2}{\frac{3}{5}}$.

4. $\frac{11\frac{2}{3}}{12\frac{3}{5}} + \frac{5\frac{7}{8}}{\frac{3}{16}} - \frac{\frac{2}{7}}{4}$.

5. $(5\frac{1}{2} + 3) \times (2 - \frac{3}{7})$.*

6. $\frac{4 - \frac{2}{3}}{3} - \frac{3\frac{1}{2} + 2}{8}$.

7. $\frac{5\frac{1}{3} + 2\frac{3}{4}}{4.5 - 1\frac{1}{3}} \div 6.25$.

8. $\frac{1\frac{1}{4} \times \frac{2}{3}}{5\frac{1}{2}} \div \frac{3 - \frac{2}{3}}{\frac{1}{2} \text{ of } 7}$.

9. $\frac{4\frac{1}{3} + 2\frac{1}{6} - 3}{\frac{3}{5} \text{ of } 1.25} \times (\frac{2}{3} + \frac{1}{2})$.

10. $\frac{5.8 \div .002}{1.6} + \frac{.45}{5}$.

11. $\frac{1 \div .0001}{.5 \div 50} \times 400$.

12. $\frac{42.68 \div .002}{\frac{1}{5} \text{ of } 13} \div .8$.

13. $\frac{500 \div (\frac{2}{3} \text{ of } .066)}{\frac{1}{2} \text{ of } .7 \div (4 - \frac{2}{20})}$.

14. $\frac{.3 + .03 + .003}{3.5 \div .07}$.

15. $\frac{56 \div .007}{\frac{1}{2} \text{ of } .04} \div \frac{.02}{20}$.

16. $\frac{(\frac{2}{3} + 4.2) \div (.125 \times \frac{5}{3})}{.375 \times (\frac{3}{5} - .16\frac{2}{3})}$.

17. $\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} \times \frac{1 + \frac{1}{3}}{1 + \frac{1}{3}}$.

18. $\frac{.05 \times 1.02}{\frac{3}{4}} \div \left(\frac{1}{6\frac{1}{2}} - \frac{.005}{50}\right)$.

19. $\frac{3.7 + 1.05 + .508}{.43 - .005} \times (\frac{2}{3} - .5)$.

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* The parenthesis used in this way indicates that the quantities within are to be taken as one. This, therefore, is $8\frac{1}{2} \times 1\frac{4}{7}$.

20. $(.0008 \div .008) \times 10{,}000.$

21. $\dfrac{.468}{200} \times \dfrac{\frac{5}{6}}{2.12\frac{1}{2}} \div 5\frac{3}{8}.$

22. $\dfrac{1 - .0001}{.5 \div 5} + 3\frac{1}{2}.$

23. $\dfrac{4 - .002}{3 \div .03} + \dfrac{.04 \div .0002}{.01\frac{1}{3}}.$

24. $\dfrac{\sqrt{.4} + \sqrt{.9}}{\sqrt{.36} + \sqrt{.16}} + \dfrac{\sqrt{.4 + .9}}{\sqrt{.36 - .16}}.$

25. $\dfrac{\sqrt{\frac{4}{9}} \div \sqrt{\frac{16}{25}}}{\sqrt{\frac{1}{5}} \times \sqrt{5}}.$

26. $\sqrt{\frac{2}{3}} : \sqrt{\frac{3}{2}} :: \sqrt{7} : ?$

27. $\dfrac{\sqrt{4.9} + \sqrt{3.6}}{\sqrt{.25} - \sqrt{.01}}.$

28. $\dfrac{\sqrt{.003\dot{6}} + \sqrt{490}}{\sqrt{.025} - \sqrt{.0025}}.$

29. $\sqrt{16} \times \sqrt{4}$, and $\sqrt{16} + \sqrt{4}.$

30. Is $\sqrt{16} \times \sqrt{4} = \sqrt{16 \times 4}$?

31. Is $\sqrt{16} + \sqrt{4} = \sqrt{16 + 4}$?

32. $\sqrt{\frac{3}{7}} \times \sqrt{\frac{7}{3}} \times \sqrt{\frac{2}{5}} \times \sqrt{\frac{5}{2}}.$

33. $\sqrt{\frac{3}{7}} + \sqrt{\frac{7}{3}} + \sqrt{\frac{2}{5}} + \sqrt{\frac{5}{2}}.$

34. $5\frac{1}{2} : 3\frac{1}{4} :: \frac{2}{3} : ?$

35. $\frac{2}{5} : 1\frac{1}{5} :: 8 : ?$

36. Why is $\frac{2}{3} : \frac{4}{5} :: \frac{15}{7} : ?$ the same as $\frac{3}{2} \times \frac{4}{5} \times \frac{15}{7} = ?$

37. $\sqrt{\frac{3}{4}} : \sqrt{\frac{3}{5}} :: 10 : ?$

38. $.05 : 3 :: 1.02 : ?$

39. $.4 : ? :: 6\frac{1}{2} : .03.$

40. $? : .001 :: .02 : .3.$

41. $4.0\frac{3}{4} : 1.00\frac{1}{2} :: .0\frac{2}{5} : ?$

42. $\sqrt{2\frac{1}{2}} + \sqrt{4\frac{1}{5}}.$

43. $\sqrt{2\frac{1}{2}} \times \sqrt{4\frac{1}{5}}.$

44. $\sqrt{16} \div \sqrt{4}$, and $\sqrt{16} - \sqrt{4}.$

45. Is $\sqrt{16} \div \sqrt{4} = \sqrt{16 \div 4}$?

46. Is $\sqrt{16} - \sqrt{4} = \sqrt{16 - 4}$?

47. $\sqrt{\frac{2}{5}} \div \sqrt{\frac{2}{5}}.$

48. $\sqrt{\frac{2}{5}} - \sqrt{\frac{2}{5}}.$

49. $\sqrt{1\frac{1}{2}} : \sqrt{2\frac{2}{3}} :: 6 : ?$

50. $1\frac{1}{2} : 2\frac{2}{3} :: 36 : ?$

What is the relation between the answers to the last two? Why?

51. $5 : \sqrt{.6} :: \sqrt{.15} : ?$

52. $\sqrt{.8} : 3 :: \sqrt{1.1} : ?$

53. $\sqrt{\frac{2}{3}} : \sqrt{\frac{9}{16}} :: \sqrt{\frac{4}{5}} : ?$

54. $\sqrt{\frac{3}{2} \times \frac{9}{16} \times \frac{4}{5}} = ?$

Are the answers to the last two alike? Why?

55. $\dfrac{3\frac{3}{5} + 5\frac{3}{4}}{5\frac{1}{2} - .025}.$

56. $\sqrt{.002} : \sqrt{.004} :: \sqrt{7} : ?$

57. $\sqrt{10} : \sqrt{5} :: \sqrt{\frac{3}{5}} : ?$

58. $\sqrt{\frac{3}{4}} \times \sqrt{\frac{4}{3}}.$

59. $\sqrt{\frac{3}{4}} + \sqrt{\frac{4}{5}}$.

60. $\sqrt{\dfrac{.005 \div .01}{3 - 2.01}}$.

61. $\dfrac{(4\frac{1}{2} - 3\frac{1}{5})\,(.2 + 3)}{.3\,(5.6 \div .07)}$.

62. $\dfrac{(3 - \frac{1}{3}) \div (1 - \frac{2}{5})}{.04\,(1 - .05)}$.

63. $\dfrac{(4 \div .04)\,(4 - .04)}{(3 \times .03)\,(3 + .03)}$.

64. $\dfrac{2.08 \div (1 - \frac{1}{5})}{(2 + \frac{1}{5}) - (\frac{1}{3} \text{ of } \frac{3}{5})}$.

65. $\dfrac{1}{\frac{2}{3}} : ? :: \dfrac{\frac{3}{5}}{2\frac{1}{2}} : 4$.

66. $2 \times 3\frac{1}{5} : (5 - 1\frac{1}{6}) :: ? : \frac{1}{2} \text{ of } \frac{3}{7}$.

67. $(2 - \frac{3}{5}) \div \dfrac{4\frac{1}{2}}{\frac{3}{7} \text{ of } \frac{5}{9}}$.

68. $(1\frac{1}{2} - \frac{5}{6}) \div (.02 - .002)$.

69. $\dfrac{5\frac{1}{2} \times 2\frac{2}{3}}{2 \div .02} \div .125$.

70. $(\frac{2}{3})^2 : (1\frac{3}{5})^3 :: 8 : ?$

71. $(5\frac{1}{2})^2 \div (\sqrt{2.5} \times \sqrt{.016})$.

72. $[(4\frac{2}{5})^3 \div (.12)^2] \times \sqrt{.4}$.

73. $(2.01)^2 + (5.3)^3 - (2\frac{1}{8})^2$.

74. $\left[5 \div \dfrac{2\frac{1}{4}}{\frac{1}{2} \text{ of } \frac{3}{8}}\right] \times \dfrac{13\frac{1}{12}}{4.2}$.

75. $\dfrac{\sqrt{3.6} - \sqrt{.16}}{(\frac{2}{3})^2 - \sqrt{\frac{1}{8}}} \div \sqrt{\dfrac{1\frac{1}{2}}{\frac{2}{3} \text{ of } \frac{5}{6}}}$.

76. $\frac{2}{3}$ of \$5 is what part of £4?

77. $\frac{1}{5}$ of $\frac{2}{3}$ of $1\frac{1}{4}$ *yd.* is what part of $2\frac{1}{5}$ *M.*?

78. $\frac{1}{5}$ of $7\frac{1}{2}$ *ft.* is $\frac{2}{3}$ of how many meters.

79. $5\frac{1}{2}$ *Kg.* is what part of 10 *lb.*?

80. $7\frac{1}{2}$ *dg.* is what part of 10 *gr.*?

81. 4*s.* 6*d.* is what part of 2 *Nap.*?

82. $5\frac{2}{3}$ *M.* is what part of \$$2\frac{1}{2}$?

83. $14\frac{2}{7}$ less $\dfrac{\frac{1}{2} \text{ of } 8\frac{2}{5}}{14\frac{7}{10}}$ is $\frac{2}{3}$ of $\frac{7}{8}$ of what number?

84. Divide $7\frac{1}{2}$ into 3 parts which shall be to each other as $1\frac{1}{3}$, $1\frac{1}{2}$, and 2.

85. Divide 12 into 3 parts which shall be to each other as $\frac{2}{3}$, $\frac{3}{5}$, and 2.1.

86. Divide 9 into 4 parts which shall be to each other as 2.5, 1.1, $\frac{1}{2}$, and $3\frac{1}{5}$.

87. Divide \$1250 into 3 parts which shall be to each other as $\frac{2}{3}$ of 6, $\frac{1}{5}$ of 20, and $\frac{1}{7}$ of 14.

88. Divide \$3800 into 4 parts which shall be to each other other as 2×60, 3×50, 4×35, and 7×16.

89. Does it increase a proper fraction, or diminish it, to extract its root? Why?

90. Does it increase a proper fraction, or diminish it, to involve it to a power? Why?

91. Answer the last two questions with reference to an improper fraction.

92. $\frac{2}{3}$ of $3\frac{3}{4}$ liquid gallons is what part of .5 *bush.*?

93. 2.6 *l.* is what part of $\frac{2}{3}$ of 5 *gal.*?

94. .4 *mm.* is what part of .01 *in.*?

95. 25 *Dl.* is what part of 10 *bush.*?

96. 3.2 *m.* is what part of 1 rod?

97. A is 1.6 *m.* in height, and B 5 *ft.* $10\frac{1}{2}$ *in.* What is the ratio of their statures?

98. What is the ratio of ʒ ij to 4 *g.*?

99. What is the difference between $\frac{3}{7}$ of $4\frac{1}{2} \times \frac{9\frac{3}{4}}{\frac{13}{20}} \times \frac{1}{15}$ of $\frac{7}{9}$ of £43 18 *s.* $11\frac{1}{2}$ *d.*, and $3\frac{8}{9} \times \frac{1}{17\frac{1}{2}}$ of .56 of $1.75 \times 6\frac{1}{2}$ times \$97.18?

100. Reduce $\frac{2}{3}$ of $4\frac{1}{2} \times 7\frac{4}{5} \times \frac{9}{19\frac{1}{2}}$ of $\frac{5}{9}$ of 3 *oz.* 4 *dr.* 2 *scr.* 5 *grains* to the decimal of $\frac{6}{11}$ of .63 of $2\frac{37}{42} \times \frac{3}{13}$ of $6\frac{1}{2}$ times 7 lbs. 3 oz. *Av.*

FIFTY TEST EXERCISES IN PERCENTAGE.

1. What % is 5 of 25? Of 10? Of 43?

2. Of what number is 11 6%? Of what 10%? Of what 7%?

3. What % of $\frac{5}{7}$ is $\frac{1}{11}$? $\frac{1}{7}$? $\frac{3}{7}$?

4. What % of $2\frac{1}{3}$ is $\frac{7}{24}$? $\frac{7}{9}$? $\frac{7}{50}$?

5. When gold is quoted 105, what would be the quotation for currency with gold par? What when gold is at 110? At 112? At 104? At $103\frac{1}{2}$?

6. The bank is discounting at 8%. I wish $200 for 60 *da.* For what must the note be made? What for $350 for 30 *da.* at 10%? For $250 for 90 *da.* at 5%?

7. I send a 6% note for $500, dated May 10, 1875, with an endorsement July 1, 1876, of $150, to a broker in Chicago for collection. He collects it March 10, 1877, and charges me $\frac{3}{4}$% for collecting. What does he remit to me?

8. I wish to buy 65 shares of Bank of Commerce N. Y. stock, at 112. How much will they cost me, exchange being $\frac{1}{10}$% premium and the brokerage $\frac{1}{4}$%? What shall I make if I hold them 2 years, receiving 4% quarterly dividends, if I sell them at 120?

9. How much better is the above than 10% annual interest, allowing 10% on the mid-year dividends to the close of the year?

10. I send an 8% note for $1000, dated July 1, 1876, to Buffalo for collection. The maker of the note having gone into bankruptcy, pays only 75 *c.* on $1. The note is collected by my broker, Apr. 30, 1877. What does he remit, charging $\frac{1}{2}$% for collecting?

11. What are the proceeds of a 60 *da.* note for $500 at bank, discount 5%?

12. I have a $350 7% note, dated Jan. 15, 1875, with an endorsement of $100 July 1, 1876. The note

is due Oct. 1, 1877, (with grace). What is it worth, discounting at 10%, Feb. 1, 1877?

13. How long does it take \$4080 to gain \$935.34, interest at 7%?

14. How long does it take a principal to double at 6%? 7%? 8%? 10%?

15. How long does it take a principal to double itself at compound interest at 5%? At 10%? At 4%?

Without a table at hand this would be solved by finding the amount at compound interest of \$1 for a series of years till it became \$2 or more. Thus at 7% \$1 amounts to $(1.07)^{10} = 1.96715$ in 10 *yr.*, and $(1.07)^{11} = 2.10485$ in 11 *yr.* Hence the question is, How much longer than 10 *yr.* is required? For this fraction of a year the case would be one of simple interest; that is, How long will it take \$1.96715 to gain \$2 − \$1.9615 = \$0.0385 at 7%?

16. What amount in currency must I invest in U. S. 5–20's at 114, to yield me semi-annually \$100 in gold? What in U. S. 10–40's at 111¾?

17. I have \$10,000 to invest in 10–40 U. S. bonds, quoted at 112. What will a Detroit draft on N. Y. cost me to cover the largest amount of bonds I can buy (the smallest bond is \$50), brokerage in N. Y. ¼%, and N. Y. exchange being $\frac{1}{10}$% premium in Detroit?

18. \$5600. PHILADELPHIA, *Jan.* 11, 1871.

For value received, on demand, I promise to pay James Jones, or order, Five Thousand Six Hundred Dollars, with interest, without defalcation.

JOHN SMITH.

Endorsements: May 19, 1871, \$500; Sept. 5, 1871, \$200; Jan. 1, 1872, \$300; April 17, 1872, \$150.

What is due Jan. 11, 1873?

19. What was due on the above by the Merchant's Rule?

20. At what % will \$240.80 amount to \$325.08 in 5 *yr.* 10 *mo.*?

21. At what % will a given principal double in 12 *yr.*? In 15 *yr.*?

22. What principal will amount to \$1617 in 3 *yr.* 6 *mo.* 15 *da.* at 8%?

23. What will be the duty in our currency, on a case of silk mantillas, invoiced in Paris at 13950 francs, the rate of duty 60 per cent ad valorem?

24. Suppose an annual premium of \$68.25 is paid for insuring a house worth \$2275, what per cent is paid?

25. At a rate of $1\frac{1}{2}$ per cent a year a warehouse is insured for $\frac{3}{4}$ of its value, paying thereon a premium of \$202.50; what is the whole value of the warehouse?

26. A tax of \$50,000 net is to be raised in a certain city on a valuation of \$2,000,000. Supposing 3% to be uncollectible, and allowing 5% for collecting, what tax must be levied? What will be a man's tax who is assessed on \$3500?

27. I invest \$2,000 in certain goods, which I sell at 50% advance, but at a cost of 3% on the sales for selling. Allowing 5% loss by selling on credit, what % do I make by the transaction?

28. A 7% note for \$460 with annual interest is dated June 3, 1874, and is payable Jan. 1st, 1878. There is a payment of \$200 endorsed Oct. 13, 1875. What was the

note worth May. 7, 1876, allowing a discount of 10% compound interest?

29. July 15, 1876, Mr. A bought a house and lot for $10,000, agreeing to pay 8% semi-annual interest, and the principal in 10 equal semi-annual installments. He finds, however, that he can pay $2000 semi-annually, and does so, the holder of the note allowing him 10% semi-annual interest on all he pays above what the note calls for. When will the note be paid, and what will be the last payment?

30. What % do I make by investing in R. R. stock at 75, which pays 3% semi-annually?

31. What is the balance of the following account, and when is it due?

Dr. JOHN SMITH in Acct. with WILLIAM ESTES. *Cr.*

1875.			1875.		
March 10.	For Sundries,	$250	April 1.	By Bal. of Acct.,	$110
April 15.	" Flour on 60 *da.*,	$420	May 21.	" Dft. on 30 *da.*	$300
June 20.	" Mdse. on 30 *da.*,	$600	July 1.	" Cash,	$560

32. I buy a bill of $540 in N. Y. on 30 *da.* cash, to be discounted at 3% if paid within 10 *da.*, and at 2% if within 20 *da.* I pay $300 the 9th day, and the balance the 19th. What was the last payment?

33. A fruit-dealer invested $25,000 in apples. Buying, packing, shipping, and storing cost him 5% on the cost. He lost 300 *bbl.* for which he paid 80 *c. per bbl.* He sells the remainder at 50% more than the first cost of the apples. What % per annum does he make on his investment, allowing that his sales average 6 *mo.* after purchase?

34. I mark down from the retail price 10%, goods which I was selling at 25% advance on cost. At what % advance on cost do I now propose to sell them?

35. In consequence of a rise of a certain article in the market, I mark up 5% on my former retail price, goods which I was selling at 20% advance on cost. At what % profit do I now propose to sell them?

36. A N. Y. bank in which I hold stock declares a 4% dividend, I draw a draft for the amount due me, and sell it at 1% premium in Omaha, receiving $707. How many shares do I own?

37. A bought 230 bales of cotton, each bale containing 450 *lb.*, at $10\frac{3}{4}$ cents a pound, on a credit of 9 *mo.* He sold the cotton immediately for $12000 cash, and paid the present worth of the debt at 8%. What was his gain?

38. At what must cloth be bought to sell it at $9 *per yd.* and make $12\frac{1}{2}$% profit?

39. A jeweller has a watch which cost him $150, he wishes to mark it so that he can fall 5% on the asking price and still make 20%. How must he mark it?

40. If I borrow at bank on 60 *da.* time at 2% *per mo.*, what % annual interest do I pay? *Ans.*, $28\frac{1}{8}$%.

If I get $100 Jan. 1, for what must I make my note? If, when this note falls due, I renew it, paying nothing, for what must this note be made? Proceeding in this manner, for what length of time will I have to borrow the last time to make the note mature Jan. 1? If I then pay the note, how much do I pay for the use of the $100 1 *yr.*?

41. If I borrow money at bank for 90 *da.* at 10% *per ann.*, and carry it 1 *yr.*, what annual interest do I pay?

42. 1% *per mo.*, on 30 *da.* paper at bank, is what % annual interest?

43. What is gold worth in currency, when currency is quoted at 95? At 93?

44. What is currency worth when gold is quoted at 106? At 110?

45. A bookseller sells a book for $1.20 and makes 25% thereby. What would he have made had he sold it at $1.28?

46. When $\frac{2}{3}$ the selling price equals the cost, what % is made? When $\frac{1}{2}$ the selling price equals the cost? When the selling price is $\frac{5}{6}$ of the cost, what % is lost?

47. How much water must be added to 1 *gal.* pure alcohol to make a mixture 75% alcohol? How much to make one 50%? 40%?

48. I buy a bill of goods in N. Y. amounting to $1500. $500 is cash in 30 *da*,* $500 in 60 *da.*, and $500 in 90 *da.* The rule of the house is to allow 3% discount on 30 *da.* paid within 10 *da.*, 4% on 60 *da.* paid within 10 *da.*, and 5% on 90 *da.* On 60 and 90 *da.* payments made within 30 *da.* are allowed 3% and $3\frac{1}{2}$%, respectively. On 90 *da.* paid within 60 *da.*, $2\frac{1}{2}$%. What amount will pay the bill if I pay the 30 *da.* and $\frac{1}{2}$ the 60 *da.* in 10 *da.*, the remainder of the 60 *da.* and $200 of the 90 *da.* in 30 da., and the balance in 60 *da.*?

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* This means that I am to have 30 *da.* credit without interest.

49. On the above terms what amount will pay \$800 on 60 *da.*, $\frac{1}{3}$ in 10 *da.*, and $\frac{1}{3}$ in 30 *da.* ?

50. A railroad has been constructed through a farm, making it necessary to build fences at a cost of \$750, which must be renewed every 15 years; what should the owner receive to meet this expenditure, at 6% compound interest? *Ans.*, \$1287.03.

TEST EXAMPLES IN MENSURATION AND DENOMINATE NUMBERS.

1. I have a cylindrical cistern 6 *ft.* deep and $6\frac{1}{2}$ *ft.* in diameter. How much shall I increase its capacity if I increase each of its dimensions 25%? 50%? 100%?

2. How much is a rectangular bin increased in capacity by increasing 2 of its dimensions 10%? By increasing all three of its dimensions 10%? If I double 2 of its dimensions? If I double all three dimensions?

3. If a solid globe of 4 *in.* diameter weigh 20 *lb.*, what will one of the same material 6 *in.* in diameter weigh?

4. How many minutes will there be in the month of February, 1880?

5. A lady bought 6 silver spoons, each weighing 3 *oz.* 3 *pwt.* 8 *gr.*, at \$2.25 an ounce, and a gold chain weighing 14 pwt., at \$1.25 a pwt.; what was the cost of both spoons and chain?

317. *The* AREA *of the surface of a sphere is* 4 *times the area of a circle of the same diameter.*

The VOLUME *of a sphere is its surface multiplied by* $\frac{1}{3}$ *its radius.*

6. How many boxes of common double tin, 100 12×17 *in.* sheets in a box, will it take to cover a hemispherical dome of 20 *ft.* diameter, allowing $\frac{1}{2}$ *in.* lap on end and side of each sheet?

7. The specific gravity of iron being 7.25, what is the weight of a 10 *in.* cast-iron cannon-ball?

8. What is the diameter of a sphere whose surface is 100 *sq. ft.*?

9. What is the diameter of a sphere whose volume is 150 *cu. ft.*?

10. How many gallons does a tub 18 *in.* deep contain, whose top is 16 *in.* in diameter, and bottom 20 *in.*?

11. If telegraph poles are 66 *ft.* apart, and a train passes one every 3 *sec.*, what is the rate per hour?

12. A physician having 1 ℔. ℥ iij ʒ iv ℈ ij *gr.* xij of a certain medicine, put it up in *gr.* xx. packages. How many did it make?

13. The U. S. "Trade Dollar" (silver) weighs 420 *gr.*, and the common half-dollar 12.5 *g.* How much more is the trade dollar worth than two common half-dollars? The dollar of 1878 is $412\frac{1}{2}$ *gr.* How does it compare with the others?

14. I sell 12 logs at $10 *per M.*, board measure. The logs are 6 12 *ft.* long and 6 14 *ft.* The first scale 28 *in.*, 30 *in.*, 40 *in.*, and 3 32 *in.*; the others, 2 30 *in.*, 3 35 *in.*, and 1 20 *in.* What do the logs bring me, reckoning by *Scribner's Log Book?*

15. How many cubic feet of hewn timber in 3 logs measuring 15 *ft.* long, 21 and 25 *in.* in diameter; 20 *ft.* long, 26 and 30 *in.* in diameter; and 32 *ft.* long, 30 and 36 *in.* in diameter, measured by *Scribner's* rule?

16. If a 2 *in.* pipe fill a cistern in a given time, how long will it take a 3 *in.* pipe with the same velocity of current to fill it?

17. A third-pitch "square roof" is to be put on a rectangular house, 36 by 42 *ft.*, with a flat deck at top 8 *ft.* above the plates. What will be the size of the deck, what the length of the side rafters, and what the 4 corner rafters?

Ans., The deck will be 12 by 18 *ft.*, the side rafters 14 *ft.* 5 *in.*+, and the corner rafters 18 *ft.* 9.1 *in.*+.

Suppose 4 posts 8 *ft.* high supporting the deck and standing on joists at the level of the plates. To give $\frac{1}{3}$ pitch to the roof, how far must the foot of one of these posts be from the plates.

18. How large a square can be cut from a circle 36 *in.* in diameter?

19. How large a cube can be cut from a sphere 2 *ft.* in diameter?

318. *The* STRENGTH (*power to support weight*) *of rectangular* BEAMS, *supported at both ends, is in the ratio of their cross sections, multiplied by their depths.*

20. How much stronger is a 3 by 4 *in.* beam when set on edge than when lying flat? A 2 by 8? A $2\frac{1}{2}$ by 10?

21. Which is the stronger beam, one 6 *in.* square or one $2\frac{1}{2}$ by 10 *in.*, set on edge? What is the ratio of their strengths?

22. The specific gravity of cast-iron being 7.25, what is the weight of a 12 *in.* cast-iron shell (hollow sphere), the shell being $1\frac{1}{2}$ *in.* thick?

23. Calling the specific gravity of boiler-iron 7.75 what is the weight of a cylinder of $\frac{3}{8}$ *in.* plate, 12 *ft.*

long and $3\frac{1}{2}$ *ft.*, outside diameter, including ends, and making no allowance for laps and rivets?

24. The specific gravity of bricks being 1.9, what is the weight of 1000 common bricks?

25. The specific gravity of common loose earth is about 1.5. A cubic yard makes a good sized load for a span of horses. What does it weigh?

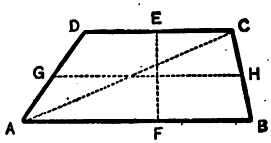
26. The specific gravity of common white marble is about 2.7. What is the weight of a slab 7 *ft.* long, $2\frac{1}{2}$ *in.* thick, and 30 *in.* wide?

27. How many barrels does a cylindrical cistern 9 *ft.* in diameter and 8 *ft.* deep contain?

28. How many bushels in a bin 3.2 *m.* long, 1 *m.* wide, and 2 *m.* deep. How many hektōl's? How many kilōg's of wheat will it contain at 60 *lb.* to the bushel?

319. **A Trapezoid is a quadrilateral (a four-sided figure) with only two of its sides parallel.**

29. Show by dividing the trapezoid ABCD into two triangles by the diagonal AC, that its area is equal to its altitude EF, multiplied by $\frac{1}{2}$ the sum of its parallel sides, (*i. e.*, by $\frac{AB + DC}{2}$).



30. What is the area of a trapezoid whose parallel sides are 30 and 50 rods, and whose altitude is 40 rods?

Note.—Half the sum of the parallel sides is the average width, as GH.

31. How many square feet in a board 14 *ft.* long and 18 *in.* wide in the middle (its ends being 20 *in.* and 16 *in.*)?

UNCLASSIFIED EXAMPLES.

1. What cost 3 piles of 4 foot wood, one 58 *ft.* long and 5 *ft.* high, another 70 *ft.* long and $5\frac{1}{2}$ *ft.* high, and the other 65 *ft.* long and 6 *ft.* high, at $5.50 per cord?

2. At $1.25 per gallon what cost 1 *bbl.* 15 *gal.* 3 *qt.* of molasses? 2 *bbl.* 12 *gal.* 2 *qt.* 1 *pt.*?

3. A ladder 30 *ft.* in length was found to reach just to the eaves of a building when its foot was 12 *ft.* from the foundation. What was the height of the building?

4. I bought 2 horses for $420, paying $48 more for one than for the other. What was the price of each?

5. What per cent of $5\frac{1}{2}$ is $\frac{1}{3}$? $\frac{1}{2}$? 1? $1\frac{2}{3}$? $1\frac{3}{8}$? 11?

6. Two bodies start together to move around the same circle in the same direction; one goes 5 times around while the other goes 7. How often will they be together? When will they be together at the point of starting?

7. I have a note for $300 given July 24, 1872, bearing 8% interest, and due October 13, 1876. On the note is endorsed $110, April 5, 1874. What is the value of the note May 20, 1875, money being worth 10% at this date?

8. What must be the depth of a cylindrical cistern whose diameter is 6 *ft.*, that it may contain 60 *bbl.*? 100 *bbl.*?

9. The scale of the common thermometer (*Fahrenheit*) is divided into 180° between the freezing point (32°) and

the boiling point (212°). The *Centigrade* scale is divided into 100°, from freezing (0°) to boiling (100°). What is the relative length of the degrees?

10. Show that 68° F. = 20° C.; 85° F. = $29\frac{4}{9}$° C.; 30° C. = 86° F.; −23° F. (*i. e.* 23° below 0) = $-30\frac{5}{9}$° C.

11. The diameter of the moon being to that of the earth as 3 : 11, what is the relation between their volumes?

12. What is the diameter of a grindstone when it is $\frac{1}{3}$ worn away, its original diameter having been 2 *ft.*? What when $\frac{2}{3}$ worn? When $\frac{1}{2}$ worn? When $\frac{1}{4}$? $\frac{3}{4}$?

13. A merchant bought goods at 25% below their nominal price, and sold them at 20% above, thereby making $1920. How much did he invest? *Ans.*, $3200.

14. A quantity of flour lasts a man and wife 9 days, and the wife alone 27 days; how long would it last the man alone?

15. If a ball of thread is 4 inches in diameter, what will be the diameter in each of three conditions—when $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of it are wound off?

Ans., 3.634 +; 3.175 −; 2.52 − inches.

16. What part of the thread will be left, when the diameter is reduced to 2 inches? *Ans.*, $\frac{1}{8}$.

17. A agreed to labor for $2.50 per day, on condition that he should forfeit 50 *c.* every day he was idle. At the end of 100 days he received $190. How many days was he idle?

18. The bank is discounting at 12%. I wish $500 for 60 days. For what must I draw the note (exact interest)?

Ans., $510.57.

19. In a mixture of gold and silver consisting of 100 *oz.* there are 6 *oz.* of silver; how much gold must be added that there may be $\frac{3}{4}$ *oz.* of silver to 10 *oz.* of gold?

Ans., 56 *oz.*

20. How many yards of carpeting $\frac{3}{4}$ *yd.* wide, must I buy to carpet a room 20 by 25 feet, the strips running lengthwise of the room and there being 4 *in.* waste on each strip in matching? *Ans.*, 76 *yd.*

21. When Gen. Tom Thumb (Chas. S. Stratton) was 5 years old he measured 2 *ft.* in height, and weighed 16 *lb.* What would be the weight of a man of similar form who was 6 *ft.* tall? *Ans.*, 432 *lb.*

22. The amount of A's money for 7 *yr.* at 10% is $244 more than its amount for 6 *yr.* at 8%; required A's money.

23. The eaves of a house are at the same height and 30 feet apart. The ridge pole is 12 feet higher than the eaves, and just midway between them. The house is 40 feet long. How many shingles will it take to cover the roof, if each shingle covers a space 6 inches long and 4 inches broad? *Ans.*, $9\frac{1}{4}$ *M.* within 5 shingles.

24. I bought 25000 feet of board at $2.25 per thousand, and sold $\frac{1}{2}$ of them for what $\frac{2}{3}$ of the whole cost. What per cent did I gain on the part sold? *Ans.*, $33\frac{1}{3}$%.

25. A man bought a bar of gold at $192 *pr. lb.* and sold it for $16 *pr. oz.*, weighing it in both cases by avoirdupois weight. How much did he gain, the true weight of the bar being 5 pounds? Did he gain, or lose, by selling by avoirdupois instead of troy? *Ans.*, $263.31 +.

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26. What are the dimensions of a rectangular granary whose width and height are each $\frac{2}{3}$ of its length, and which will contain 1500 bushels of wheat?

Ans., 10.756 *ft.* wide; 10.756 *ft.* high; 16.134 *ft.* long.

27. Required the difference between the simple interest and compound interest of $800 for 16 *yr.* 8 *mo.*, at 7%.

28. Required the time of day, provided the time past noon equals $\frac{2}{3}$ of the time to midnight.

Ans., 48 *min.* past 4, P. M.

29. What is $\sqrt{.1}$? $\sqrt[3]{.8}$? $\sqrt{1.44}$? $\sqrt{.251}$? $\sqrt[3]{7}$?

30. If a 2 *in.* pipe will fill a cistern in 6 hours, how long will it take a 3 *in.* pipe to fill it, the water flowing at the same velocity? *Ans.*, $2\frac{2}{3}$ *hr.*

31. Bought a $500 10% note, due 6 *mo.* from the time I bought it, which was 2 *yr.* 8 *mo.* from its date. I paid $580 for the note but had to borrow $400 at bank for $^{30}/_{33}$ *da.* at 2% a month. How much did I make, allowing money to be worth 7%?

Ans., $26.34 at the maturity of the note.

Amount of note at maturity $633.33 +. Amount of $180 for 6 *mo.*, at 7%, $186.30. Bank note $409. Amount of this for 4 *mo.* 27 *da.* at 7%, $420.69.

32. The amount of B's money for 8 *yr.* at 6% is $5100 more than the interest of his money for 10 *yr.* at 8%; required B's money. *Ans.*, $7500.

33. In the tomb of one of the Incas, £150000 in gold was found. What was the weight of the whole, estimating the average fineness of the gold at 18 carats,* £1 sterling containing 113.0001 grains of pure gold?

Ans., 1 *T.* 1228 *lb.* 9 *oz.*+.

34. Gold is quoted to-day (Feb. 17, 1875) at 116. What will an English book for which I have to pay £1 2*s.* 6*d.* gold, cost me in our paper currency? *Ans.*, $6.35.

35. With gold at 115 what will a bill of German books amounting to 25 marks including duty, cost me in our paper currency? *Ans.*, $6.84.

* A carat is $\frac{1}{24}$. 18 carats fine is $\frac{18}{24}$ gold, or $\frac{3}{4}$ gold and $\frac{1}{4}$ alloy.

36. French exchange being 117, what will it cost me in our paper currency to pay 1000$^{fr.}$ in Paris.

Ans., $225.81.

37. A wishes to draw $1000 from a bank for 60 days; for what sum must he give his note, discounting at 10%?

Ans., $1017.56.

38. If I buy U. S. bonds bearing 5% interest in gold and pay $120 in currency for $100 in bonds, what per cent in currency shall I realize on my investment, gold being worth $115\frac{1}{8}$? *Ans.*, $4\frac{4}{5}$%.

39. A holds a note for $500 dated May 10, 1873, and bearing 7% annual interest, the principal payable in 5 equal annual installments. September 18, 1875, B proposes to buy the note (all previous payments having been made as agreed), and pay such a sum for it as will allow him 10% per annum on his money. What must B pay for the note? (No grace.) *Ans.*, $296.20.

The note yields $121 7 *mo.* 22 *da.* after the purchase; $114 1 *yr.* 7 *mo.* 22 *da.* after; and $107 2 *yr.* 7 *mo.* 22 *da.* after. The present worths of these sums for the respective times at 10% are $113.674+, $97.901+, and $84.622+.

40. If I buy bonds for 85 cents on a dollar which pay 3% semi-annual interest on their face, what per cent per annum does this give me on my investment, money being worth 10%? *Ans.*, $7\frac{6}{17}$%.

41. The ratio of the circumference of a circle to its diameter being 3.1416, how many revolutions in a second does a 6 foot drive-wheel of an engine make when the engine is running at the rate of 30 *mi.* an hour?

Ans., A little more than $2\frac{1}{3}$ times a second.

42. Cleveland, O., is in longitude 81° 47′ W., and Bos-

ton in 71° 4′ 9″ W. When it is 5 A. M. at Boston what time is it at Cleveland. *Ans.*, 4 o'clock, 17 *min.* 8 *sec.*+.

43. What per cent is made by buying stocks at 15% below par (their nominal value), and selling at 10% above? *Ans.*, $29\frac{7}{17}$%.

44. What per cent is lost by buying bonds at 8% premium and selling them at 5% discount? *Ans.*, 12%.

45. If a man $5\frac{1}{2}$ feet high weighs 121 *lb.*, what should be the weight of a man of similar proportions 6 feet high?

46. Bought a piece of cloth containing 36 *yd.*, at $\$2\frac{1}{2}$ per yard, and sold $\frac{4}{5}$ of it at what the whole cost me, and the remainder at $3 per yard. What per cent did I make?

47. What does it cost to excavate a cellar 20 *ft.* by 18 *ft.* and 5 *ft.* deep, at 15 *c.* per cubic yard?

48. What is the difference in longitude between two places whose difference in time is 52 *min.* 18 *sec.*?

49. A and B together can do a piece of work in 6 days, but B alone requires 10 days to do it. In what time can A do it alone? *Ans.*, 15 days.

50. Three men, A, B, and C, agree to do a certain piece of work. A and B can do the work in $6\frac{2}{3}$ days; B and C in 12 days; and A and C in 10 days. How long will it take each separately to do it?

Subtract the part A and C can do in a day from the sum of what A and B, and B and C can do in a day, and the remainder, $\frac{2}{15}$, is twice what B can do in a day. *Ans.*, A 12 days, B 15, and C 60.

51. What are the dimensions of a rectangular box containing 3000 cubic feet, the dimensions being to each other as 2, 3, and 4? *Ans.*, 10, 15, and 20 feet.

52. Divide 88 into 2 parts which shall be to each other as 4 : 7. Into 3 which shall be as 2 : 3 : 4.

53. Divide 56 into 2 parts so that the less part shall be to 56 as 3 : 7.

54. Is $\frac{3}{5}$ increased, or diminished, by adding 4 to each term? How much? $\frac{5}{5}$? $\frac{6}{5}$?

55. Two houses are built on exactly the same ground-plan for corresponding stories; the stories are of the same height; but the first has 2 stories and the second 3; while the corresponding ground measurements are as 2 to 3. If the material for the small house cost $3000, what was the cost of that of the larger? *Ans.*, $6750.

56. The sun's diameter being 852,573 *mi.*, and the earth's radius 3962 *mi.*, how many times as large as the earth is the sun?

57. A 6% note for $346.36 was dated March 26, 1863; and had endorsements, July 20, 1863, $54.75; April 8, 1864, $10; Sept. 26, 1864, $5.50; Jan. 6, 1865, $150.46. What was due Nov. 2, 1865. *Ans.*, $166.18.

58. On a note for $340, bearing 10% annual interest, the interest was paid one year, and $20 on the next year's interest. After this no more interest was paid. What was the amount of the note 3 *yr.* 8 *mo.* 15 *da.* from date, allowing simple interest on accrued interest?

59. January 1, 1872, the German Reichstag passed a a new coinage act making the Mark the unit of the currency of the Empire, and enacting that "An imperial gold coin will be struck, of which 139½ pieces will contain a pound (1.1025 *lb. Av.*) of fine gold, and the tenth part of this gold coin will be named a *Mark.*" Now the standard U. S. gold dollar weighs 25.8 *gr.*, and is 900 parts in 1000 pure gold. On this basis justify the value of the Mark as given by the Director of the U. S. mint, 23.8 cents.

60. French Coin is of the same fineness as our own. A franc is 19.3 cents. Our gold dollar weighs 25.8 *gr.*, and our silver half-dollar 192.9 *gr.* What should be the weight of a silver franc in grams? What of a Napoleon?

APPENDIX I.

It may be well for the teacher to explain somewhat at length how our public lands are divided and described. Thus when a new territory is to be surveyed, the first thing the surveyor does is to run one (or more) north and south lines through some convenient parts of it. These are run with great care, are carefully marked, by posts, stones, marks upon trees, or other means, throughout their entire length, and are called PRINCIPAL MERIDIANS. In a similar way one or more east and west lines, called BASE LINES, are run and marked. After this the whole country is checked up into townships by running north and south lines parallel to the Principal Meridian and six miles apart, and east and west lines in a similar manner parallel to the Base Line. The north and south rows of these townships are called *Ranges*, and are numbered east and west from the *Principal Meridian.* The townships in each row are numbered north and south from the *Base Line.* Thus in Ohio the western boundary of Pennsylvania is the eastern principal meridian from which ranges are numbered up to Range XX West, which reaches the western boundary of Huron county. Again the western boundary of Ohio is another principal meridian from which ranges are numbered eastward to Range XVII East. (Let the pupils tell from this the entire width of the State of Ohio, 222 miles.) The Base Line in this State runs along the southern boundary of Paulding, Seneca, and Huron counties. From this line townships are numbered, as Town 1 North, Town 2 North, etc.; Town 1 South, Town 2 South, etc. Give the pupils exercise in locating townships from such descriptions as T. 2 N., R. 4 E.; T. 8 S., R. 5 W.; etc. The first of these is read "Town 2 north, Range 4 east," and locates Brown township in Paulding county, 6 miles north of the Base Line, and 18 miles east of the West Meridian, or west line of the State. Such questions as the following will also suggest themselves; How far from the west line of Pennsylvania does a man in Ohio live who is in Range 10 West? How far one who is in Range 15 West? Similar questions can be framed with reference to the Base Line and the number of the township.

The Base Line in Michigan runs along the north line of the second tier of counties, viz., Wayne, Washtenaw, etc. The Principal Meridian commences at the Ohio line and runs north between Lenawee and Hillsdale counties. The fact should be understood that the *names* which we apply to the townships form no part of a description of them as given in deeds; this is done exclusively by Range and number. Sometimes (usually) the *political* divisions we name townships correspond to these surveyed and recorded towns, but often other divisions are made for political purposes; otherwise all the townships would be regular (except those along lake and river boundaries, etc.).

The following cuts illustrate the division of a township into sections and the subdivisions of sections. These sections are desig-

A TOWNSHIP.

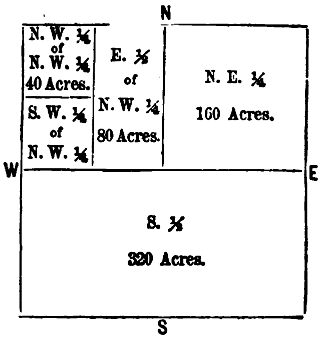
N

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

W — E

S

A SECTION.



nated by number, beginning at the N.E. corner and numbering across to the West, then back to the East, and thus back and forth. The division of the section is not made in the original survey, but the middle of the sides on the boundary lines are marked so that all that is afterward necessary is to run the lines across in order to divide the sections into quarters. For purposes of sale these quarters are again divided into halves and quarters. The following designations will be readily understood: "Northwest quarter of the northwest quarter of Section 10," written, N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$ of

Sec. 10. See cut of Section. So the east half of the northwest quarter is designated E. ½ of N.W. ¼ of Sec. 10, etc.

The teacher may give exercises in describing the position of the following pieces of land, and by diagram on the blackboard illustrate them. A few descriptions might be written on the board and the pupils required to illustrate the location by diagrams on their slates.

N.E. ¼, of S.E. ¼, Sec. 20, T. 7 N., R. 9 E.
W. ½, S.W. ¼, Sec. 17, T. 10 S., R. 6 W.
S. ½, Sec. 28, T. 1 S., R. 15 E.
E. ½, N.E. ¼, Sec. 8, T. 6 N., R. 18 W.
W. ½, Sec. 16, T. 13 N., R. 11 E.
E. ½, S.E. ¼, Sec. 32, T. 5 S., R. 13 E.

APPENDIX II.

CONTRACTIONS.

IN MULTIPLICATION.

1. *To multiply by* 25.—Multiply by 100 (**59**), and then divide by 4.

2. *To multiply by* 125.—Multiply by 100 (**59**), and add to this product ¼ itself.

Ex. Multiply 2346 by 25.

$$\begin{array}{r} 234600 \\ \hline 58650 \end{array}$$

Ex. Multiply 5082 by 125.

$$\begin{array}{r} 508200 \\ 127050 \\ \hline 635250 \end{array}$$

3. *To multiply by a number represented by 2 digits, one of which is* 1.—Multiply by the digit which is not 1, and write the product

under the multiplicand, removing this product one place to the left if this digit is 10's, and 1 to the right if it is units, adding these numbers.

Ex. Multiply 78579 by 81.

$$\begin{array}{r} 78579 \\ 628632 \\ \hline 6364899 \end{array}$$

Ex. Multiply 78579 by 18.

$$\begin{array}{r} 78579 \\ 628632 \\ \hline 1414422 \end{array}$$

4. *To multiply by a number represented by 9's.*—Multiply by 1 with as many 0's at the right as there are 9's (**59**), and then subtract the multiplicand. If the right-hand figure of the multiplier is 8, subtract twice the multiplicand, etc.

Ex. Multiply 857639 by 999.

$$\begin{array}{r} 857639000 \\ 857639 \\ \hline 856781361 \end{array}$$

Ex. Multiply 857639 by 998.

$$\begin{array}{r} 857639000 \\ 1715278 \\ \hline 855923722 \end{array}$$

Ex. Square 9999.

$$\begin{array}{r} 99990000 \\ 9999 \\ \hline 99980001 \end{array}$$

5. *To square any number ending in* $\frac{1}{2}$.—Multiply the integral part by 1 more than itself and to the product add (annex) $\frac{1}{4}$.

Ex. Square $34\frac{1}{2}$.

$$\begin{array}{r} 34 \\ 35 \\ \hline 170 \\ 102 \\ \hline 1190\frac{1}{4} \end{array}$$

Ex. Square $17\frac{1}{2}$. $128\frac{1}{2}$.

Ex. Square 145.

$(145)^2 = (14.5)^2 \times 100 = (14\frac{1}{2})^2 \times 100.$

$$\begin{array}{r} 14 \\ 15 \\ \hline 70 \\ 14 \\ \hline 21025 \end{array}$$

Ex. Square 595.

DEM.—$(m + \frac{1}{2})^2 = m^2 + m + \frac{1}{4} = m(m + 1) + \frac{1}{4}.$

6. *To find the product of two numbers involving decimals, to within a unit of any proposed order.*

Ex. An example will illustrate the process; Multiply 763.05403-698956 by 25.4463057845, to within 1 ten-thousandth.

Operation.

```
  763.05403698956
5487503644.52
  ---------------
    1526108074
     381527018
      30522161
       3052216
        457832
         22892
           382
            58
             6
    -----------
    19416.90634
```

Explanation.—Using either number as multiplier, write it with the order of its digits reversed, and so that its units figure will fall under the specified order in the multiplicand (in this case under ten-thousandths). Then multiply by the digits of this multiplier, commencing in each case with the figure immediately over the multiplier, and writing the partial products so that the first figure in each shall stand under the first digit multiplier, observing in each case to "carry" the nearest approximate number of units from the product of the next lower order in the multiplicand than the one used.

Note.—For many methods of *Approximate* computations, see a little work by Skinner, recently published by Henry Holt & Co., N. Y. The Author does not think very highly of such approximations, especially when they prevent checks on the work, as they usually do.

IN DIVISION.

7. *To divide by* 25.—Multiply by 4 and divide by 100, *i. e.*, remove the decimal point 2 places to the left.

8. *To divide by* 15, 35, 45, or 55.—Double the dividend and divide by 30, 70, 90, or 110.

9. *To divide by* 125.—Multiply by 4 and divide by 1000.

10. *To divide by* $3\frac{1}{3}$.—Multiply by 3 and divide by 10.

11. *To divide by* $12\frac{1}{2}$, *or* $16\frac{2}{3}$.—Multiply by 8, or by 6, and divide by 100.

12. *When the division is required to be extended only till some specified order of decimals is reached.* We may observe when an order is reached as many places to the left of this order as there are figures in the divisor less 1, and after this drop a figure from the divisor instead of bringing down one from the dividend at each division. In doing this we should observe in multiplying to "carry" from the product of the last order (or two orders) cut off.

```
754.337385 | 61'.3'4'7
613 47     |----------
---------- | 12.2962
140 867
122 694
-------
 18 173
 12 269
 ------
  5 904
  5 521
  -----
    383
    368
    ---
     15
     12
     --
      3
```

The accents are used to mark off the figures as they are dropped.

Ex. Divide 785.64 by 84.37216, extending the quotient to 10-millionths

As 10-millionths is the 7th order of decimals, and 1 less than the number of figures in my divisor is 6, I may begin to drop off after having obtained the tenths of the quotient, *i. e.*, after the 2d step in the division.

PROGRESSIONS.

13. An Arithmetical Progression is a series of numbers which increase or decrease by a common difference, as 3, 5, 7, 9, 11 ; or 28, 23, 18, 13, 8.

14. The Last Term of an *increasing* arithmetical progression is evidently equal to the first term + the common difference taken as many times as there are terms less 1. Thus the 5th term of the 1st series above is 3 + 4 times 2 = 11.

The last term of a *decreasing* arithmetical progression is equal to the first term − the common difference multiplied by the number of terms less 1. Thus the 5th term of the 2d series above is 28 − 4 times 5 = 8.

15. The Sum of an arithmetical progression is ½ the sum of the extremes multiplied by the number of terms.

This will be evident from an inspection of this operation. Hence

$$\begin{array}{r} 3+\ 5+\ 7+\ 9+11 = \text{Sum.} \\ 11+\ 9+\ 7+\ 5+\ 3 = \text{Sum.} \\ \hline 14+14+14+14+14 = \textit{Twice} \text{ the Sum.} \end{array}$$

the *Sum* $= \left(\frac{3+11}{2}\right) \times 5 = 35$, the Sum of the Series.

Ex. 1. First term 7, common difference 4, series increasing, find the 10th term and the sum.

2. First term 134, com. diff. 7, series decreasing, find the 8th term and the sum.

3. A 10% note for $300, bearing annual interest, has been running 8 *yr.* and no interest has been paid. What is due, allowing simple interest on the deferred payments of annual interest?

The 8th year's interest is $30, the 7th $33, the 6th $36, etc. Hence the interest is an arithmetical progression of 8 terms of which 30 is the first and 3 the com. diff. The last term is therefore 51, and the sum $\left(\frac{51+30}{2}\right) \times 8 = 324$. Amount of note $624.

---

16. A Geometrical Progression is a series of numbers which increase or decrease by a common multiplier, called the *rate.* If the rate is more than 1 the series is increasing; if less than 1 it is decreasing. Thus 3, 9, 27, 81, 243, is an increasing geometrical progression, rate 3. 6561, 729, 81, 9, is a decreasing geometrical progression, rate $\frac{1}{9}$.

17. The Last Term of a *geometrical progression* is the first multiplied by the rate raised to a power whose index is 1 less than the number of terms. This appears when we consider that the 2d term is the first multiplied by the rate, the 3d is the first multiplied 2 times in succession by the rate, etc.

18. The Sum of a *geometrical progression* is the difference between the last term multiplied by the rate and the first term, divided by the *rate* − 1 if the series is *increasing*, and by 1 − *the rate* if it is decreasing.

Thus, taking the series 3, 9, 27, 81, 243, of which the rate is 3,

the sum is $\frac{3\times 243-3}{3-1}$, or $\frac{729-3}{2} = 363$. An inspection of the following will indicate the reason for the rule:

$$\begin{array}{ll} 729+243+81+27+9 & = 3 \text{ times the sum.} \\ \underline{\quad\quad 243+81+27+9+3} & = \text{the sum.} \\ 729 \quad\quad - \quad\quad 3 & = (3-1) \text{ times the sum.} \end{array}$$

Again,

$$\begin{array}{ll} 6561+729+81+9 & = \text{the sum.} \\ \underline{\quad\quad 729+81+9+1} & = \tfrac{1}{9} \text{ the sum.} \\ 6561 \quad\quad - \quad\quad 1 & = (1-\tfrac{1}{9}) \text{ times the sum.} \end{array}$$

Ex. 1. First term of a *geom. prog.* 7, rate 4; what is the 8th term? What the sum?

2. First term 6250, rate $\frac{1}{5}$; what is the 6th term? What the sum?

ALLIGATION.

19. Ex. 1. A grocer mixes together 12 *lbs.* of tea at 50 cents, 16 *lbs.* at 72 cents, 12 *lbs.* at 65 cents, 18 *lbs.* at 85 cents, and 100 *lbs.* at 42 cents. How much per *lb.* is the mixture worth? *Ans.*, $53\frac{1}{31}$ *c.*

How many pounds of the mixture were there. What was the entire mixture worth?

2. Having melted together 7 *oz.* of gold 22 carats fine, $12\frac{1}{2}$ *oz.* 21 carats fine, and 17 oz. 9 carats fine, I wish to know the fineness of each ounce of the mixture? *Ans.*, $15\frac{44}{73}$ carats.

A carat is $\frac{1}{24}$ part. 7 *oz.* of alloyed gold 22 carats fine contains $7 \times \frac{22}{24}$, or $\frac{154}{24}$ *oz.* pure gold. 17 *oz.* 9 carats fine contains $\frac{153}{24}$ *oz.* pure gold, etc. The question is, How many 24ths of the mixture was pure gold?

Such examples as the above were formerly classed under *Alligation Medial*, but it is evident that there is no propriety in dignifying so simple a case of the first principles of arithmetic in any such way.

20. Alligation Alternate is of little or no practical use, and is with propriety now dropped from most of our courses of training in arithmetic, even when treated in the text-book. Moreover, the arithmetical solution is very cumbrous, while the algebraic is exceedingly simple. We give a few examples with the algebraic solution.

1. A merchant has sugar worth 10 cents, 12 cents, 14 cents, 15 cents, 16 cents, 17 cents, and 18 cents per pound, and wishes to form a mixture worth $12\frac{1}{2}$ cents a *lb.* How many pounds of each must he use.

Let v, x, y, z, w, r, and s represent respectively the No. *lbs.* of each.

Then $10v+12x+14y+15z+16w+17r+18s = 12\frac{1}{2}\ (v+x+y+z+w+r+s)$.

From this $1\frac{1}{2}y+2\frac{1}{2}z+3\frac{1}{2}w+4\frac{1}{2}r+5\frac{1}{2}s = 2\frac{1}{2}v+\frac{1}{2}x$, or $3y+5z+7w+9r+11s = 5v+2x$.

Now we may give any values we please to all *but one* of these letters, *provided they do not make that one negative*, and find a corresponding value for the other. Thus let y, z, w, and r each be 1, and $s = 2$, and $v = 3$. Then we have $3+5+7+9+22 = 15+2x$, and $x = \frac{31}{2} = 15\frac{1}{2}$. Again, let y and z each $= 4$, w and r each 7, s and $x = 10$, and we have $12+20+49+63+110 = 5v+20$; whence $v = \frac{234}{5} = 46\frac{4}{5}$, and so on *ad infinitum.*

2. If the quantity of one or more of the ingredients in the above example were given, as 10 *lb.* of the 18 *c.* and 15 of the 14 *c.*, it would in no way complicate the problem, as we would then proceed to assign values to all the others, save one, at pleasure, as above.

3. If the entire amount of the mixture was given, as for example in Ex. 1, if there were to be 120 *lb.* we should simply substitute for $v + x + y + z + w + r + s$ 120, and then proceed as before.

Thus if any one desires a knowledge of Alligation Alternate, he can obtain it in the least time, and get by far the most comprehensive view of it, by getting a little knowledge of the simple equation. The teacher can give all the knowledge of the equation that is needed in a single lesson.

APPENDIX III.

21. The *Metric System*, originally devised and adopted by the French, makes *The Meter* the fundamental unit. It was designed that the Meter should be $\frac{1}{10000000}$ part of a quadrant of a meridian of the earth. With this design an arc of the meridian, starting from the parallel of Dunkirk in the extreme north of France, and running the entire length of France, and terminating in the parallel of Barcelona in the north of Spain, was measured by Delambre and Méchain, as directed by the French government. From this measurement the whole quadrant was computed, and the Meter established as $\frac{1}{10000000}$ part of it. It is now known that there are irregularities in the form of the earth which would make such measurements give different results when taken in different places, and that the Meter thus established is about $\frac{1}{5000}$ of an inch too short.

The meter being thus established, the *liter* is made a cubic *decim*, and this amount of pure water at the temperature of melting ice is made the kilŏg.

The Metric System has now come to be adopted by most civilized nations, although generally only permissively, as a system legally recognized, but which may be used by the people, or not, as they see fit. Nevertheless, all nations, except the French (and they to a considerable extent), continue to use their various and older systems. It is, however, coming to be pretty generally accepted for scientific and philosophical purposes, and its cosmopolitan character makes it specially desirable that it should be understood by all who lay any claim to general intelligence.

In attempting to teach the metric system, it is of first importance that the pupils be made familiar with the measures themselves. *The Metric Bureau*, Boston, is organized for the purpose of furnishing apparatus for teaching, and information upon this subject.

22. APPENDIX IV.

Value of Foreign Coins in U. S. Money (gold) as proclaimed by the Secretary of the Treasury, January 1, 1878.

Country.	Unit.	Metal.	U. S.
Argentine Republic*	Peso fuerte	G.	$1.00
Austria	Florin	S.	.45,3
Belgium	Franc	G. & S.	.19,3
Bolivia	Dollar	G. & S.	.96,5
Brazil	Milreis of 1,000 reis	G.	.54,5
Bogota	Peso	G.	.91,2
Canada*	Dollar	G.	1.00
Central America	Dollar	S.	.91,8
Chili	Peso	G.	.91,2
Cuba*	Peso	G.	.92,5
Denmark	Crown	G.	.26,8
Ecuador	Dollar	S.	.91,8
Egypt	Pound of 100 piasters	G.	4.97,4
France	Franc	G. & S.	.19,3
Great Britain	Pound Sterling	G.	4.86,6½
Greece	Drachma	G. & S.	.19,3
German Empire	Mark	G.	.23,8
Hayti	Dollar	S.	.95,2
India	Rupee of 16 annas	S.	.43,6
Italy	Lira	G. & S.	.19,3
Japan	Yen	G.	.99,7
Liberia	Dollar	G.	1.00
Mexico	Dollar	S.	.99,8
Netherlands	Florin	S.	.38,5
Norway	Crown	G.	.26,8
Paraguay*	Peso	G.	1.00
Peru	Dollar	S.	.91,8
Porto Rico*	Peso	G.	.92,5
Portugal	Milreis of 1,000 reis	G.	1.08
Russia	Rouble of 100 copecks	S.	.73,4
Sandwich Islands	Dollar	G.	1.00
Spain	Peseta of 100 centimes	G. & S.	.19,3
Sweden	Crown	G.	.26,8
Switzerland	Franc	G. & S.	.19,3
Tripoli	Mahbub of 20 piasters	S.	.82,9
Tunis	Piaster of 16 caroubs	S.	.11,8
Turkey	Piaster	G.	.04,3
U. S. of Columbia	Peso	S.	.91,8
Uruguay*	Patacon	G.	.94,9

* Taken from the Treasury circular for 1875, as they are not mentioned in 1878.

APPENDIX V.

Bank Discount, or Expeditious Methods of Computing Interest for 33, 63 and 93 days.

GENERAL METHOD.—FOR 12 PER CENT.

For 33 da. take 11-1000ths of the Principal.

For 63 da. take 21-1000ths of the Principal.

For 93 da. take 31-1000ths of the Principal.

Demonstration.—For 12% for 33 *da.*, letting P represent the Principal, we have

$$\frac{\overset{11}{\cancel{33}} \times \cancel{12} \times P}{\underset{\underset{10}{\cancel{120}}}{\cancel{360}} \times 100} = \frac{11}{1000} \text{ of } P.$$

The others are demonstrated in the same manner.

Observe that to take $\frac{11}{1000}$ is to multiply by 11, and remove the decimal point three places to the left.

To multiply by 11 write the Principal under itself, removing it one place to the left, and add; to multiply by 21, write 2 times the Principal in the same way; to multiply by 31, write 3 times the Principal in the same way.

Example.—Find the interest on $5872 at 12% for 33 *da.*, 63 *da.*, 93 *da.*

$5 872	$5 872	$5 872
58 72	117 44	176 16
$64.59, Int. for 33 *da.*	$123.31, Int. for 63 *da.*	$182.03, Int. for 93 *da.*

FOR OTHER RATES PER CENT. THAN 12.

First find 12% as above. Then,

For 6%.—Take $\frac{1}{2}$ of 12%.

For 7%.—Add $\frac{1}{6}$ to 6%.

For 8%.—Deduct $\frac{1}{3}$ from 12%.

For 9%.—Deduct $\frac{1}{4}$ from 12%.

For 10%—Deduct $\frac{1}{6}$ from 12%.

Note.—When the Principal is a round number of hundreds of dollars, 12% can be told at a glance: Thus 12% on $300 for 33 *da.* is $3.30; on $500, $5.50; on $700, $7.70, etc. Again, 12% on $300 for 63 *da.* is $6.30; on $800, $16.80; on $700, $14.70, etc. For 93 *da.* 12% on $200 is $6.20; on $100 is $3.10; on $400, $12.40. Thus it will be seen that for 33 *da.* the dollars in the interest are the hundreds of dollars in the Principal; for 63 *da.*, twice the hundreds; for 93 *da*,, three times the hundreds; the cents in each case being the Principal with the right hand 0 dropped.

Example.—Find the interest on $600 at 6% for 33 *da.*, 63 *da.*, 93 *da.*

$6.60	$12.60	$18.60
$3.30 for 33 *da.*	$ 6.30 for 63 *da.*	$ 9.30 for 93 *da.*

Example.—Find the interest on $200 at 8% for 33 *da.*, 63 *da.*, 93 *da.*

$2.20	$4.20	$6.20
.73	1.40	2.07
$1.47 for 33 *da.*	$2.80 for 63 *da.*	$4.13 for 93 *da.*

SPECIAL METHOD FOR 7 PER CENT.

For 33 ***da.***—**From 7-1000ths of the Principal take 1-12.**

For 63 ***da.***—**From 1-80th of the Principal take 1-50 of the 1-80.**

For 93 *da.*—Take $\frac{18\frac{1}{12}}{1000}$ of the Principal.

Demonstration.—For 33 *da.* we have

$$\frac{\overset{11}{\cancel{33}} \times 7 \times P}{\underset{120}{\cancel{360}} \times 100} = \tfrac{11}{12} \text{ of } \tfrac{7}{1000} \text{ of } P = .$$

$$(\tfrac{7}{1000} \text{ of } P) - (\tfrac{1}{12} \text{ of } \tfrac{7}{1000} \text{ of } P).$$

For 63 *da.*
$$\frac{\overset{7}{\cancel{63}} \times 7 \times P}{\underset{40}{\cancel{360}} \times 100} = \tfrac{49}{4000} \text{ of } P =$$

$$(\tfrac{50}{4000} - \tfrac{1}{4000}) \text{ of } P = (\tfrac{1}{80} - \tfrac{1}{50} \text{ of } \tfrac{1}{80}) \text{ of } P.$$

For 93 *da.* $\dfrac{\overset{31}{\cancel{93}} \times 7 \times P}{\underset{120}{\cancel{360}} \times 100} = \frac{217}{12000}$ of $P = \dfrac{18\frac{1}{12}}{1000}$ of P.

Example.—Find the interest on \$456 at 7%, for 33 *da.*, 63 *da.*, 93 *da.*

\$456	\$456	456
3.192	5.70	3648
.266	.11	38
\$2.93 for 33 *da.*	\$5.59 for 63 *da.*	\$8.25 for 93 *da.*

SPECIAL METHOD FOR 8 PER CENT.

For 33 *da.*—Take $7\frac{1}{3}$-1000ths of the Principal.

For 63 *da.*—Take 14 1000ths of the Principal.

For 93 *da.*—From 21-1000ths of the Principal take $\frac{1}{3}$ of $\frac{1}{1000}$ of it.

Demonstration.—For 33 *da.* $\dfrac{33 \times 8 \times P}{360 \times 100} = \dfrac{7\frac{1}{3}}{1000}$ of P.

So $\dfrac{63 \times 8 \times P}{360 \times 100} = \dfrac{14}{1000}$ of P; and $\dfrac{93 \times 8 \times P}{360 \times 100} = \dfrac{20\frac{2}{3}}{1000}$ of P$=$ $\left(\dfrac{21}{1000} - \dfrac{1}{3} \text{ of } \dfrac{1}{1000}\right)$ of P.

Example.—Find the interest on \$1426 at 8 %, for 33 *da.*, 63 *da.*, 93 *da.*

\$1 426	\$14 26	\$1 426
9.982	5 70	28 52
.475	\$19.96 for 63 *da.*	\$29.946
\$10.46 for 33 *da.*		.475
		\$29.47 for 93 *da.*

SPECIAL METHOD FOR 10 PER CENT.

For 33 *da.*—Deduct 1-12 and divide by 100.

For 63 *da.*—From twice the Principal deduct 1-4 of it and divide by 100.

For 93 *da.*—To 1-40 of the Principal add 1-30 of the 1-40.

Example.—Find the interest on $3486 at 10% for 33 *da.*, 63 *da.*, 93 *da.*

	$34 86	$348 6
$34 86	69 72	87.15
2 91	8 72	2.91
$31.95, for 33 *da.*	$61.00, for 63 *da.*	$90.06, for 93 *da.*

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24. In computing interest or discount, "a year" is a calendar year, and "a month" a calendar month. To this there are no exceptions in the States. Also, when years, months and days are mentioned in the contract, the days are reckoned as 30ths of a month. But in transactions with the General Government, the month unit is dropped, and the time is reckoned in years and days, the days being called 365ths of a year. In New York, when time is specified in days, the days are to be reckoned as 365ths of a year.

25. Notes falling due on Sunday, or on a legal holiday, are in most of the States required to be paid on the preceding day. In Connecticut if the day of maturity is a legal holiday falling on Sunday, the note is due on Monday. In Maine and Nebraska, if the day of maturity is a legal holiday falling on Monday, the note is payable on Tuesday; and in New York a note maturing on a legal holiday, or Monday observed as such holiday, is payable the following day.

26. In the following States simple interest can be collected on unpaid annual interest, viz.: Michigan (same rate as borne by the note), Ohio, Wisconsin, Vermont, New Hampshire, Iowa (6%). In Pennsylvania, and probably in Georgia, Illinois and Indiana, by special contract (only). In Massachusetts such annual interest can be sued for when due, but no interest can be collected on it.

27. In Pennsylvania a note for 30 *da.* is discounted at bank for 34 *da.*; one for 60 *da.*, for 64 *da.*; one for 90 *da.*, for 94 *da.* This practice comes from counting both the day on which the note is drawn and the day on which it falls due. In the ordinary practice, only one of these is counted.

ANSWERS.

Pages 35–36. *1.* 2353. *2.* 24371. *3.* 292587. *4.* 689983. *5.* 1084474. *6.* 3618913. *7.* 9901906. *8.* 1245441. *9.* 969754. *10.* 1234567890. *11.* 200000. *12.* 111010. *13.* 412524. *14.* 26072. *15.* 207996. *16.* 8696260. *17.* 4177075. *18.* 710891. *19.* 100185371743.

Pages 37-40. *7.* 188. *8.* 287. *9.* 21. *14.* 38. *15.* 296. *16.* 175. *17.* 284, 106, 46, 115. *18.* 163. *19.* 88967, 1880. *24.* 101500. *25.* 16624, 31000, 7334, 40290, 47624. *26.* 4020. *28.* 148, 48. *29.* 2927.

Page 50. *1.* 207861, 30796. *2.* 173054, 40286. *3.* 72217, 44531. *4.* 441189, 9217. *5.* 67058, 1600. *6.* 18816, 518217. *7.* 48383, 7251. *8.* 9454, 1. *9.* 42942. *10.* 224087. *11.* 329463, 769247. *12.* 14551, 64712. *13.* 94944, 294. *14.* 997, 9792. *15.* 5188, 8023.

Pages 51–55. *3.* 88. *6.* 2235. *7.* 642. *8.* 1116. *9.* 2825. *10.* 2530. *11.* 1900. *14.* 8992. *15.* 284. *16.* 85. *18.* 1714. *20.* 1116942. *21.* 88. *22.* Lost, 22*c*. *23.* 41. *24.* 111. *25.* 33. *26.* Gain $50. *27.* Gained $96. *28.* $6. *29.* 43, 48. *32.* 4, 9. *35.* 601. *36.* 830. *37.* 785. *40.* 275. *46.* 1618. *47.* 2747. *48.* 492.

Pages 66–68. *5.* 366, 482, 246, 484, 642. *8.* 2934. *9.* 2268. *10.* 40135. *11.* 2292. *12.* 23504. *13.* 59822. *14.* 4518. *15.* 1230125. *16.* 4101632. *17.* 4536. *18.* 9738. *19.* 20200. *20.* 219138. *21.* 21416. *22.* 8827101.

Page 71. *2.* 11730. *3.* 6084. *8.* 17963. *13.* 981480. *17.* 11457760. *19.* 39077316. *22.* 333055302665. *30.* 2634951456554888.

Page 73. *4.* 424480000. *5.* 394240000. *6.* 1759200. *7.* 22404000000.

Pages 74–78. *4.* Last two, 2048, 27424. *5.* 7700. *6.* 896, 4736. *12.* 727776. *13.* 57.75, 68.25, 26.25. *14.* 34.56. *15.* 168, 672, 720, 744. *16.* 1440. *19.* 4.80. *24.* 10800. *25.* 14688. *28.* 11.31. *40.* 90, 138, 276.

(To give the answers to problems designed to teach the operation of division is to destroy the value of the exercise. Hence such are omitted.)

Pages 105–110. *7.* 182. *9.* 13. *12.* 1.50. *18.* 26, 50. *28.* 468. *29.* 290.50. *31.* 48, 2. *33.* 2, 16. *39.* 48. *40.* 465.50. *41.* 68. *42.* 8. *45.* Last, $832\frac{5}{7}$. *47.* Last, $11\frac{19}{21}$. *55.* 5826. *56.* 539. *57.* 784. *58.* 95. *59.* 8500. *60.* 85.

Page 121. *7–18.* $\frac{1}{2}$, $\frac{2}{5}$, $\frac{4}{7}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{8}{9}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{5}$, $\frac{4}{5}$. *32–39.* $\frac{19}{20}$, $\frac{15}{23}$, $\frac{3}{5}$, $\frac{88}{91}$, $\frac{171}{280}$, $\frac{391}{763}$, $\frac{7}{8}$, $\frac{7}{8}$.

Page 122. *12–21.* $56\frac{7}{15}$, $56\frac{7}{11}$, $9\frac{11}{17}$, $77\frac{3}{17}$, $2704\frac{5}{21}$, $8\frac{93}{206}$, $6\frac{2324}{3395}$, $12\frac{3}{5}$, $8\frac{1}{2}$, $93\frac{18}{25}$.

Page 123. *10.* $\frac{21}{8}$, $\frac{16}{5}$, $\frac{13}{2}$, $\frac{91}{7}$, $\frac{399}{19}$, $\frac{46002}{874}$, $\frac{115515}{765}$.

Page 124. *11–28.* $\frac{95}{4}$, $\frac{37}{7}$, $\frac{131}{8}$, $\frac{209}{12}$, $\frac{680}{11}$, $\frac{734}{9}$, $\frac{249}{2}$, $\frac{1028}{8}$, $\frac{1805}{9}$, $\frac{3794}{7}$, $\frac{629}{18}$, $\frac{2591}{985}$, $\frac{105}{71}$, $\frac{1342}{607}$ $\frac{74}{19}$, $\frac{52}{10}$, $\frac{708}{100}$, $\frac{18021}{1000}$.

Pages 127, 128. *19.* 2400. *20.* 120. *21.* 180. *22.* 252. *23.* 60. *24.* 280. *25.* 2730. *26.* 120. *27.* 1560. *28.* 714. *29.* 4180. *30.* 2520.

Page 129. *7.* $\frac{105}{231}$, $\frac{297}{231}$, $\frac{154}{231}$. *8.* $\frac{351}{585}$, $\frac{260}{585}$, $\frac{135}{585}$. *9.* $\frac{252}{315}$, $\frac{225}{315}$, $\frac{70}{315}$. *10.* $\frac{55}{165}$, $\frac{33}{165}$, $\frac{15}{165}$. *11.* $\frac{168}{1092}$, $\frac{91}{1092}$, $\frac{156}{1092}$. *12.* $\frac{225}{495}$, $\frac{33}{495}$, $\frac{330}{495}$. *13.* $\frac{105}{210}$, $\frac{140}{210}$, $\frac{126}{210}$, $\frac{150}{210}$. *14.* $\frac{228}{513}$, $\frac{270}{513}$. *15.* $\frac{12875}{32125}$, $\frac{18621}{32125}$. *16.* $\frac{147}{19257}$, $\frac{131}{19257}$.

Page 130. *2.* $\frac{15}{24}$, $\frac{8}{24}$. *3.* $\frac{14}{30}$, $\frac{27}{30}$, $\frac{24}{30}$. *4.* $\frac{45}{90}$, $\frac{60}{90}$, $\frac{30}{90}$, $\frac{27}{90}$. *5.* $\frac{8}{30}$, $\frac{1}{30}$, $\frac{25}{30}$. *6.* $\frac{8}{96}$, $\frac{15}{96}$, $\frac{42}{96}$. *7.* $\frac{6}{10}$, $\frac{7}{10}$. *8.* $\frac{10}{100}$, $\frac{1}{100}$. *9.* $\frac{192}{240}$, $\frac{330}{240}$, $\frac{20}{240}$, $\frac{135}{240}$. *10.* $\frac{297}{495}$, $\frac{165}{495}$, $\frac{385}{495}$, $\frac{360}{495}$. *11.* $\frac{2}{12}$, $\frac{5}{12}$. *12.* $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$. *13.* $\frac{45}{240}$, $\frac{156}{240}$, $\frac{100}{240}$. *14.* $\frac{80}{600}$, $\frac{105}{600}$, $\frac{114}{600}$. *15.* $\frac{24}{160}$, $\frac{40}{160}$, $\frac{55}{160}$, $\frac{100}{160}$. *16.* $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$.

Pages 133, 134. *45.* $10\frac{1}{12}$. *46.* $24\frac{12}{35}$. *48.* $18\frac{1}{4}$. *49.* $56\frac{1}{4}$. *52.* $3\frac{1}{3}$. *54.* $10\frac{2}{5}$, $7\frac{5}{7}$. *57.* $46\frac{42}{77}$. *58.* $\frac{11}{12}$. *59.* $1\frac{11}{13}$. *60.* $7\frac{12}{23}$. *61.* $8\frac{3}{16}$. *62.* $285\frac{403}{488}$.

Page 135. *5.* $\frac{31944}{347} = 92\frac{20}{347}$. *6.* $\frac{213}{7} = 30\frac{3}{7}$. *7.* $\frac{134}{2} = 67$. *8.* $\frac{251}{2} = 125\frac{1}{2}$. *9.* $\frac{7}{4} = 1\frac{3}{4}$. *10.* $\frac{28}{5} = 5\frac{3}{5}$. *11.* $\frac{516}{17} = 30\frac{6}{17}$. *12.* $\frac{4038832}{34771} = 116\frac{11196}{34771}$. *14.* $\frac{27}{7} = 3\frac{6}{7}$. *15.* $\frac{11}{3} = 3\frac{2}{3}$. *16.* $\frac{74}{5} = 14\frac{4}{5}$. *17.* $\frac{36}{5} = 7\frac{1}{5}$. *18.* $\frac{16}{3} = 5\frac{1}{3}$. *19.* $\frac{28}{5} = 5\frac{3}{5}$. *20.* $\frac{372}{7} = 53\frac{1}{7}$. *21.* $\frac{85}{4} = 21\frac{1}{4}$. *22.* $\frac{217}{1331}$. *24.* $260\frac{1}{3}$. *25.* $1647\frac{3}{48}$. *26.* $12\frac{1}{2}$. *27.* 32. *28.* $24309\frac{1}{4}$. *29.* $2904\frac{121}{137}$. *30.* $35\frac{3}{7}$. *31.* $21\frac{1}{2}$. *32.* 60768. *33.* 14. *34.* 63. *35.* 132.

Page 139. *28.* $\frac{13}{65}$. *29.* $\frac{3}{22}$. *30.* $\frac{4}{7}$. *31.* $122\frac{2}{3}$. *32.* 500. *33.* 16. *34.* $156\frac{28}{45}$. *35.* 152. *36.* 6. *37.* $\frac{187}{2451}$. *38.* $\frac{847}{3477}$. *39.* $3\frac{24477}{41035}$.

Page 141. *45–51.* $\frac{6}{35}$, $2\frac{8}{15}$, $10\frac{1}{2}$, $\frac{28}{243}$, *, *, *, *, $\frac{1}{6}$. *59.* $\frac{1}{6}$, 1, 1, 108.

Page 142. *1–15.* $\frac{3}{35}$, $\frac{11}{12}$, $\frac{5}{18}$, $\frac{2}{13}$, $\frac{3}{13}$, $\frac{128}{959}$, $\frac{5}{78}$, $\frac{848}{113082}$, $\frac{2}{163}$, $\frac{1}{10}$, $\frac{5}{7}$, $\frac{1}{29}$, $\frac{148}{86802}$, $\frac{1}{152}$, $\frac{171}{10000}$.

Page 144. *16–31.* $26\frac{2}{5}$, $\frac{195}{517}$, $\frac{10}{63}$, $482\frac{3}{5}$, 86, $53\frac{1}{4}$, $1\frac{13}{15}$, *, *, *, *, *, $2\frac{24}{111}$, *, *, $\frac{68}{31106}$.

Page 145. *33–44.* $3\frac{11}{39}$, $2\frac{16}{39}$, $2\frac{1}{7}$, $1\frac{1}{2}$, 9, $\frac{14}{3219}$, $\frac{18}{25}$, $5\frac{29}{140}$, 12, 16, *, *.

Page 146. *9–18.* 2, $\frac{1}{6}$, $\frac{63}{88}$, $\frac{7}{13}$, $\frac{2}{55}$, $\frac{486}{1311}$, $\frac{4}{669}$, $12\frac{3}{5}$, $19\frac{247}{339}$, 4.

Pages 147-161. *9.* *, *, *, *, *, $\frac{8}{27}$, $\frac{7}{24}$. *10.* *, $\frac{2}{3}$, $\frac{5}{3}$, $\frac{7}{6}$. *11.* *, $1\frac{3}{8}$, $2\frac{13}{21}$. *13.* *, *, $\frac{3}{8}$, $\frac{3}{16}$. *14.* $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$. *15.* 5280, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{66}$, $\frac{25}{132}$. *17.* $\frac{13}{17}$, $\frac{5}{7}$, $\frac{7}{25}$, $\frac{17}{128}$, $\frac{6}{13}$, $\frac{4}{5}$, $\frac{11}{32}$. *18.* *, *, $\frac{6}{35}$. *19.* *A*, $\frac{1}{21}$. *20.* *A*, $\frac{1}{6}$, $\frac{1}{18}$. *23.* 35. *24.* 90, 63. *25.* 8, 64. *27.* 8, 40, 15, 15, 39, 50. *28.* 308. *30.* 2, 22, 27, 10, 2. *35.* $2\frac{1}{3}$. *39.* 24, 13, 40, 15. *40.* $153\frac{3}{5}$, 25, $1\frac{41}{64}$. *43.* $\frac{27}{40}$, $6\frac{3}{4}$. *44.* 42. *45.* $20\frac{17}{80}$. *46.* 20, $7\frac{1}{2}$, *47.* 26. *53.* 2, $2\frac{1}{2}$, 3, $2\frac{3}{4}$. *54.* \$$2.21\frac{1}{4}$ in my favor. *56.* $5\frac{5}{7}$ lbs. *57.* I lost $85\frac{5}{9}$*c.* *59.* $36\frac{1}{4}$. *60.* $2\frac{17}{45}$, \$$1.18\frac{8}{9}$. *62.* \$30.25. *66.* $11\frac{1}{5}$, 20, 52, $\frac{4}{5}$, $38\frac{13}{25}$. *68.* Lost \$$1453\frac{3}{4}$. *70.* 30, $34\frac{2}{7}$, $54\frac{6}{11}$. *71.* 15, 40. *72.* 9 P.M. *73.* 8 P.M. *74.* 2, 4, 6, 2, 4, 6, 210, 150. *75.* 100, 60. *76.* 11 P.M., 320. *81.* $10\frac{5}{12}$. *89.* \$500. *90.* 50 *m.* *91.* The second. $4\frac{52}{115}$ acres. *92.* $\frac{7}{8}$. *93.* 132 by 66. *96.* 40. *97.* Increased. *98.* Diminished. *100.* 125, 73; 206, 89. *103.* A, \$$32\frac{1}{4}$, B, \$$24\frac{3}{4}$.

Page 167. *2.* .15, .019, .0006, .024, .500, .000039, .100, .49, .0000010, .052, .00008, .800, .71, .000091, .0017, .2845, ,316. *3.* 69.903. *5–19.* 5.000263, 980.004, 2.000000085, 200.0074, 8200.032, .00452, 65.521, 82.000000065, 763020.000108, .007529, .00475, 45.0375, 3755226000.000543, 3.1416, 927364500.0002568. *20.* 405.17, 300.5, 1.027, 57.000802, 1002.0001804, 7.0005, 6.0007.

Page 170. *19–36.* 13.4375, 40.9375, .285714+, 13.555555+, 567567+, 12.666666+, 1.833333+, 3.2, 10.04, 128.272727+, 13.416666+, .444444+, .555555+, .090909+, .181818+, .272727+, .617777+, .010101+, .020202+, .030303+, 7.846154+, 73.882352+.

Page 172. *1–24.* $\frac{1}{2}$, $\frac{7}{20}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{4}$, $\frac{3}{4}$, $23\frac{5}{8}$, $10\frac{3}{10}$, $2\frac{3}{5}$, $\frac{1}{400}$, $\frac{1}{2000}$, $1\frac{3}{50}$, $1\frac{1}{10}$, $1\frac{7}{100}$, $1.\frac{3}{25}$, $1\frac{1}{8}$, $\frac{9}{40}$, $3\frac{3}{4}$, $3\frac{3}{40}$, $15\frac{3}{5}$, $15\frac{3}{5}$, $15\frac{3}{5}$, $1\frac{1}{8}$.

Page 173. *2–5.* 2224.9083, 1516.002, 1674.986, \$1083.395

Pages 174, 175. *5–20.* 921.984, 677.584, 47.454, .997, 4.553, 44.744, 7.76, .4563, 37.666, 9.9, .999, .85, 4.51, *, \$211.09, 24.05.

Pages 175, 176. *1-11.* *, 32.9, $10.26, *, 12.12⅛, *, I owed $2.32, 21.25, 82, 5⅛ *yd.*, $6089.25.

It is deemed inexpedient to give more answers than are found in the text, to the examples in the fundamental operations in Decimal Fractions. To give the answers to such problems, is to tell the student exactly the thing—the position of the decimal point—which he needs to learn by solving the example.

Pages 182-187. *6.* 12.2 *hr.*+. *7.* 20 *m.* *12.* $7.18. *13.* $15.49. *15.* $489.834. *16.* $197.041. *17.* $13000. *19.* $41.56+. *20.* $294.50. *21.* $11.72−. *22.* $78.83. *23.* $9.92. *24.* $11½. *25.* 1725. *26.* $58.12½. *27.* $15500. *28.* $7.50. *29.* $8.03−. *30.* 22½ *c.* *31.* 9⅛ *c.* *32.* To pay freight. *33.* $84.37½. *35.* $5600. *36.* It will cost $32.50 more by rail. *37.* 15 *c.*−. *38.* $186.60. *39.* $1071.04.

Page 190. *1.* 24 *c.*+, 2 *c.*+, ½ *c.*+, $1.22−, $5.11−. *2.* $27.25+. *4.* $2.55+, $1.42−.

Page 191. *6.* $193000, $4.90−, $3.04−, $49.91−, $386, 2849fr.74.

Page 192. *4.* $3.28−, 15 *c.*−, $2.54+, $1.18−, $5.77−, *5.* 420.17 marks−. *6.* 105+. *7.* 4.20+.

Pages 202-204. *8.* $47.95. *9.* $34.475. *10.* ⅝ *mi.* *11.* 90 *sq. rd.* *12.* 1215, 2430. *13.* 2880. *14.* $34.32. *15.* 20 *yds.* *16.* $81.42. *17.* 200, $27\frac{8}{11}$, $\frac{4355569}{4356000}$ (nearly 1). *20.* *, 8.4. *2.* 120 *ft.* *3.* 3. *4.* 16⅔.

Pages 205-209. *1.* 5. *4.* 4½. *6.* $14\frac{13}{16}$. *7.* 43¾, $32\frac{13}{16}$, 21⅞; 37½, 28⅛, 18¾; 31¼, $23\frac{7}{16}$, 15⅝; 50, 37½, 25. *11.* 1173⅓. *12.* $28.89−. *13.* $10.01+. *14.* $39.12. *15.* 38500 *cu. yd.* *16.* 31⅔ *cu. ft.* *17.* $693.41. *18.* 24 *cu. ft.* *19.* 348¾. *20.* 13⅓ *min.* *21.* 28181. *22.* 31674. *1.* 196, 256, 676. *2.* 300, 547, 972, 817. *3.* 1152, 960, 640, 1280.

Pages 211, 212. *4.* 64, 16. *5.* 2492.2+, 27.19−. *6.* 103144.5, 3274.4+. *15.* About $\frac{6}{7}$ (more exactly $\frac{5775}{6720}$), 37.2+.

Page 215. *5.* 53⅓. *7.* 440.811 +. *10.* 3 *pwt.* *11.* $65 *14.* $5.04, $2.40, $3.34, 2 *c.*

Pages 220–223. *1.* 8 A. M., 1 P. M. *4.* 7½°. *5.* 52½°, 37½′. *7.* 12 *m.* 42.5 *sec.*, 5 *hr.* 12 *m.* 42.5 *sec.* P. M., 7 *hr.* 47 *m.* 17.5 *sec.* P. M. *9.* 90° 15′ 15″. *10.* 2° 45′. *12.* 10 *hr.* 22 *m.* 8 *sec.* P. M. *13.* 31 *m.* 36 *sec.* *14.* 52.2+. *15.* 189.375 *mi.*, 252.5 *mi.*, 378.75 *mi.*, 170.4375 *mi.* *16.* It will be 19 *m.* slow, 87° 43′.

Pages 228, 229. *57.* 30 *ft.*, 42⅔ *fath.* *58.* 3710 *yd.* *59.* 1 3/22 *mi.* *60.* 1760 *yd.*, 220 *yd.* *61.* 18 2/11 *rd.*, 4/33 *rd.* *62.* ¼ *A.* *63.* 18150 *sq. yd.*, 1/32 *A.* *66.* 51⅓ *cu. ft.*, 3/32 *cd.* *67.* 28 4/27 *cu. yd.* *68.* *, 27/128 *cd.* *69.* 458 *qt.* *79.* 41 *bbl.* 30 *gal.* 1 *qt.* 0 *pt.* *80.* 6288 *pt.*

Pages 234, 235. *3.* £4 4*s.* 2*d.* *5.* 6 *bu.* 2 *pk.* 2 *qt.* *6.* 2 *pk.* 7 *qt.* 1 *pt.* *16.* 268 *A.* 115 *sq. rd.* *18.* 20 *gal.* 2 *qt.* 1 *pt.* *19.* 3 *oz.* *20.* 2 *in.*

Page 236. *4.* 21 *yr.* 8 *mo.* 19 *da.* *5.* 19 *yr.* 4 *mo.* 21 *da.* *6.* 254 *yr.* 0 *mo.* 27 *da.* *7.* 1 *yr.* 2 *mo.* 10 *da.*

Page 238. *5.* 90 *gal.* 1 *pt.*, 1287 *gal.* 2 *qt.* *6.* 6 *lb.* 2 *oz.* 6 *pwt.* 51 *lb.* 19 *pwt.* 12 *gr.* *8.* 552 *mi.* 20 *rd.* 14 *ft.*, 1086 *mi.* 237 *rd.* 9 *ft.* 6 *in.* *9.* 1921 *da.* 6 *hr.* 46 *min.* 40 *sec.* *10.* 459° 10′ 30″, 61° 13′ 24″, 132° 39′ 2″, 102° 2′ 20″. *11.* 14 *lb.* ℥ ix ʒ j ℈ j, 44 *lb.* ℥ iij ʒ iv, 88 *lb.* ℥ vij.

Page 240. *5.* 5 *lb.* 12⅔ *oz.* *6.* ℥ vij ʒ ij *gr.* 6⅔. *7.* 2 *A.* 10 *sq. rd.*, 122 2/9 *sq. rd.*, 1 *A.* 93 11/13 *sq. rd.* *12.* 1 *yd.* 1 *ft.* 2 *in.* *14.* 63, 4.6+.

Pages 241, 242. *4.* 2.13, 14, 15, 20, 25, 50, 75, $3, $5, $10. *5.* 7, 8, 9, 10, 11, 12, 13, 14, 15, 25, 50. *7.* 11.26, 28, 30, 40, 50, $12, $14, $16, $18, $20, $30, $40, $50. *8.* 15, 16, 18, 20, 30, 40, 50; 45, 47, 49, 50, 60, 70, 80, 90, $1; 8, 9, 11, 13, 15, 25. *10.* 32, 33, 34, 35, 36, 37, 38, 39, 40, 50, $1, $1.50, $2; 37, 38, 39, 40, 50, $1, $1.50, $2; 12, 13, 14, 15, 16, 17, 18, 19, 20, 30, 40, 50, $1.

Pages 242–244. *2.* $7, $2.62½. *3.* $11.66⅔, $8.75, $17.50, $7. *4.* $16.50, $22.50, $18.75, $16.87½, $18. *5.* 8 *lb.*, 24 *lb.*, *. *6.* 3 *lb.*, *, 15 *lb.* *7.* $48, $31.50, $65. *8.* *, $85, $281.25. *9.* $1.06⅔; $1.20, $1.50. *10.* $5, $14, $2, $27.50. *12.* $45. *17.* $3.11¼. *18.* $256.66⅔, $1054.66⅔, $1415.55 5/9. *19.* $1.89, $1.76¼, $1.11, $2.50. *20.* $1.80¼, $1.21⅝, $1.39. *21.* $1231.25, $651.25.

Pages 247–249. *22–30.* $0.105, $21, $9.80, 8 *c.*, $9, ½ *c.*, 8⅓ *m.*, $9.625, $12.50. *33.* $1417.50, $381.70, $5.20, $132.50. *34.* 434, 266. *35.* $81.396, $1.596. *36.* $825. *37.* $5785. *39.* $4.40, $4.50, $5, $5.20, $4.60, $4.80. *40.* .0648, .060375, .0606, .0615, .0603, .06105, .063, .0627.

Pages 250–253. *12.* 25, 15, $17\frac{1}{17}$, 50, 10. *13.* 20, 80, 100. *20.* 25, 75. *23.* 100, 20, 10. *24.* 25, $12\frac{1}{2}$, $6\frac{1}{4}$, $37\frac{1}{2}$. *3.* $3.04+. *4.* 25 *c.*

Pages 254–256. *1.* $5.76. *2.* $4.35. *3.* 4%. *4.* $22.73. *5.* Gain 12%. *6.* $17632. *7.* $1.25. *8.* $48. *9.* 19%+. *10.* 5% Dis. *11.* 10% loss, 20% loss, cost $62\frac{1}{2}$ *c.* *12.* $\frac{1}{4}$%. *13.* Infinity %. *14.* $8888.89−. *15.* $16\frac{4}{11}$%, $14\frac{1}{16}$%. *16.* $108\frac{1}{3}$%. *17.* $6\frac{3}{7}$ *c.* *18.* $228.26. *19.* $38\frac{16}{43}$%, $249.42. *20.* 68%, $64600.

Pages 260, 261. *15.* $384.09+. *21.* $9.22. *22.* $132.37. *23.* $759.30. *24.* $15.82. *25.* $45.34. *26.* $0.955. *27.* $0.088. *28.* $504. *29.* $4.72. *30.* $22.46. *31.* $80. *32.* $3.50. *33.* $36.51. *34.* $455.39. *35.* $8.05. *36.* $4.66. *37.* $0.606. *38.* $24.90. *39.* $71.48. *40.* $0.46. *41.* $11.25. *42.* $45. *43.* $9.41. *44.* $179.64. *45.* $113.05. *46.* $15.40. *47.* $412.78. *48.* $28.75. *49.* $30.85. *50.* $37.

Pages 263–265. *5.* $4.29. *6.* 6.81. *7.* $118.40. *8.* $1.46, $21.97. *9.* $14.16, $45.04. *10.* $400, $705.54. *11.* $10.82. *12.* $45.59. *13.* $10.55. *16.* $148.18. *17.* $614.05. *18.* $39.83. *19.* $788.88. *20.* $427.50. *21.* $1680. *22.* $757.50. *23.* $148.37, $98.03, $107.25.

Page 266. *4.* $29.73; $37.53. *5.* $3.94. *6.* $4.64. *7.* $7.84. *8.* $68.07. *9.* $136.60. *10.* $14.93. *11.* $3.07. *12.* $0.34.

Page 268. *3.* $258.67. *4.* $185.34. *5.* $618.27. *6.* $437.51. *8.* $440.03. *9.* $805.27. *10.* $501.54.

Pages 272, 273. *6.* $134.98, $78.22, $61.02. *7.* $53.22. *8.* $99.51. *9.* $101.77. *11.* $15.50. *12.* $394.44. *13.* $7.21. *14.* $1.35. *14.* $54.12. *16.* $724.68.

Pages 274, 275. *3.* $393.63. *4.* $108.24. *6.* $441.82. *8.* $95.07. *9.* $320.96. *10.* $643.46.

Page 281. *6.* $172.46. **Page 285.** *9.* $1025.51.

Pages 289, 290. *9.* $4038.46. *12.* $49.12. *15.* $3.02, $1.58, $6.37, $2.26.

Pages 291–294. *1.* $2.75. *2.* $6. *3.* $383.84. *4.* $490.03. *5.* $487.04. *6.* $3756.57. *7.* $729.49. *15.* $3670.61.

Page 295. *1.* $381.21, $385.14. *2.* $470, $475. *3.* The first is $63\frac{7}{8}$, or 63%, *per ann.*, and the second $45\frac{5}{8}$, or 45%, (as a day is called $\frac{1}{365}$ *yr.*, or $\frac{1}{360}$.) *4.* $54.

Page 296. *8.* $803.21, $3.21. *9.* $311.10, $3.244+.

Pages 300, 301. *5.* \$25000. *10.* \$2811.06. *11.* \$12,670,-357.75; \$54,301,533.21. *12.* \$50,314.29. *13.* \$162,764.83.

Pages 302–311. *4.* \$5.17$\frac{1}{2}$. *5.* 38$\frac{8}{9}$, 33$\frac{1}{3}$, 22$\frac{2}{9}$, 11$\frac{1}{9}$. *7.* 10 *c.* *8.* \$77.76. *13.* 3 *yr.* 9 *mo.* 18 *da.* *14.* 10%. *15.* 10$\frac{1}{5}$%. *16.* 4 *yr.* 3 *mo.* 6 *da.*, 5 *yr.* 1 *mo.* 13 *da.*, 8 *yr.* 6 *mo.* 12 *da.* *17.* \$520.23, \$1250, \$681.82. *18.* \$77777.78–, \$58333.33+, \$47945.21–. *19.* 20 *yr.*, 16$\frac{2}{3}$, 14$\frac{2}{7}$, 10, The same. *21.* 133$\frac{1}{3}$, 21, 45$\frac{5}{11}$. *22.* \$4700. *24.* 11 *yr.* 9 *mo.* 13 *da.* *25.* 990 *yr.* *26.* 2 *c.* *28.* \$44.64, \$45.45, \$50. *31.* \$0.95$\frac{5}{21}$. *36.* \$1.00$\frac{93}{146}$; \$503.18; \$454.71; \$102.26. *49.* 5$\frac{1}{2}$+. *50.* 5$\frac{1}{2}$+. *51.* 7$\frac{1}{7}$. *52.* \$1080; 7$\frac{29}{53}$%; \$1120; 7$\frac{11}{27}$%. *53.* 9$\frac{19}{109}$%; 9$\frac{1}{11}$%.

Page 320. *1–12.* 17$\frac{1}{2}$, 7$\frac{1}{3}$, 3$\frac{1}{3}$, 32, .538+, 4$\frac{4}{15}$, 1.38+, 32571.43–, 210, $\frac{3}{4}$, 7, $\frac{150}{203}$.

Pages 321–323. *4.* 1152 *bu.* *5.* \$130. *8.* \$315.91. *11.* \$244.80. *13.* $\frac{1}{2}$ teaspoonful (ʒ $\frac{1}{2}$). *14.* 6$\frac{2}{3}$ *gr.* *16.* 1089.4 *ft.*+. *17.* 3.1 *mi.* nearly. *18.* 185,485 *mi. per sec.* *19.* 7$\frac{3}{4}$ nearly. *20.* 17 *yr.* 34 *da.* 21 *hr.* 32 *min.* 33 *sec.* *21.* 180, 204; 147$\frac{9}{13}$, 236$\frac{1}{13}$; 109$\frac{5}{7}$, 274$\frac{2}{7}$. *22.* 27, 63, 45; 30, 45, 60. *23.* 150 *ft.*

Pages 333–338. *1.* 7$\frac{2}{3}\frac{1}{2}$ *A.* *2.* 80 *rd.*, 126.49 *rd.*+, 40 *rd.*, 56.56 *rd.*+. *3.* 54. *8.* 10.09 *rd.*, 3.99 *ft.*, 180.54 *rd.* *12.* 512 *cu. ft.*, 411.4+. *15.* 59.9+ *in.* on an edge, 3 *cu. ft.* 960 *cu. in.*, or 2.9 *bu.*+. *18.* 7 *ft.* 5.7 *in.*+. *23.* 408.8 *cu. ft.*+, in all. *28.* 150.8 *cu. ft.*–. *29.* 77.59 *bbl.* *30.* 261.86 *bbl.* *31.* 2,986,811,024,414,968,223,812 tons.

Pages 341–343. *1.* Yard; 5.03 *m.*; 3.65 *m.*; 6.09 × 7.31 *m.* *3.* About $\frac{2}{3}$ as long; about $\frac{5}{6}$ as long. *7.* 233.357 *Km.*; 463.49 *Km.* *8.* About $\frac{5}{8}$ *mi.* *9.* 24.854 *mi.* *10.* 1 *Km.* in 1 *min.* 17 *sec.*; 1 *Km.* in 3 *min.* 6 *sec.* *11.* The latter is 1 *Km.* in 1 *min.* 39 *sec.*; and hence is the faster. *12.* .03937, or .04 nearly. *13.* About .0004, and .000004 respectively. *14.* $\frac{1}{1000}$, $\frac{1}{2000}$, $\frac{1}{80}$, $\frac{1}{5000}$ *in.* respectively.

Pages 343–345. *1.* 31 *c.*; 13$\frac{2}{3}$ *c.*; 11$\frac{1}{3}$ *c.* *2.* 27.21 *Kg.*; 14$\frac{1}{2}$ *Kg.*; 25.4 *Kg.*; 88.9 *Kg.* *3.* .4536 *Kg.*; 2.2046 *lb.* *5.* The latter. *6.* 6$\frac{1}{2}$ *cg.*; 13 *dg.*; 4 *g.* *7.* 28.35 *g.*; 113.4. *8.* 8 *mg.* *10.* 2.06 *Kg.* *12.* \$9.92, \$13.23.

Pages 345, 346. *1.* Our liquid quart is a very little less than a liter, and our dry quart a very little more; A little more than 3$\frac{3}{4}$; 119.2 nearly; 9$\frac{1}{2}$ nearly. *2.* 8.8+; 17.6+. *3.* 170$\frac{1}{4}$ *lb.* *4.* \$1.23. *5.* \$3.32. *8.* .8. *9.* 10.32 *Kg.* *10.* 1.018. *11.* 1.842.

Page 347. *1.* 259 very nearly. *2.* \$101.17. *3.* 10$\frac{3}{4}$+. *4.* \$12.50–. *5.* 40. *6.* 12. *1.* 78.54 *Hl.* *2.* 55.18. *3.* 882.26+. *4.* 12,000.

Pages 348–351. *1.* $23\frac{1}{3}$. *2.* $\frac{38}{50}$. *3.* $1\frac{17}{25}$. 4. $32\frac{71}{378}$. *5.* $13\frac{5}{14}$. *6.* $\frac{61}{144}$. *7.* $\frac{124}{475}$. *8.* $\frac{5}{22}$. *9.* $5\frac{67}{185}$. *10.* 1812.59. *11.* 400,000,000. *12.* $10259\frac{8}{15}$. *13.* $129445\frac{5}{11}$. *14.* .00666. *15.* 40,000,000. *16.* $95\frac{168}{195}$. *17.* $\frac{1}{3}$. *18.* $\frac{8840}{19987}$. *19.* 2. $\frac{79}{1275}$. *20.* 1000. *21.* $\frac{78}{456875}$. *22.* 13.499. *23.* $13333.37331\frac{1}{3}$. *24.* 4.1306+. *25.* $\frac{5}{8}$. *26.* 3.9686+. *27.* 10.2774+. *28.* 205.5176–. *29.* 8 and 6. *30.* Yes. *31.* No; $\sqrt{16}+\sqrt{4}$ is 6, while $\sqrt{16+4}$ is $\sqrt{20}$, or 4.47+. *32.* 1. *33.* 4.3958–. *34.* $\frac{13}{33}$. *35.* 24. *36.* $2\frac{4}{7}$. *37.* $6\frac{2}{3}$. *38.* $61\frac{1}{5}$. *39.* $\frac{8}{1625}$. *40.* $.00006\frac{2}{3}$. *41.* $.0098\frac{106}{163}$. *42.* 3.6305+. *43.* 3.2404–. *44.* 2. *45.* Yes; each is 2. *46.* No; the first is 2, and the second $\sqrt{12} = 3.4+$. *47.* 1. *48.* 0. *49.* 8. *50.* 64. *51.* $\frac{3}{50}$. *52.* 3.5178+. *53.* .8216–. *54.* .8126–. *55.* $1\frac{42}{73}$. *56.* 3.7417–. *57.* .5477+. *58.* 1. *59.* 2.0207+. *60.* .7107–. *61.* $\frac{13}{75}$. *62.* $116\frac{164}{171}$. *63.* $1452\frac{44}{303}$. *64.* 1.3. *65.* 25. *66.* $\frac{288}{805}$. *67.* $\frac{2}{27}$. *68.* $37\frac{1}{27}$. *69.* $1\frac{13}{75}$. *70.* $73\frac{91}{125}$. *71.* $151\frac{1}{4}$. *72.* 3741.3258–. *73.* 148.401475. *74.* $1\frac{1151}{3024}$. *75.* 10.0259+. *76.* $\frac{5000}{29199}$. *77.* $\frac{8000}{48807}$. *78.* $\frac{2700}{3937}$. *79.* 1.21253. *80.* $1\frac{787}{5000}$. *81.* $\frac{87597}{617600}$. *82.* $\frac{2023}{3750}$. *83.* 27. *84.* $2\frac{2}{29}$, $2\frac{19}{58}$, $3\frac{3}{29}$. *85.* $2\frac{83}{101}$, $2\frac{14}{101}$, $7\frac{49}{101}$. *86.* $3\frac{6}{73}$, $1\frac{26}{73}$, $\frac{45}{73}$, $3\frac{69}{73}$. *87.* 500, 500, 250. *88.* \$873.563, \$1091.954, \$1019.157, \$815.325. *92.* $\frac{5775}{10752}$. *93.* $\frac{412113}{2000000}$. *94.* $1\frac{1487}{2500}$. *95.* $\frac{227}{320}$. *96.* $\frac{7874}{12375}$. *97.* $\frac{15748}{17625}$. *98.* The ratio of 1 to .5144. *99.* 183.245+. *100.* .1385+.

Pages 351–358. *1.* 20%, 50%, $11\frac{27}{43}$%. *2.* $183\frac{1}{3}$, 110, $157\frac{1}{7}$. *3.* $12\frac{8}{11}$%, 20%, 60%. *4.* $12\frac{1}{2}$%, $33\frac{1}{3}$%, 6%. *5.* $95\frac{5}{21}$, $90\frac{10}{11}$, $89\frac{2}{7}$, $96\frac{2}{13}$, $96\frac{128}{207}$. 6. \$202.84, \$353.24, \$253.27. *7.* \$397.19. *8.* \$7303.55, \$2576.45 gain. *9.* $8\frac{3}{10}$%+. *10.* \$795.83. *11.* \$495.625. *12.* \$291.31. *13.* 3 *yr.* 3 *mo.* 9 *da.* *14.* 16 *yr.* 8 *mo.*, 14 *yr.* 3 *mo.* 13 *da.*, 12 *yr.* 6 *mo.*, 10 *yr.* *15.* 14 *yr.* 2 *mo.* 13 *da.*, 7 *yr.* 3 *mo.* 5 *da.*, 17 *yr.* 8 *mo.* 1 *da.* *16.* \$3800, \$4470. *17.* \$9944.06. *18.* \$5060.54. *19.* \$5031.37. *20.* 6%. *21.* $8\frac{1}{3}$%, $6\frac{2}{5}$%. *22.* \$1260. *23.* \$1615.41. *24.* 3%. *25.* \$18000. *26.* \$54259.36, \$94.95. *27.* 38%. *28.* \$301.92. *29.* July 15, 1879, \$1278.50. *30.* 8%. *31.* May 23. *32.* \$226.11. *33.* 83% –. *34.* $12\frac{1}{2}$%. *35.* 26%. *36.* 175. *37.* \$1697.92. *38.* \$8. *39.* \$189.473+. *41.* $10\frac{23}{100}$%, or practically 11%. *42.* $13\frac{11}{1000}$%, or 13%+. *43.* $105\frac{5}{19}$%, $107\frac{49}{93}$%. *44.* $94\frac{18}{53}$%, $90\frac{10}{11}$%. *45.* $33\frac{1}{3}$%. *46.* 50%, 100%, $16\frac{2}{3}$%. *47.* $\frac{1}{3}$ *gal.*, 1 *gal.*, $1\frac{1}{3}$ *gal.* *48.* \$1453. *49.* \$772.

Pages 358, 361. *1.* The first nearly doubles it ($\frac{125}{64}$), the 2d $3\frac{3}{8}$ times, the 3d 8 times its former capacity. *2.* 21%, or it is made to hold $1\frac{21}{100}$ times as much as it did at first; $33\frac{1}{10}$%, or 1.331 times as much as at first; 300%, or 4 times as much; 700%, or 8 times as much. *3.* $67\frac{1}{2}$ *lb.* *4.* 41760. *5.* \$60.25. *6.* $4\frac{3}{4}$+. *7.* 137 *lb.* $1\frac{1}{2}$ *oz.* *8.* 5.642 *ft.* *9.* 6.59 *ft.*+. *10.* 19.9 *gal.* *11.* 15 *mi.* *12.* $374\frac{3}{8}$. *13.* The Trade Dollar is actually worth $\frac{171}{1920}$, or about $8\frac{1}{2}$% more than the common silver dollar. *14.* \$76.05.

15. 180$\frac{1}{3}\frac{3}{4}\frac{3}{4}$ *cu. ft.* *16.* $\frac{4}{9}$ as long. *18.* 648 *sq. in.* *19.* 1.154 *ft.* + on an edge. *20.* 1$\frac{1}{3}$, 4, 4. *21.* The latter, in the ratio 108 : 125. *22.* 136.956 *lb.* *23.* 2257 *lb.* nearly. *24.* 4391 *lb.*, or 2.4 tons—. *25.* 1$\frac{1}{4}$ tons+. *26.* 614.3 *lb.* *27.* 120.8 *bbl.* *28.* 181.6 *bush.*, 64 *Hl.*, 4942.4 *Kg.* *30.* 10 acres. *31.* 21 *sq. ft.*

Pages 362-368. *1.* $183.05. *2.* 59.06\frac{1}{4}$, $94.58+. *3.* 27.49 *ft.*+. *4.* $186, $234. *5.* 6$\frac{2}{33}$, 9$\frac{1}{11}$, 18$\frac{2}{11}$, 25$\frac{5}{11}$, 25, 200. *6.* Every time the 2d goes 3$\frac{1}{2}$, and the first 2$\frac{1}{2}$ times around; after the first has gone 5 times, and the 2d 7 times around. *7.* $243.30—. *8.* 8 *ft.* 11.23 *in.*, 14 *ft.* 10.7 *in.*+. *9.* 5 : 9. *11.* 27 : 1331. *12.* 1.633—. 1.154+, 1.414+, 1.732+, 1. *14.* 13$\frac{1}{3}$ *da.* *17.* 20 *da.* *22.* $1109.09. *27.* $738.61+. *29.* .316+, .928, 1.2, .158+, 1.91+. *45.* 157$\frac{1}{11}$ *lb.* *46.* 40%. *47.* $10. *48.* 13° 4$\frac{1}{2}$'. *52.* 32, 56; 19$\frac{5}{9}$, 29$\frac{1}{3}$, 39$\frac{1}{9}$. *53.* 24, 32. *54.* Increased, $\frac{8}{15}$; decreased, $\frac{4}{15}$; neither. *56.* 1245550.9+. *58.* 416.88\frac{1}{3}$. *60.* 4.825 grams, 6.44 grams+.

NOTE.—A French silver franc really weighs 5 grams, and hence is actually worth in silver 20 *c.* The value, 19.3 *c.*, is the relative value of the French and American *gold* coins.

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